International Journal of Mechatronics, Electrical and Computer Technology

Dec. 2012, Special Number, pp 158-168, **ISSN**: 2305-0543

Available online at: http://www.aeuso.org

Parameter Estimation of Loranz Chaotic Dynamic System Using Constriction factor approach in Particle Swarm Optimization (CFAPSO)

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Abstract

An important problem in nonlinear science is the unknown parameters estimation in Loranz chaotic system. Clearly, the parameter estimation for chaotic systems is a multidimensional continuous optimization problem, where the optimization goal is to minimize mean squared errors (MSEs) between real and estimated responses for a number of given samples. The Constriction factor approach in particle swarm optimization (CFAPSO) is a new member of meta-heuristics. This paper focuses on using the CFAPSO to solve this problem. Simulation results demonstrate the merit, effectiveness and robustness of CFAPSO Algorithm.

Keywords: Loranz chaotic system, Parameter estimation, CFAPSO Algorithm, Mean squared errors.

1. Introduction

The synchronization and control of chaotic systems have been investigated intensely in various fields during recent years [1]-[3]. Many of the proposed approaches only work under the assumption that the parameters of chaotic systems are known in advance. In the real world, the parameters may be difficult to determine owing to the complexity of chaotic systems. Therefore, parameter estimation for chaotic systems has become a hot topic in the past decade [4]-[8]. The least-squares method is a basic technique often used for parameters estimation. It has been successfully used to estimate the parameters in static and dynamical systems, respectively [9]. But, the least-squares method is only suitable for the model structure of system having the

Dec. 2012, Special Number, pp 158-168, **ISSN**: 2305-0543
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property of being linear in the parameters. Once the form of model structure is not linear in the parameters, this approach may be invalid. Heuristic algorithms especially with stochastic search techniques seem to be a more hopeful approach and provide a powerful means to solve this problem. They seem to be a promising alternative to traditional techniques, since 1) the objective function's gradient is not required, 2) they are not sensitive to starting point, and 3) they usually do not get stuck into so called local optima. Particle Swarm Optimization (PSO) is a recently invented high performance optimizer that possesses several highly desirable attributes, especially the ease with which the basic algorithm can be understood and implemented.

PSO is a technique modeling swarm intelligence that was developed by Kennedy and Eberhart in 1995 [10], who were inspired by the natural flocking and swarming behavior of birds and insects. This technique has been used in addressing several optimizations and engineering problems [11]. In order to ensure the convergence of the PSO algorithm, we follow suggestions put forth by Clerc and incorporate a constriction factor because it may be necessary to ensure the convergence of the particle swarm [12], [13]. By incorporating a constriction factor, the PSO presented herein can be considered an adaptation of the conventional PSO, which takes into account weight inertia.

In the current version, the particles have information on the entire population. Such an algorithm which is known as CFAPSO is used in this paper in order to estimate the Loranz chaotic system parameters. The remainder of this paper is organized as follows. The parameter estimation is formulated as a multi- dimensional optimization problem in Section 2. In section 3, CFAPSO Algorithm is described. Simulation results are presented in section 4. Finally, conclusion is reported in Section 5.

2. Nonlinear System Estimation

If we do not have a priori knowledge about the real system, then structure identification becomes a difficult problem and we have to select the structure by trial and error. Fortunately, we know a great deal about the structures of most engineering

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systems and industrial processes; usually it is possible to derive a specific class of models that can best describe the real system. As a result, the system identification problem is usually reduced to that of parameter estimation. In order to explore the problem of parameter estimation in this paper, the following *n*-dimensional nonlinear system is considered:

$$\dot{X} = F(X, X_0, q) \tag{1}$$

Where

$$X = [x_1, x_2, \ldots, x_n]^T \in R^n$$

is the state vector, X_0 denotes the initial state,

$$q = [q_1, q_2, \ldots, q_n]^T \in R^m$$

is the unknown parameters vector and

$$F : R^n \times R^m \rightarrow R^n$$

is a given nonlinear vector function. In order to estimate the unknown parameters in (1), an estimated model is defined below:

