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A New Approach of Fuzzy Theory with Uncertainties in Geographic Information Systems

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Abstract

Until now, fuzzy logic has been extensively used to better analyze and design controllers for chemical processes. It has also been used for other applications like parameter estimation of nonlinear continuous-time systems but in general fuzzy logic has been intensively used for heuristics based system. Recently, fuzzy logic has been applied successfully in many areas where conventional model based approaches are difficult or not cost effective to implement. Mechanistic modeling of physical systems is often complicated by the presence of uncertainties. When models are used as purely predictive tools, uncertainty and variability lead to the need for assessment of the plausible range of model outcomes. A systematic uncertainty analysis provides insight into the level of confidence in model estimates, and can aid in assessing how various possible model estimates should be weighed. In this paper, generalized fuzzy α-cut is used to show the utility of fuzzy approach in uncertainty analysis of pollutant transport in ground water. Based on the concept of transformation method which is an extension of α-cuts, the approach shows superiority over conventional methods of uncertainty modeling. A 2-D groundwater transport model has been used to show the utility of this approach. Results are compared with commonly used probabilistic method and normal Fuzzy alpha-cut technique. In order to provide a basis for comparison between the two approaches, the shape of the membership functions used in the fuzzy methods are the same as the shape of the probability density function used in the Monte-Carlo method. The extended fuzzy α-cut technique presents a strong alternative to the conventional approach.

Keywords: Temporal, Uncertainty, Fuzzy Set, Model, Reasoning, Geographic Information Systems.

1. Introduction

Geographic information systems (GIS) have been popularly applied in modeling environmental and ecological systems. However, real-world system analysis is typically

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challenged by difficulties in handling temporal change and uncertainty. Current GIS provide little or no support for modeling dynamic and uncertain phenomena. The GIS, more broadly, lack the ability to effectively model change and vagueness. There has been much research activity over the past decade focused on dynamic applications with geographic reference and handling uncertain features of geographic reference and handling uncertain features of geographic applications [1, 2].

Fuzzy set theory, as a promising logical foundation for an intelligent GIS [3], had drawn a lot of attention in its application to handling uncertainty and temporal reasoning in spatial domain. The fuzzy approach has been applied in geography on problems related to soil classification and definitions [4, 5, 6, 7, 8]. The fuzzy aspect of boundaries in geographic space is now fully recognized [9, 10]. A general spatial data model was extended to deal with the uncertainty with geographic entities in a database repository [11]. The fuzzy set theory was also applied to perform temporal interpolation in a raster GIS database [12]. There is little research trying to handle uncertainty in temporal presentation in spatial domain. This seems to be due to the fact that the handling of time and the management of uncertainty are two distinct important issues.

Each of both possesses its own specific problems, and their solutions require a lot of research effort separately. Some modeling issues in spatial domain require a representation of time and of the temporal relationships between events. Most systems rely on a mechanism in which time is associated with each piece of knowledge. In spatial data models currently used, temporal information is stored through a series of snapshots associated to particular instants in time. Relationships are then the time ordering. In the complex environmental and ecological systems modeling with GIS, the current time mechanism is not sufficient. One must be able to represent situations with relative knowledge like “precedes” or “during”. Real – world modeling requires a unified mold of time and uncertainty. Here is an example of the scenario in wildlife migration modeling analysis: All animals of species S1 moved away from area A before October 1st. animals of species S2 entered the area A after June 1 st. But between mid-July and mid-August, we seldom saw both species of animal appeared at same time in the area. This gives a sample

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description of temporal definitions and relationships necessary for modeling wildlife migration property.

One obvious characteristic with the description is that those definition and relationships are vague. A major task of the analysis is to translate it and capture the knowledge in a form which we can correctly process and reason over. The purpose of this paper is to introduce a novel fuzzy approach, fuzzy temporal data in spatiotemporal information processing. Section 2 presents a brief introduction of the fuzzy sets theory as the basis of the model. Section 3 describes the FTCN method with its main algorithms. Section 7 addresses the issues of incorporating the fuzzy approach into current spatiotemporal processing model, and of FTCN inferences in a GIS domain. Section 5 concludes the discussion by summarizing the contribution of the paper and giving some directions for future research.

