



Digital IIR Filter Design Using Genetic Algorithm and CCGA

Method

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Abstract

The Cooperative Coevolutionary Genetic Algorithm is an optimization method which can be used to design digital filter with infinite impulse response (IIR). In addition to magnitude response, this method calculates linear approximation of phase response in pass band and filter lowest order simultaneously. This method finds good and sometimes different solution instead of one best solution. However, designing a filter is strongly dependent on its functional application. The proposed method in this paper is based on CCGA advantages and helps designing IIR filter considering applied usage such as phase response optimization and magnitude response optimization.

Keywords: IIR Digital Filter, Genetic Algorithm, CCGA

1. Introduction

The traditional digital IIR filter design involves the analog IIR filter design and the analog-to-digital transformation [1]. Genetic algorithm was first used for IIR digital filter design in Etter et al. (1982) [2]. Although the results were not acceptable, design method using genetic algorithm showed its distinction. It was a direct digital design which didn't need analog-to-digital transformation and it avoided coefficient quantization error. Multi-objective problems could be solved using this method and there was more than one solution. Kosir and Tasik designed a filter with semi-linear phase using GA (1995). However, their method might result in limited input/output stable problem and additional



calculations [3]. In this paper, we used CCGA which was suggested by Yang yu for IIR filter design (2007) [4]. This method calculates linear approximation of phase response in pass band and lowest order as well as magnitude response. The proposed method in this paper is based on CCGA advantages and helps designing IIR filter considering applied usage such as phase response optimization and magnitude response optimization. We will describe the proposed method in section 2 and implement results in section 3. We will come to a conclusion in section 4.

2. IIR digital filter design using genetic algorithm

2.1. CCGA method

A filter with first and second order components can be shown in cascade form:

$$H(z) = K \frac{(1 + b_1z^{-1})(1 + b_2z^{-1})(1 + d_{11}z^{-1} + d_{12}z^{-2})(1 + d_{21}z^{-1} + d_{22}z^{-2})}{(1 + a_1z^{-1})(1 + a_2z^{-1})(1 + c_{11}z^{-1} + c_{12}z^{-2})(1 + c_{21}z^{-1} + c_{22}z^{-2})}$$

In this method filter structure and coefficients are coded separately and grow up simultaneously [4]. These two species are named “control” and “coefficients” respectively. Control species determines structure and filter order, so plays the master role and coefficients species is a derivative part. Non-dominated sorting genetic algorithm (NSGA II) was used to grow up control species and simulated annealing was used to grow up coefficient species. Control gene and coefficient gene are shown in figure 1 and 2 respectively.

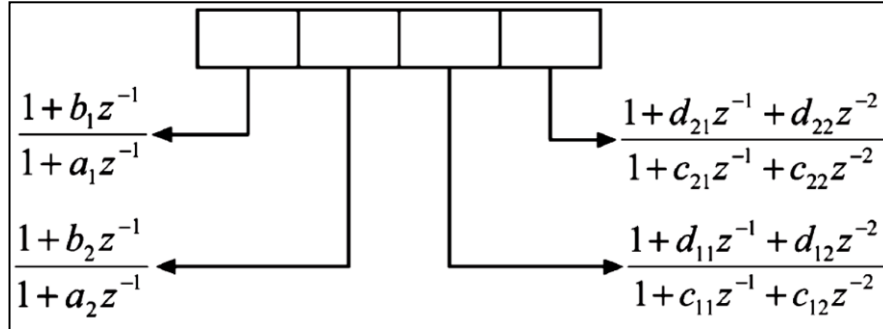


Figure 1. Control gene [4]

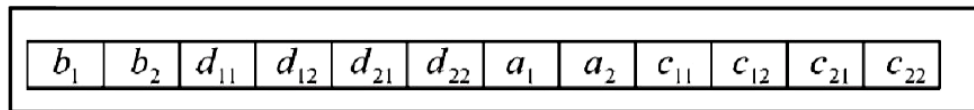


Figure 2. Coefficient gene [4]

Control genes are in binary form and decide for each block activation. Coefficient genes are real integers and define the coefficient values in each block. When the filter structure presented by equation (1), transform function:

$$H(z) = \frac{(1-0.1z^{-1})(1+0.5z^{-1}+0.6z^{-2})}{(1+0.4z^{-1})(1-0.9z^{-1}+0.1z^{-2})} \tag{1}$$

Involves following chromosome:

{1,0,1,0, -0.1,*, ,0.5,0.6,*,*, ,0.4,*, -0.9,0.1,*,*}

Figure 3 shows CCGA algorithm in which C and X are Control and coefficient respectively.