$$\dot{\hat{X}} = F(\hat{X}, X_0, \hat{q}) \tag{2}$$

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Where

$$\hat{X} = [\hat{x}_1, \hat{x}_2, ..., \hat{x}_n]^T \in R^n$$

And

$$\hat{q} = [\hat{q}_1, \hat{q}_2, ..., \hat{q}_n]^T \in R^m$$

are the estimated state vector and the estimated parameter vector, respectively. Since heuristic algorithms depend only on the objective function to guide the search, it must be defined before these algorithms are initialized. In this paper, the mean squared errors (MSEs) between real and estimated responses for a number of given samples are considered as fitness of estimated model parameters. Hence, the objective function is chosen as follows:

$$MSE = \frac{1}{N}\sum_{k=1}^{N} e^2 = \frac{1}{N}\sum_{K=1}^{N}\left[X(k) - \hat{X}(k)\right]^2 \tag{3}$$

Where N is the sampling number and $X(k)$ and $\hat{X}(k)$ are real and estimated values at time k, respectively. The contribution of this paper is to apply the CFAPSO Algorithm to minimizing the MSE value such that the actual nonlinear system parameters are accurately estimated. Fig. 1 presents a block diagram of nonlinear system parameter estimation. Considering Fig. 1, the initial state is given to both the real system and the estimated model.

Then outputs from the real system and its estimated model are input to the optimization algorithm, where the objective function (MSE) will be calculated. Owing to the unstable dynamic behavior of chaotic systems, the parameters are not easy to obtain. In addition, there are often multiple variables in the problem and multiple local optima in the landscape of MSE , so traditional optimization methods can easily be trapped in local optima and it is difficult to achieve the global optimal parameters. Here, the CFAPSO Algorithm method will be adopted to overcome these drawbacks.

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3. Constriction Factor Approach in PSO (CFAPSO)

In the basic PSO technique proposed by Kennedy and Eberhart, a great number of particles moves around in a multidimensional space, with each particle memorizing its position vector and velocity vector as well as the time at which it reached its highest level of fitness. The inspiration underlying the development of this algorithm was the social behavior of animals, such as the flocking of birds and the schooling of fish, and the swarm theory. The advantages of PSO are that it involves no evolution operators, such as crossover and mutation operators, and it does not require the adjustment of too many free parameters. PSO begins with a random population and searches for optima by continually updating this population. Moreover, each potential solution is assigned a randomized Velocity.

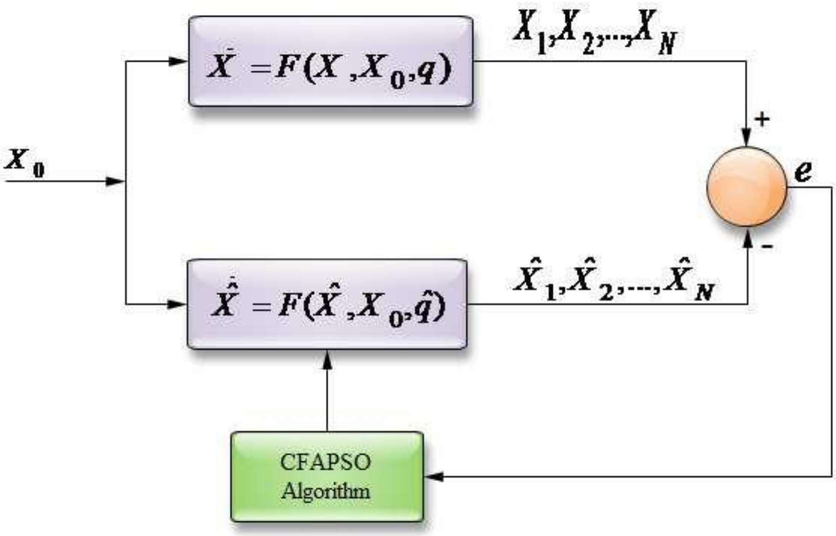

Figure 1. The principle of parameter estimation for nonlinear systems

The potential solutions, called particles, are then ‘‘flown” through the problem space. Related particles can share data at the best-fitness time. The velocity of each particle is updated according to the best positions reached by all particles through iterations, and the best positions are determined by the related particles over the course of multiple generations.