2. Fuzzy set theory

Fuzzy theory is a method that facilitates uncertainty analysis of systems where uncertainty arises due to vagueness or "fuzziness" rather than due to randomness alone [13]. This is based on a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth-truth values between "completely true" and "completely false". It was introduced by Zadeh as a means to model the uncertainty of natural language [14]. Fuzzy theory uses the process of "fuzzification" as a methodology to generalize any specific theory from a crisp (discrete) to a continuous (fuzzy) form. Classical set theory has a "crisp" definition as to whether an element is a member of a set or not. However, certain attributes of systems cannot be ascribed to one set or another.

For example, an attribute of a system can be specified as either "low" or "high". Is such a case, uncertainty arises out of vagueness involved in the definition of that attribute. Classical set theory allows for either one or the other value. On the other hand, fuzzy theory provided allows for a gradual degree of membership. This can be illustrated as follows:

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In the classical set theory, the truth value of a statement can be given by the membership function $\mu_A(X)$, as

$$\mu_A(X) = 1 \quad if \;\; X \in A \quad and \; \mu_A(X) = 0 \quad if \;\; X \notin A \qquad (1)$$

On the other hand, fuzzy theory allows for a continuous value of μ_A, between 0 and 1, as

$$\mu_A(X) = \begin{cases} 1 & if\ X \in A \\ 0 & if\ X \notin A \\ P & 0 < P < 1 \quad if\ xpartially\ belong\ to\ A \end{cases} \qquad (2)$$

In fuzzy theory, statements are described in terms of membership functions that are continuous and have a range [0, 1]. For example, given the measured value of a parameter, the membership function gives the "degree of truth" that the parameter is "high" or "low". Further, fuzzy logic is defined by the following the set relationships:

$$\begin{cases} \mu_{\acute{A}}(X) = 1 - \mu_A(X) \\ \mu_{A\cap B}(X) = \min(\mu_A(X), \mu_H(X)) \\ \mu_{A\cup B}(X) = \max(\mu_A(X), \mu_H(X)) \end{cases} \qquad (3)$$

Using fuzzy arithmetic, based on the grade of membership of a parameter of interest in a set, the grade of membership of a model output in another set can be calculated. Fuzzy theory can be considered to be a generalization of the classical set theory. Is must be noted that if the membership grades are restricted to only 0 and 1, the fuzzy theory simplifies to classical set theory.

3. Fuzzy Temporal Constraint Network

Temporal constraint network algorithm was proposed to handle time reasoning in artificial intelligence systems with explicit temporal representation [15]. However, it is not enough for domains where knowledge about time is highly pervaded with vagueness and uncertainty. Several pieces of work have considered the representation of approximate temporal knowledge. Described an extended method by applying fuzzy set theory, the

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Fuzzy Temporal Constraint Network (FTCN) [16]. Timeline is assumed to be a discrete and totally ordered set as the basis of temporal entities, $\tau = \{t_0, t_1, \ldots, t_i, \ldots\}$, where each t_i, represents a precise point of time and t_0, represents the time origin. Time points are equally spaced and the unit of time is the distance between two consecutive time points. An imprecise time point a is represented by means of a possibility distribution π_a over τ. Given any precise time point t$\in \tau$, π_a(t)$\in$ [0,1] represents the possibility (membership) of a being precisely t. the extreme values 1 and 0 represent, respectively, the absolute and null possibilities of a being t. There is always a fuzzy set A associated with a possibility distributionπ_a, whose membership function concise with π_a. The concept of imprecise time extent was introduced to represent fuzzy quantities of time. An imprecise time extent m is represented by a possibility distribution π_m over the set of integer numbers I, where the elements of I represent units of time. Thus, given an $n \in I, \pi_m(n) \in [0,1]$ represents the possibility of the quantity of time m being precisely n units of time. The duration of the time interval between tow fuzzy time points is an imprecise time extent. Given an ordered pair of imprecise time points (a, b), the temporal distance from a to b, D (a, b), is represented by means of the time extent possibility distribution, $\pi_{d(a,b)}$:

$$\forall n \in I, \pi_{d(a,b)}(n) = \max \min\{\pi_a(s), \pi_b(t)\}, n = t - s; t, s \in \tau \qquad (4)$$

