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Apply Simultaneous Tensor Completion based on the Inequality Constrained Convex Optimization Denoising for Image Recovery

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***Abstract*—In this paper, we applied the tensor completion method based on the denoising optimization with Inequality Constrained Convex for recovering images. Convex optimization, rather than a non-convex approach, still play important roles in many computer science applications because of its exactness and efficiency. we consider a tensor completion problem with noise based on the convex optimization. We applied this method on some noisy destructed images for restoring to original states. Our experiments showed that this hybrid method more than effective older classical tensor completion methods.**

***Keywords*—Tensor Completion; Image Recovery; Constrained Convex Optimization; Denoising.**

I. Introduction

The concept of tensors was introduced by Gauss, Riemann and Christoffel, etc., in the 19th century in the study of differential geometry. Operations with tensors, have become increasingly prevalent in recent years [1]. Tensors play an important role in physics, engineering, and mathematics. There are many application domains of tensors such as data analysis and mining, information science, image processing, and computational biology, etc [4].

Tensor completion is a problem of filling the missing or unobserved entries of partially observed tensors [1]. Due to the multidimensional character of tensors in describing complex datasets, tensor completion algorithms and their applications have received wide attentions and achievements in areas like data mining, computer vision, signal processing, and neuroscience [3]. Multidimensional datasets are often raw and incomplete owing to various unpredictable or unavoidable reasons such as mal-operations, limited permissions, and data missing at random [7]. On the other hand, in practice, due to the multiway property of modern datasets, tensor completion natural arises in data-driven applications such as image completion, image processing and video compression. The tensor completion is a generalized version of matrix completion. Because of some limitations of matrix completion for analyzing big data, the tensor completion was emerged, see figure 1. Intuitively, the tensor completion problem could be solved with matrix completion algorithms by downgrading the problem into a matrix level, typically by either slicing a tensor into multiple small matrices or unfolding it into one big matrix [7]. Tensor factorization of incomplete data is a powerful technique for imputation of missing entries (also known as tensor completion) by explicitly capturing the latent multilinear structure [10]. In this paper, after introducing tensor completion problem, we state the famous method for image recovery:" Simultaneous Tensor Completion and Denoising by Noise Inequality Constrained Convex Optimization (STCDN)" and implement it on some examples. Tensors (i.e., multiway arrays) provide an eff ective and faithful representation of structural properties of the data, especially for multidimensional data or data ensemble aff ected by multiple factors. For instance, a video sequence can be represented by a third order tensor with dimensionality of height*width* time; an image ensemble measured under multiple conditions can be represented by a higher order tensor with dimensionality of pixel *person*pose* illumination [9].

Figure.1. The schematic view of tensor completion problem.

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II. PRELIMINARIES

In this section, we briefly state some preliminaries for tensor calculus and tensor completion. For more details and information, please read [5, 7, 8].

Definition 1.2. A tensor is a multidimensional array $\mathcal{A} = (a_{i_1 \ldots i_m})$, from entries $a_{i_1 \ldots i_m} \in F$, where $i_j = 1, \cdots, n_j$ $j=1,\ldots,m$ and F is a field, the dimensionality of it is described as its order. An Nth-order tensor is an N -way array, also known as N -dimensional or N -mode tensor, denoted by X. We use the term order to refer to the dimensionality of a tensor (e.g., Nth-order tensor), and the term mode to describe operations on a specific dimension (e.g., mode- n product) [1, 5].

The tensor A is called symmetric if its entries are invariant under any permutation of their indices. Like square matrices, this class of tensors can be regarded as square tensors. Note that a zero-order tensor is a scalar, a first order tensor is a vector, and a matrix is a second order tensor. Actually, the representation of a second order tensor is a square matrix [4].

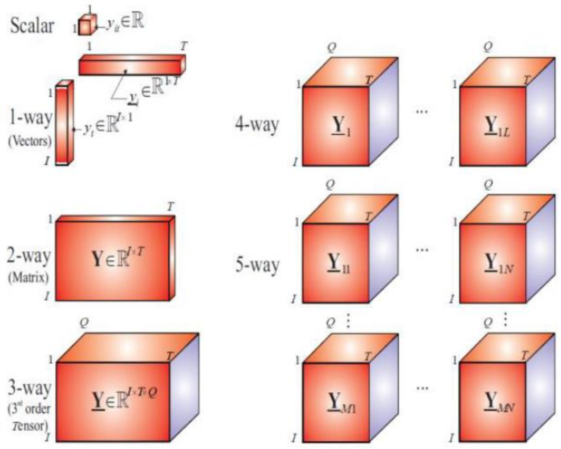

Figure.2. Visualization of the zero, one, two and three-way tensors.