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In order to ensure the convergence of the PSO algorithm, we follow suggestions put forth by Clerc and incorporate a constriction factor because it may be necessary to ensure the convergence of the particle swarm [12], [13]. By incorporating a constriction factor, the PSO presented herein can be considered an adaptation of the conventional PSO, which takes into account weight inertia. In the current version, the particles have information on the entire population. Assume that each particle is considered in the d -dimensional space, $X_i(t) = (X_{i_1}(t), X_{i_2}(t), \ldots, X_{i_D}(t))$ denotes the ith particle's position, $V_i(t) = (V_{i_1}(t), V_{i_2}(t), \ldots, V_{i_D}(t))$ denotes the ith particle's velocity. The best previous position of the ith particle P_{best} is represented as among all particles in the population G_{best} is represented as $P_i(t) = (P_{i_1}(t), P_{i_2}(t), \ldots, P_{i_D}(t))$ and the best particle $P_g(t) = (P_{g_1}(t), P_{g_2}(t), \ldots, P_{gd}(t))$. The velocity and position updating equations of the CFAPSO, for $d=1,2,\ldots,D$ are given below [14]:

$$V_{id}(t+1) = X^*(V_{id}(t) + c_1^* r_1^* (P_{id}(t) - X_{id}(t)) + c_2^* r_2^* (P_{gd}(t) - X_{id}(t))) \quad (4)$$

$$X_{id}(t+1) = X_{id}(t) + V_{id}(t+1) \quad (5)$$

Where X is constriction factor, given by:

$$\chi = \frac{2}{\left|2 - \varphi + \sqrt{\varphi^2 - 4\varphi}\right|}, \ \varphi = c_1 + c_2, \ \varphi > 4 \quad (6)$$

4- Simulation Results

A chaotic system is a nonlinear deterministic system and its prominent characteristic is the sensitive dependence on initial conditions. Due to the complexity and unpredictable behavior of chaotic systems it is difficult to determine parameters of

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these systems. So a Lorenz system which is a known chaotic system is considered to show the performance of the CFAPSO algorithm in parameter estimation of chaotic nonlinear systems. The mathematical description of the Lorenz system is as follows:

$$\begin{cases} \dot{x}_1 = q_1(x_2 - x_1) \\ \dot{x}_2 = q_2 x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 = x_1 x_2 + q_3 x_3 \end{cases} \tag{7}$$

Where $q_1 = 10$, $q_2 = 28$, and $q_3 = 2.6667$ are the original parameters. The searching ranges are set as follows: $5 \le q_1 \le 15$, $20 \le q_2 \le 30$, and $0 \le q_3 \le 5$. Initial conditions are chosen as: $(x_1(0), x_2(0), x_3(0)) = (10, -10, 10)$. In the Lorenz system (Eq. 7), three-dimensional parameters are unknown and need to be estimated. According to (3), in this case, the objective function is chosen as

$$MSE = \frac{1}{N}\sum_{K=1}^{N}\left([X_1(k) - \hat{X}_1(k)]^2 + [X_2(k) - \hat{X}_2(k)]^2 + [X_3(k) - \hat{X}_3(k)]^2\right) \tag{8}$$

Where N is the sampling number and $X(k)$ and $\hat{X}(k)$ are real and estimated values at time k, respectively. The parameters of the CFAPSO Algorithm are set as shown in Table 1. The sampling time and sampling number (N) in this simulation are 0.01 and 1000, respectively.

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Table 1: Parameters used in the CFAPSO

Population size	100
Acceleration constant c_1	2.05
Acceleration constant c_2	2.05
constriction factor χ	0.729
Number of iterations	130

Table 2: Estimated parameters obtained using CFAPSO Algorithm

	q_1	q_2	q_3	MSE
Real Parameters	10.0000	28.0000	2.6667	-
CFAPSO (Run1)	9.999999687890258	27.999996860352955	2.666700870033932	2.192528803833660e-10
CFAPSO (Run2)	9.999999710969814	27.999992646658560	2.666704029737875	2.250990917407529e-08
CFAPSO (Run3)	9.999999882309535	27.999999155923362	2.666700195911218	1.786496484142039e-11
CFAPSO (Run4)	10.000003556236084	28.000035510340552	2.666688541049144	2.763591038262844e-08
CFAPSO (Run5)	10.000000121062028	28.000001239956248	2.666699609967542	2.717704352468503e-11
CFAPSO (Run6)	10.000000700302197	28.000008578681925	2.666696955965875	2.430691039455422e-09
CFAPSO (Run7)	**10.000000077318278**	**28.000000616239312**	**2.666699857867356**	**1.323486040908375e-11**
CFAPSO (Run8)	9.999999938941630	27.999999231254836	2.666700259845471	1.342260055733996e-11
CFAPSO (Run9)	10.000000274604188	28.000001610447790	2.666699732658722	1.710168390687514e-10
CFAPSO (Run10)	9.999999833630673	27.999997777469808	2.666700796933658	1.594580854119093e-10