Corresponding to the fuzzy difference D(a, b)=B⊙A

An imprecise time interfacial I may be defined as a triplet (a, b, d), in which a and be are fuzzy time points representing, respectively, the initial and final time points of the intervals, and d is a fuzzy time extent repressing its duration. Assume we have a set of instantaneous events whose time points of occurrence are completely unknown. Each of them is represented by means of a temporal variable X that, in absence of any other information, can take as its value any precise time point t_i. When any imprecise information is added about the position of event X_i, the information can be represented as the possibility distribution of an imprecise time point π_i and establishes a constraint over the possible values forX_i. It is called unary constraint L_i over the variableX_i. Then assume some

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imprecise information is added about the relative position of two variables X_i andX_j, that is, over the relative position of two variables X_i and X_j, that is, over the time elapsed between the two events they represent. This information can be represented as the possibility distribution of an imprecise time extent and π_i, establishes a constraint over the possible values of other constraints, the assignments $X_i = t_i$ and $X_j = t_j$ are possible if $\pi_j(t_j - t_i) > 0$. This is called binary constraint L over variables X_i andX_j. Based on the above arbitrary set of information, a fuzzy temporal constraint network (FTCN) can be defined as follows: A FTCN L is a finite set of n + 1 temporal variables $X_0, X_1, \dots X_n$, and a finite set of binary constraints L_j, over those variables, where X_0 is a precise origin of times.

Minimized imprecision in the position X_i can be obtained when all the constraints over X_i are combined. The formalization of concept of minimizing imprecision is also supported by a few more definitions on L. Here we just introduce how to obtain the minimal network M corresponding to network L. between each pair of L, X_i, X_j there is direct constraint X_j, but there will also be induced constraints. Each induced constraint will correspond to a possible path connecting to them time points applied with X_i and X_i. In order to obtain the minimal constraint between tow nodes X_i and X_i, it is necessary to obtain the intersection of the direct constraint $L_j$and the induced constraints between X_i and X_j given a path k from a network node of index I to another one of index j:

$$i_0 = i,\ i_1, i_2, \dots i_k = \qquad (5)$$

The induced constraint is represented as $C^k_{i0,i1,\dots ik}$. It is given by the composition of the direct constraints between each pair of consecutive nodes belonging to the path.

$$C^k_{i_1,i_2,\dots i_k} = L_{i_0 i_1} + L_{i_1 i_2} + L_{i_{k-1}k = \sum_{p=1}^{k} L_{i_{p-1} i_p}} \qquad (6)$$

L^k_{ij} is used as the intersection of the induced constraints corresponding to all the paths of length k from the node with index i to the one with index j:

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$$L_{ij}^{k} = \cap_{\,i_0=i,i_1=0,\ldots n,\, i_k=j} C_{i_1,i_2,\ldots i_k}^{k}\, z \qquad (7)$$

Finally $M_j$is used to present the intersection of all induced constraints corresponding to paths of nay length between 1 and n going from X_i toX_j: $M_j = \cap_{k=1}^{n} L_{ij}^{k}$

It can be proved that a network L is consistent if and only if the constraints M_j obtained by means of this expression are normalized. Also in this case, the constraints M_j correspond to the minimal network M associated with L. These results have practical interest from the computational viewpoint. It is easy to operate with normalized distributions which are mostly common. Figure 1 show a trapezoidal distribution represented by means of the four parameters $(\alpha, \beta, \gamma, \delta)$. The M_j can be obtained by solving constraint static problems with path-consistency algorithms [17, 18].