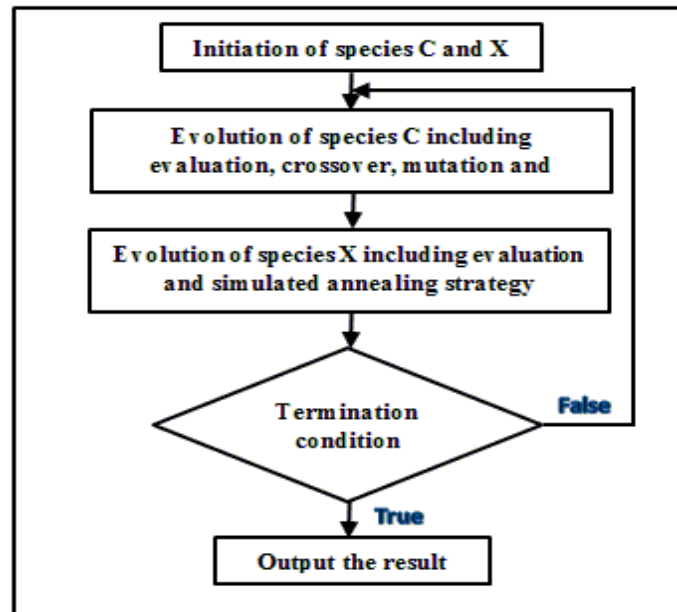


Figure 3. CCGA process to design IIR digital filter [4]

2.2. Evaluation

There are three objective functions to filter design and minimizing each one can lead to intended purpose:

- 1- Magnitude response error
- 2- Linear phase response error
- 3- Order

In this part we describe how to implement fitness functions.

2.2.1. Magnitude response error: Following assumptions are necessary to design:

- Attenuation in pass band should not exceed δ_1
- Attenuation in stop band should not be less than $1 - \delta_2$



Magnitude response error for pass band and stop band is shown as follows:

$$eH_p(\omega) = \begin{cases} 1 - \delta_1 - |H(e^{j\omega})|, & |H(e^{j\omega})| < 1 - \delta_1 \\ 0, & |H(e^{j\omega})| \geq 1 - \delta_1 \end{cases}$$

Where ω is pass band.

$$eH_s(\omega) = \begin{cases} |H(e^{j\omega})| - \delta_2, & |H(e^{j\omega})| < \delta_2 \\ 0, & |H(e^{j\omega})| \geq \delta_2 \end{cases}$$

Where ω is stop band.

So, the first objective function is:

$$\min f_1 = \sum_{\omega_i} eH_p(\omega_i) + \sum_{\omega_j} eH_s(\omega_j) \tag{2}$$

Where ω_i and ω_j are sampling frequency in pass band and stop band respectively.

2.2.2. Linear phase response error: Linear phase response is simplified as pass band linear phase response and transmission band linear phase response. Transmission band linear phase response is usually considered because magnitude response on some points of transmission band may be the same value as pass band. If phase response deviates from linear form in these points, it may lead to a significant distortion. In order to simplification, frequency corresponding phase response is considered and phase string is presented as below:

$$\text{Phases} = \{\theta_1, \theta_2, \dots, \theta_n\}$$

This can be shown equally as below:

$$\Delta\text{Phases} = \{\Delta\theta_1, \Delta\theta_2, \dots, \Delta\theta_{n-1}\}, \quad \text{where } \Delta\theta_i = \theta_{i+1} - \theta_i$$

Part of phase response can be estimated using ΔPhases string variance and then regarded as phase response error. So the second objective function is:



$$\min f_2 = \text{variance} \{ \Delta\theta_i | \theta_i \in \text{pass band } \cup \text{ transition band} \} \tag{3}$$