Definition 2.2. The inner product of two tensors X and Y of same size is defined as < X, Y > [5]. Unless otherwise specified, we treat it as dot product defined as follows [6]:

$$< X, Y >= \sum_{i_1 \ldots i_m=1}^{n} x_{i_1 \ldots i_m} y_{i_1 \ldots i_m}$$

Definition 3.2. Generalized from matrix Frobenius norm, the F-norm of a tensor X is defined as [2]:

$$\|X\|_F = \sqrt{< X, X >}$$

Definition 4.2. Given a low-rank (either CP rank or other ranks) tensor T with missing entries, the goal of completing it can be formulated as the following optimization problem:

$$\text{Minimize}_X \ rank*(X)$$
$$\text{subject to } X_\Omega = T_\Omega$$

where rank∗(X) denotes a specific type of tensor rank based on the rank assumption of given tensor T, X represents the completed low rank tensor of T and Ω is an index set of observations [3]. For this paper, the specific rank is completed positive (CP) rank [10].

III. ALGORITHM IMPLEMENTATION

The method discussed in this paper is Simultaneous Tensor Completion and Denoising by Inequality Constrained Convex Optimization. This advanced method includes Tucker decomposition, Primal-Dual Splitting (PDS), and low rank tensor completion methods. In this method, the tensor completion problem and denoising by total variation and nuclear norm are done. The main optimization problem as follows:

$$\begin{aligned} &\underset{\mathcal{X}}{\text{minimize}} \ \alpha f_{\text{TV}}(\mathcal{X}) + \beta f_{\text{LR}}(\mathcal{X}), \\ &\text{s.t. } v_{\min} \le \mathcal{X} \le v_{\max}, \\ &\quad D_\Omega(\mathcal{X}, \mathcal{T}) \le \delta, \end{aligned}$$

Where, $0 \le \alpha \le 1$, and $\beta := 1 - \alpha$ are weight parametrs between nuclear norm and total variation terms. For the first time, this method proposed by Tatsuya Yokota and et al, in 2018 [7]. The extended version of this algorithm proposed by Tatsuya Yokota and Hidekata Hontaniin 2019 [7]. The main algorithm as follows:

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input : $\mathcal{T}, \mathcal{Q}, \delta, \nu_{\min}, \nu_{\max}, \alpha, \boldsymbol{w}, \beta, \boldsymbol{\lambda}, \gamma_1, \gamma_2$;
initialize : $\mathcal{X}^0, \mathcal{U}^0, Y^0, \mathcal{Z}^{(n)0}\ (\forall n), k = 0$;
repeat
$\quad \boldsymbol{v} \leftarrow \boldsymbol{u}^k + \sum_{n=1}^{N} z^{(n)k} + \sum_{n=1}^{N} \sqrt{w_n} \boldsymbol{D}_n^T \boldsymbol{y}_n^k$;
$\quad \boldsymbol{x}^{k+1} = \text{prox}_{i_\delta}\left[\boldsymbol{x}^k - \gamma_1 \boldsymbol{v}\right]$;
$\quad \boldsymbol{h} \leftarrow 2\boldsymbol{x}^{k+1} - \boldsymbol{x}^k$;
$\quad \tilde{\boldsymbol{u}} \leftarrow \boldsymbol{u}^k + \gamma_2 \boldsymbol{h}$;
$\quad \boldsymbol{u}^{k+1} = \tilde{\boldsymbol{u}} - \gamma_2 \text{prox}_{i_v}\left[\frac{1}{\gamma_2}\tilde{\boldsymbol{u}}\right]$;
$\quad \widetilde{\boldsymbol{Y}} \leftarrow \boldsymbol{Y}^k + \gamma_2[\sqrt{w_1}\boldsymbol{D}_1\boldsymbol{h}, \ldots, \sqrt{w_N}\boldsymbol{D}_N\boldsymbol{h}]$;
$\quad \boldsymbol{Y}^{k+1} = \widetilde{\boldsymbol{Y}} - \gamma_2 \text{prox}_{\frac{\alpha}{\gamma_2}||\cdot||_{2,1}}\left[\frac{1}{\gamma_2}\widetilde{\boldsymbol{Y}}\right]$;
$\quad \widetilde{\boldsymbol{Z}}^{(n)} \leftarrow \boldsymbol{Z}_{(n)}^{(n)k} + \gamma_2 \boldsymbol{H}_{(n)}; (\forall n)$
$\quad \boldsymbol{Z}_{(n)}^{(n)k+1} = \widetilde{\boldsymbol{Z}}^{(n)} - \gamma_2 \text{prox}_{\frac{\beta\lambda_n}{\gamma_2}||\cdot||_*}\left[\frac{1}{\gamma_2}\widetilde{\boldsymbol{Z}}^{(n)}\right]; (\forall n)$
$\quad k \leftarrow k + 1$;
until convergence