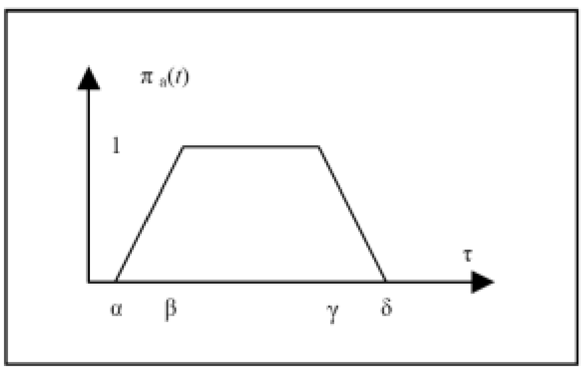

FIGURE 1. TRAPEZOIDAL POSSIBILITY DISTRIBUTION FOR AN IMPRECISE TIME POINT

4. Results and Fuzzy Time Layer: Model and Reasoning

A critical issue for this research is how to incorporate fuzziness into a general spatio-temporal data model in GIS with the fuzzy temporal constraint network method. The most usual way of handling the time factor in current GIS is to look on time as an attribute to the objects in the same way as for other attributes. This view corresponds with the usual way of presenting spatial data, and can thus be realized for both vector and raster data [19]. Temporal information is stored through a series of time layers (snapshots) associated to particular instants in time. To accommodate uncertain time information, the current data

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model must be extended to represent the partial memberships (possibilities) of each time point. Thus the fuzzy time layer concept is introduced here. Corresponding to the definition in the FTCN method, a fuzzy time layer is an imprecise time interval with initial with initial and final time points and possibility distribution function (Figure 2). The fuzzy information can be specified in various ways based on practical observation and individual type of possibility distribution.

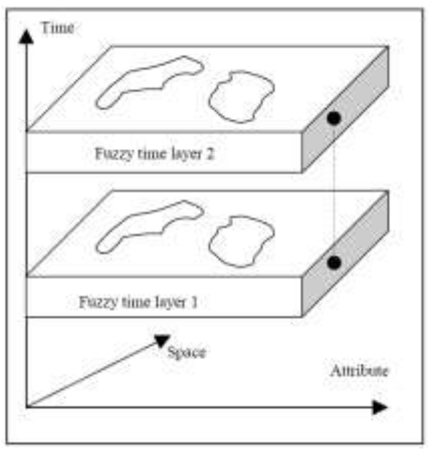

FIGURE 2. UNCERTAIN TIMES ARE REALIZED IN FUZZY LAYER (DASHED LINE BETWEEN TWO LAYERS INDICATES FUZZY RELATIONSHIP)

New components and related definitions have to be brought into data model for incorporation of the temporal fuzziness. Also the model structure has to be rebuilt to fit the change. Obviously, the number of layers increases for characterizing a temporal uncertainty. More complicated structure will be seen due to the requirements for storing and retrieving fuzzy membership functions, and if necessary, reasoning results with the FTCN method as to be discussed later. The specific concept of fuzzy temporal layer should not be unique. It may vary depending on specific uncertainty pattern on individual time layer and query requirements.

The temporal reasoning regarding fuzzy relationships among time layers has to be tackled following reconstruction of data model. This involves the incorporation of FTCN algorithms into basic data interpretation operations in GIS modeling packages. Let us still

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use the wildlife migration modeling example mentioned in Section 1. As we can see now, it is an ideal trapezoidal distribution problem. As shown in Figure 1, α June 1st, β July 15th, γ August 15th, and δ October 1st. A logical translation of the fuzzy temporal problem starts with construction of separate possibility distributions (Figure 3). A simplified inference procedure in mathematical form can be as follows:

The fuzzy temporal possibility distributions π^*, objects of fuzzy subsets:

$$\pi_1^* = \tau 2 \cap \tau 4 \cap t5$$
$$\pi_2^* = \tau 2 \cap \tau 3$$
$$\pi_3^* = \tau 1 \cap \tau 4 \quad (8)$$
$$\pi_4^* = \tau 1 \cap \tau 3 \cap t5$$