2.2.3. Order: For a given control chromosome structure, order can be expressed as below:

$$\text{order} = \sum_{i=1}^m p_i + 2 \sum_{j=1}^n q_j$$

Where $m+n$ is control chromosome length, p_i and q_j are control bits which control activity of i^{th} first-order block and j^{th} second-order blocks. The filter's maximum allowed order is $m+2n$. Consequently, the third objective function would be:

$$\text{Min } f_3 = \text{order} \tag{4}$$

2.3. Proposed Method

As stated before, in addition to magnitude response, linear approximation of phase response in pass band and transmission and the lowest filter order can be calculated using CCGA. Making decision to design is strongly dependent on its applications. For example, if we want to minimize domain phase error and design a band-pass filter with minimized domain response error, we should use first fitness equation (Eq.3). To design a filter considering intended application, instead of Non-dominated Sorting Genetic Algorithm (NSGA II) for control species and simulated annealing for coefficient species, multi-objective simulation was used with all fitness functions through genetic algorithm.

3. Experiments and results

Consider filter structure as below:

$$H(z) = K \prod_{i=1}^3 \frac{(1 + b_i z^{-1})}{(1 + a_i z^{-1})} \prod_{j=1}^4 \frac{(1 + b_j z^{-1} + b_{j2} z^{-2})}{(1 + a_j z^{-1} + a_{j2} z^{-2})}$$



Maximum designed filter order is 11. Control chromosome length is 7 and coefficient chromosome length is 22. The constraint for first fitness function is that objective function value equals to zero and for the two others is that objective function value be less than given value. Suppose designing a band-pass filter with minimum domain response error is concerned. In this case first fitness function (Eq.2) is used. The given characteristics are:

Normalized pass band cut-off frequency: [0.4 0.6]

Normalized stop band cut-off frequency: [0.25 0.75]

Chromosome length: 7

Population size: 20

Crossover: two-point crossover

Crossover rate: 0.8

Mutation: bit-flip mutation

Mutation rate: 0.3

There are several ways to optimization. One way is using genetic algorithm advanced toolbox [5]. To optimize the fitness function, we should set the parameters in a way that genetic algorithm can solve the problem in order to minimize domain response error. Figure 4 shows domain and phase responses.

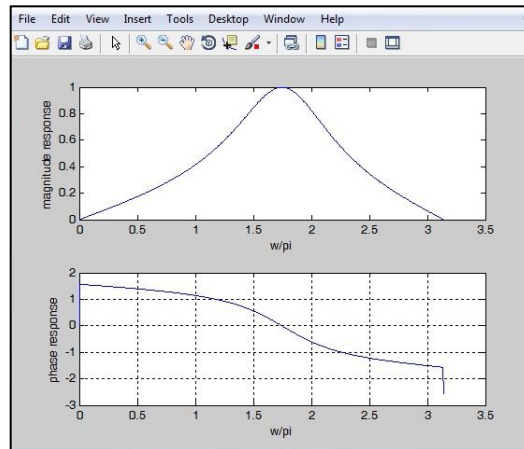


Figure 4. Domain and phase response for first experiment

In second experiment, designing a band pass filter with minimum phase response error is concerned. Here, the second fitness function (Eq.3) should be used. Domain and phase response shown in figure 5 confirms this minimization. Phase response approximation is near to linear approximation, domain response is not acceptable though. Best individual in population and best fitness function is plotted in figure 6.