Table 2 lists estimated parameters obtained by CFAPSO, when algorithm is implemented 10 times independently. From Table 2, it can be seen that all of the estimated parameters obtained by CFAPSO are very close to the true values in all experiments. It can be seen from table 2 that experiment 7 is able to generate better solutions than other experiments for parameter estimation of Loranz system. Figs 2 and 3 show results obtained by this experiment.

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Figs. 2(a)–2(c) depict the great success of optimization process by using CFAPSO algorithm for the identified parameters q_1, q_2 and q_3, respectively. Moreover, the convergence of the optimal MSE at each generation is plotted in Fig. 2(d). It confirms the superiority of CFAPSO Algorithm in terms of convergence speed without the premature convergence problem. Figures 3(a)-3(c) show the observed time response of the model with estimated parameters. The synchronization of the model response with that of the actual system is clearly evident.

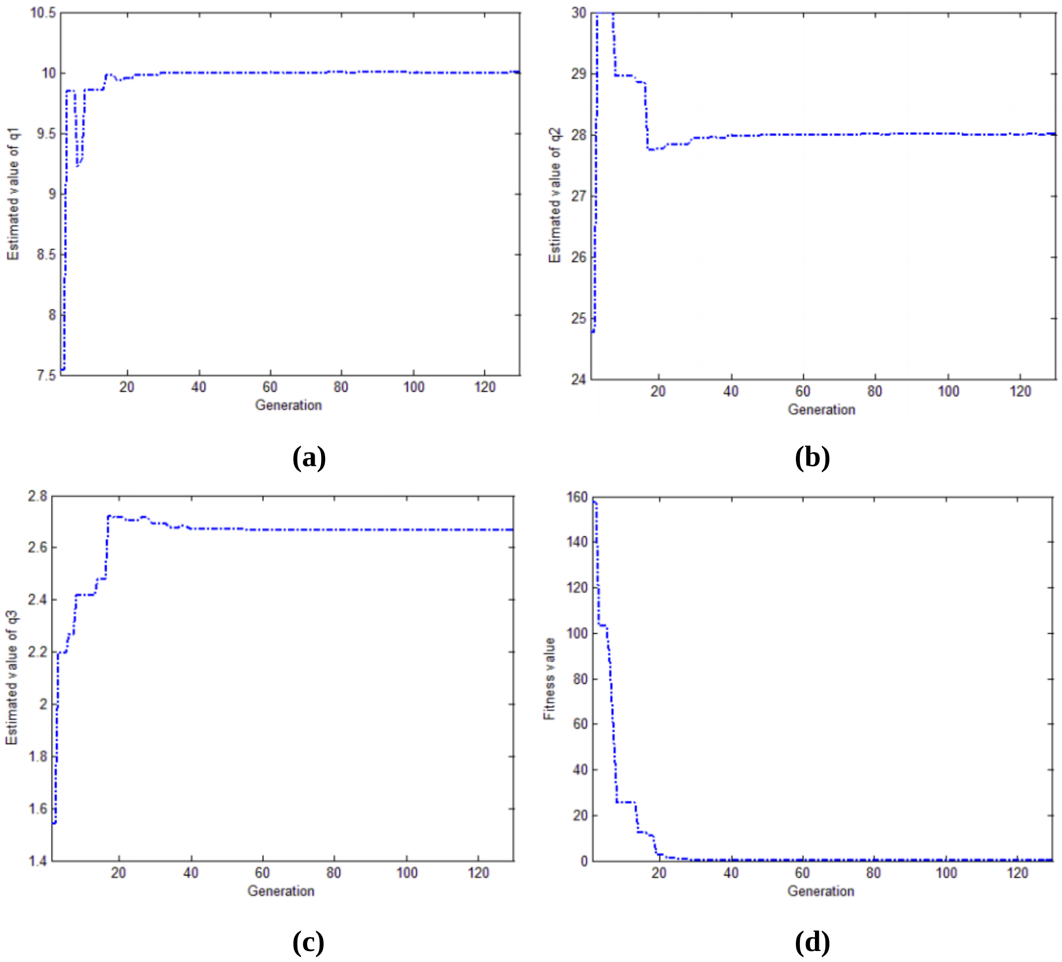

Figure 2. (a) The convergence process of q_1, (b) The convergence process of q_2, (c) The convergence process of q_3, (d) The convergence process of fitness value