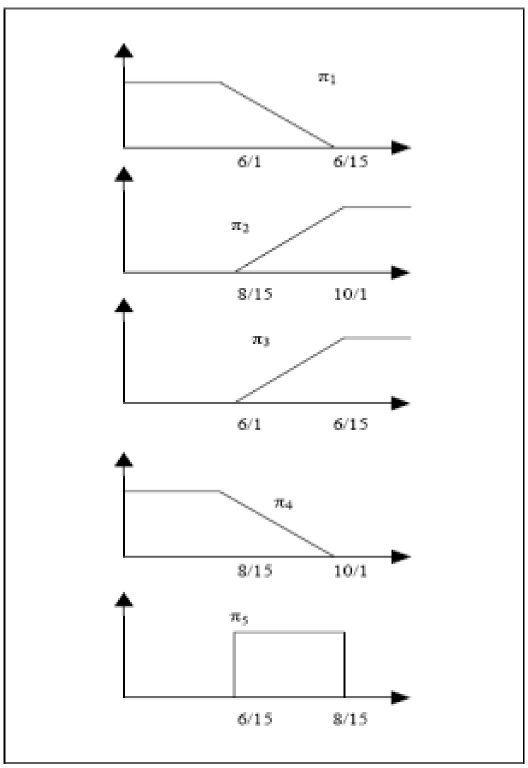

FIGURE 3: SEPARATED POSSIBILITY DISTRIBUTIONS

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Then we can have the fuzzy complementary of the union of these four fuzzy subsets:

$$\hat{U} = \pi_1^* \cup \pi_2^* \cup \pi_3^* \cup \pi_4^* = (\tau 2 \cap \tau 4 \cap t5) \cap (\tau 2 \cap \tau 3) \cap (\tau 1 \cap \tau 4) \cap (\tau 1 \cap \tau 3 \cap t5) = (\tau 2 \cap \tau 4 \cap t5) \cap \tau 1 \cap \tau 3 \cap (\tau 1 \cap \tau 3 \cap t5) = \tau 1 \cap \tau 3 \cap (\tau 2 \cap \tau 4 \cap \tau 5) = (\tau 1 \cap \tau 2 \cap t3) \cup (\tau 1 \cap \tau 3 \cap t4) \cup (\tau 1 \cap \tau 3 \cap t5) = \tau 1 \cap \tau 3 \cap t5 = \tau 1 \cap t5 \quad (9)$$

Now we can compute temporal necessity functions $N^*(1)$ and $N^*(2)$ induced by π^* for the following two expressions in the example: (1) Species S_1 has certainly left the area, and (2) Species S_2 has certainly left the area.

$$N^*(1) = (\tau 1 \cap \tau 4) \cup (\tau 1 \cap \tau 3 \cap \tau 5)$$
$$= (\tau 1 \cup \tau 4) \cap (\tau 1 \cup \tau 3 \cup \tau 5) = (\tau 1 \cup \tau 4) \cap T = \tau 1 \quad (10)$$
$$N^*(2) = (\tau 2 \cup \tau 3) \cap (\tau 1 \cup \tau 3 \cup \tau 5) = \tau 3$$

Generally, this approach makes possible to compute the fuzzy set of instants when we are more or less certain that a proposition is true, and on the other hand, to compute the fuzzy set of instants when we are more or less certain that proposition is false. The above translation procedure and algorithm can be built into GIS as one of spatiotemporal operation functions. This would be possible if we have a well-defined fuzzy data model with much knowledge on uncertainties for specifying possibility distributions and fuzzy relationships. It is necessary for large-scale dynamic modeling analysis which may have a lot of uncertainty in time dimension and interactions among events. Expected queries to related information in a GIS database should also be considered before constructing a built-in system. A set of computer programs are being developed to implement the FTCN approach to model the uncertain temporal reasoning for a case study we are working on. The major steps and tasks in the program include:

- Generation of fuzzy time layers;
- Quantification and construction of fuzzy relationships between timed events;
- Verification of generalized fuzzy parameters;

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- Implementation of the FTCN process to parse the fuzzy data, and storage of results in a GIS database.