Algorithm1. Simultaneous Tensor Completion and Denoising by Noise Inequality Constrained Convex Optimization (STCDN).

For more information and details, we refer the reader to [8, 9].

For experiments, we run MATLAB R2018b on the laptop system with following configuration in Table 1.

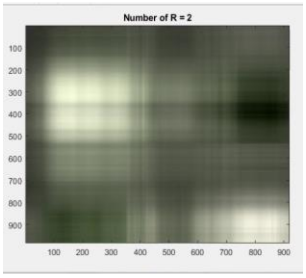
Table.1. Experiment system Configuration.

CPU	Ci7-2670QM (2.2 up to 3.3)
Cache	6 MB
RAM	16 GB (DDR3)
GPU	Geforce GT 530M (2GB)
OS	Win 10 Pro
H.D.D	750GB (7200 rpm)

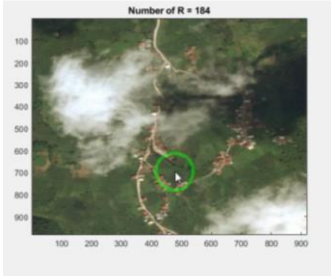
In MATLAB R2018b, the codes of this method use some toolboxes such as: tensor toolbox 2.6, CFDA toolbox, GLMC toolbox, GRBF toolbox, and Hankel-Tucker. We have implemented the algorithm by missing rates from 70% to 95%. The result of 90% missing data as shown in the figure 3.

Figure 4 show results of comparison between proposed method and some famous recovery methods, for more details please see [8].

(a)



(b)



(c)

Fig.3. Image recovery: (a): Original image, (b): noisy image up to 90% (Type: dissolved), (c): Reconstructed image (after image processing by STCDN.

Based on these results, STCDN (the proposed method) is an effective method for restore and recovery missing noisy colorful images. The recovery rate is 95% and this method can recover images, that noisy by various methods (In our experiments, we applied 8 noisy methods, such as: dissolve, row, column, block, linear, stochastic, Gaussian, Periodic).

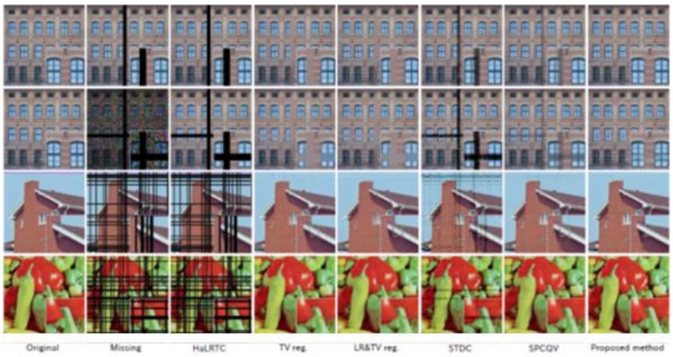


Figure.4. Color images completed with various methods. Four missing color images were artificially generated comprising: Missing case rate is 95%.

4 CONCLUSION

Overall, STCDN is a powerful method for recovery noisy images. Based on our experiments, STCDN is useful up to 90% noising and destruction, in some cases, images noised up to 95% is recovered with good accuracy.

Several real-world applications, such as image completion and image synthesis, demonstrate the superiorities of this method over the new tensor factorization and tensor completion approaches.

Computational bottleneck: The proposed method has an issue with data volume expansion due to MDT. An N-th order tensor is converted into a 2N-th order tensor by MDT and its data size increases roughly $\prod_{n=1}^{N} \tau_n$-fold. This issue makes it difficult to apply the proposed method to large-scale tensors and this problem should be consider in the future research. In summary, due to several interesting properties, this method would be attractive in many potential applications [8].

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