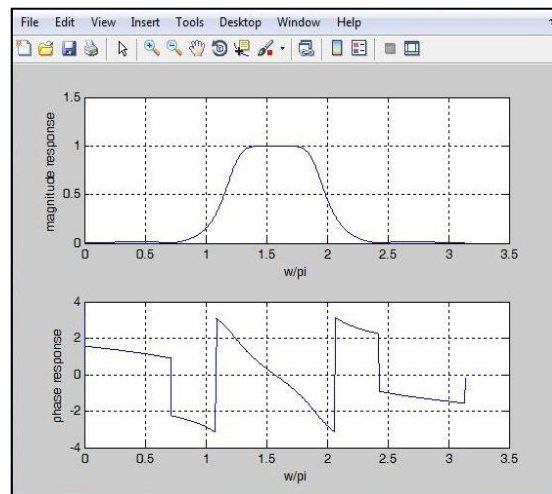


Figure 5. Domain and phase response for second experiment

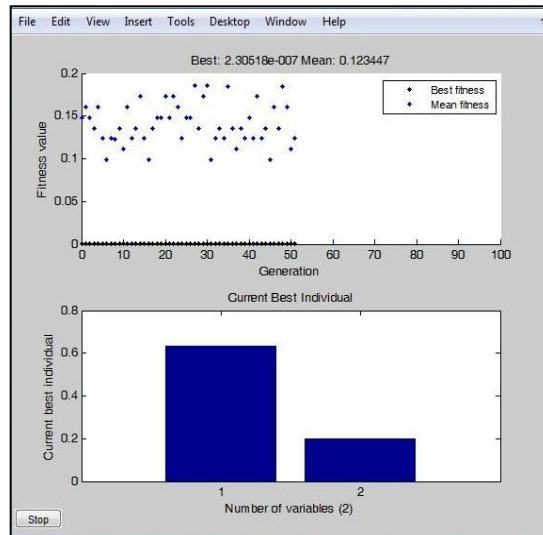


Figure 6. Best individual and best fitness function

To design a band pass filter with minimum order, we should use Eq.4. In this case, domain and phase response is shown as figure 7.

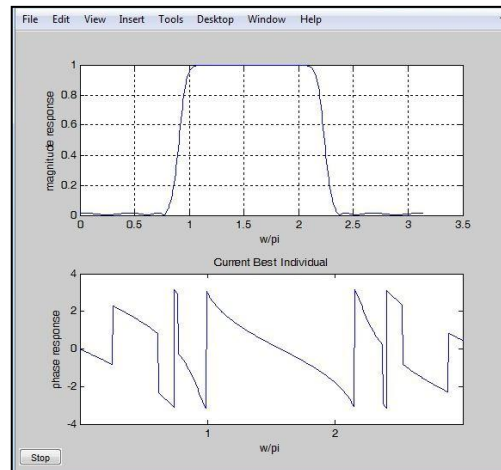


Figure 7. Domain and phase response for third experiment

To compare current results with results from CCGA method, we consider control and coefficient species together using NSGA II and simulated annealing for optimization. In annealing algorithm, the result for minimum sum of phase and domain response is which is just close to zero. Although it is not intended in annealing algorithm to find a general optimized value, but an approximately optimized value could be estimated using this method [6]. The minimum order using this method is 4. The results are shown in figure 8.

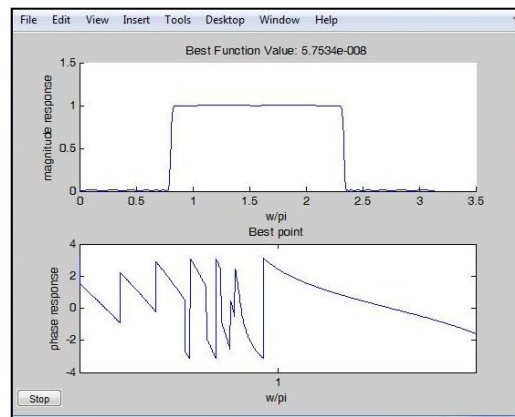


Figure 8. Domain and phase response for CCGA-based method

Conclusion

In current study, a method is proposed based on Cooperative Coevolutionary Genetic Algorithm (CCGA) to design digital IIR filter. In this method, fitness functions used in CCGA were applied to design the filter considering intended applications such as phase response or domain response optimization, or minimizing filter order. Instead of Non-dominated Sorting Genetic Algorithm (NSGA II) for control species and simulated annealing for coefficient species, multi-objective simulation was used with all fitness functions through genetic algorithm. Although the results of current study and CCGA are similar, proposed method is more accurate in some special cases and some traditional complications in filter design using GA is lost in our suggested method.



References

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