Conclusion

Fuzzy set theory has been proved to be effective in supporting GIS-based decision-making analysis. The fuzzy temporal constraint network method is examined in this paper with regards to its application to handling uncertain temporal information in spatio-temporal modeling. It shows the FTCN concept can be adopted as a useful tool for representation and reasoning of temporal uncertainty in real-world applications. The paper discusses how the general temporal data model is extended to accommodate uncertainty with temporal data and relationships among events. A theoretical FTCN process of fuzzy translation for the imprecise information is introduced with an example. The research is going on with an experimental study of the application of the method to environmental systems modeling. The focus will be on streamlining the FTCN reasoning procedure for various circumstances of temporal uncertainty. Then effort will be made to develop a versatile computer program built into a GIS database.

References

[1] R. Banai, International Journal of Geographical Information Science, contribution from the analytic hierarchy process. (1993) 4: pp. 315-329.

[2] T. Bernhardsen, Editor, Geographic Information Systems: An Introduction, New York, (1999) John Wiley.

[3] P. A. Burrough, Journal of Soil Science, Fuzzy mathematical methods for soil survey and land evaluation. (1989) 40: pp. 477-492.

[4] T. Cheng, M. Molenaar, and H. Lin, International Journal of Geographical Information Science, (2001) 15 (1): pp.27-42.

[5] D. A. Davidson, S. P. Theocharopoulos, and R. J. Bloksma, International Journal of Geographical Information Science, (1994) 4: pp.369-384.

[6] R. Dechter, I. Meiri, and J. Pearl, Editors, Temporal constraint networks. Artificial Intelligence,(1991) 49: pp.61-95.

[7] S. Dragicevic, and D. J. Marceau, International Journal of Geographical Information Science, (2000) 14 (3): pp.225-245.

[8] G.W. Evans, W. Karwowski, and M.R. Wilhelm, editors, Applications of Fuzzy Set Methodologies in Industrial Engineering, Elsevier, New York. (1986) pp. 3-11.

Dec. 2013, Special Number, pp 410-421, **ISSN**: 2305-0543

Available online at: http://www.aeuso.org

[9] M. M. Fischer, Editor, Expert systems and artificial neural networks for spatial analysis and modeling. Geographical Systems Journal,(1994) 1: pp.221-235.

[10] V. J. Kollias, and A. Voliotis, International Journal of Geographical Information Science, (1991) 2: pp.209-223.

[11] Y. Leung, and K. S. Leung, International Journal of Geographical Information Science, (1993) 7: pp.189-213.

[12] D. M. Mark, and F. Csillag, The nature of boundaries on "Area-Class" maps. Cartographical, (1989) 1: pp.65-78.

[13] J. Rit, Editor, Propagating temporal constraints for scheduling. Proc. AAAI-86, (1986) Philadelphia, pp. 383-388.

[14] E. Stefanakis, Department of Electrical and Computer Engineering, National Technical University of Athens,(1997) Greece.

[15] E. Stefanakis, M. Vazirgiannis, and T. Sellis, International Journal of Geographical Information Science, (1999) 13 (7): pp.657-675.

[16] V. Beek, P, Editor Exact and Approximate Reasoning about Qualitative Temporal Relations. Ph.D. Thesis. University of Waterloo, Ontario, Canada (1999).

[17] L.Vila, and L. Godo, Editors, On fuzzy temporal constraint networks, Math ware and Soft Computing,(1994) 3:pp. 315-334.

[18] F. Wang, and G. Hall, International Journal of Geographical Information Science, (1996) 5: pp.573-590.

[19] L.A. Zadeh, Editor, Fuzzy Sets, Information and Control,(1965) 8: pp.338-253.