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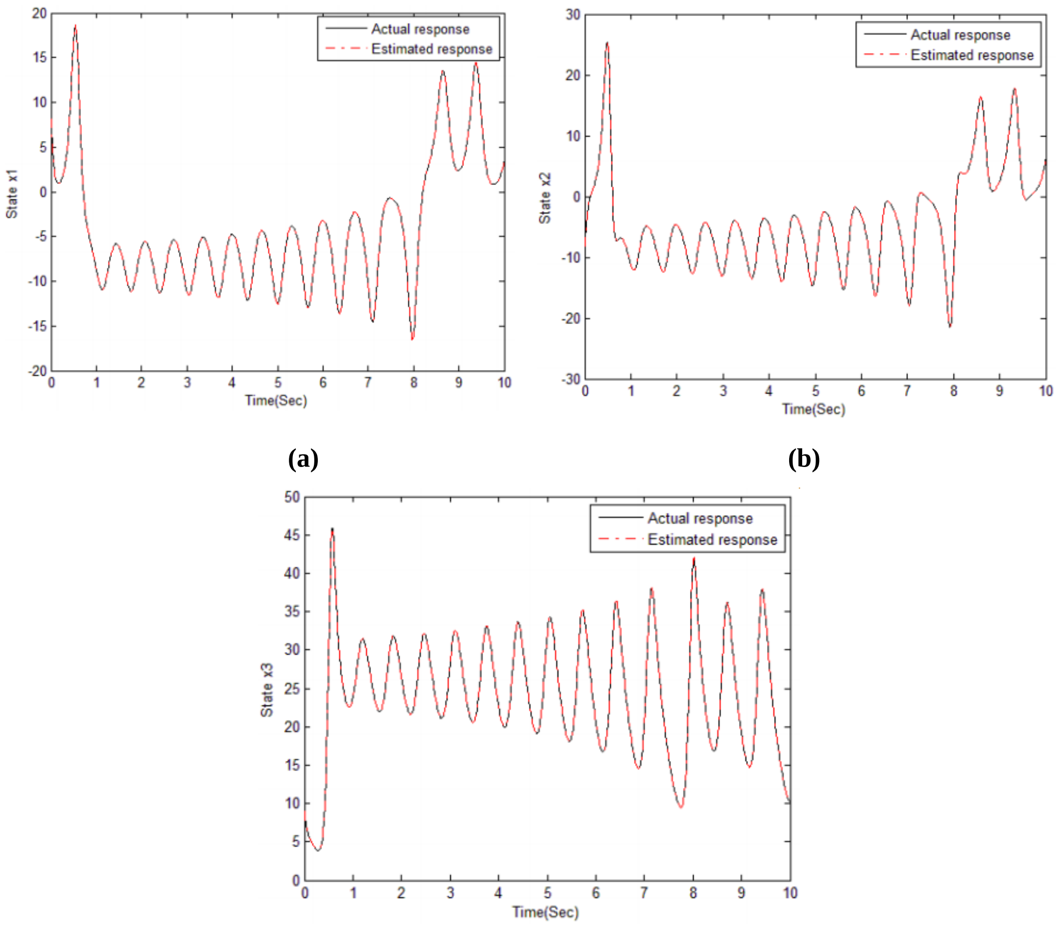

(a) (b) (c)

Figure 3. (a) Actual and estimated responses of X1 in (7), (b) Actual and estimated responses of X2 in (7), (c) Actual and estimated responses of X3 in (7)

Conclusion

Parameter estimation for the Loranz chaotic system has been formulated as a multidimensional optimization problem in this paper. A heuristic evolutionary algorithm, CFAPSO Algorithm, has been applied to solve such as issue. Minimized fitness function is associated with the Mean Squared Errors (MSEs). Numerical simulation results demonstrated that the proposed method is efficient and robust in parameter estimation of Loranz chaotic system.

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