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A New Survey of types of Uncertainties in Nonlinear System with Fuzzy Theory

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Abstract

This paper is an attempt to introduce a new framework to handle both uncertainty and time in spatial domain. The application of the fuzzy temporal constraint network (FTCN) method is proposed for representation and reasoning of uncertain temporal data. A brief introduction of the fuzzy sets theory is followed by description of the FTCN method with its main algorithms. The paper then discusses the issues of incorporating fuzzy approach into current spatio-temporal processing framework. The general temporal data model is extended to accommodate uncertainties with temporal data and relationships among events. A theoretical FTCN process of fuzzy transition for the imprecise information is introduced with an example. A summary of the paper is given together with outlining some contributions of the paper and future research directions.

Keywords: Fuzzy α-cut, transformation method, transport model, Uncertainty, Variability.

1. Introduction

Uncertainty is intrinsically imbedded in system modeling and parameters being the main cause of uncertainty in output values. Mechanistic modeling of physical systems is often complicated by the presence of uncertainties. Commonly environmental models are calibrated to field data to demonstrate their ability to reproduce contaminant behavior at site. However, solute transport modeling presents a big uncertainty due to the lack of reliable field data. On the other hand, specific field situations cannot be extrapolated over

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larger distances, even in the same site [1]. Several approaches to sensitivity and uncertainty analysis in solute transport have been developed [2]. Among them probabilistic approach (like Monte Carlo Simulation) is quite common and has been commonly used in the treatment and processing of uncertainty for solution of system modeling. This technique requires knowledge of parameter values and their statistical distribution. However generally site investigation is not detailed to determine values for some of the parameters and their distribution pattern and sufficient data may not be collected for calibrating a model. These approaches suffer from obvious lack of precision and specific site which make it difficult to determine how much error is introduced into the result due to assumptions and prediction.

Also Monte Carlo method for uncertainty propagation typically requires several model runs that use various combinations of input values, resulting in substantial computational demands. Fuzzy set approach has been applied recently in various fields, including decision making, control and modeling [3]. However, the application of standard fuzzy arithmetic turns out to be very problematic. Normally, the calculated results of the problem do not only reflect the natural uncertainties, which are directly induced by the uncertainties in the model parameters, they also show some additional, artificial uncertainties, generated by the solution procedure itself [4]. The fuzzy alpha-cut analysis is based on fuzzy logic and fuzzy set theory which is widely used in representing uncertain knowledge.

Uncertain model parameters can be treated as fuzzy numbers that can be manipulated by specially designed operators. But this approach is also doing the same mistake of treating independent and strictly dependent variable together result in overestimation effect arises from evaluating the arithmetical expression for unreal combination of elements of support of the fuzzy numbers. The present study aims to show the utility of fuzzy approach in uncertainty analysis comparing the features, advantages and drawbacks of conventional Monte-Carlo with the transformation method and normal Fuzzy alpha-cut technique when applied to the analysis of uncertainty of solute transport modeling in saturated porous media.

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2. Methodology

2.1 Fuzzy Modeling

Basic principal of fuzzy modeling is based on Zadeh's extension principle [5]. If all input parameters in a mathematical model are known, also the dependent variables are defined with crisp values and we assume that the input parameters are imprecise and represented by fuzzy numbers, the resulting outputs of the model will also be fuzzy numbers characterized by their membership functions.

2.2 Fuzzy Alpha-Cut (FAC) technique

An alpha-cut $A\alpha$ of a fuzzy number A is defined as the set $\{x \in R|A(x)\geq\alpha\}$. A is completely determined by the collection $(A\alpha)$ $\alpha\in[0, 1]$. An alpha cut is the degree of sensitivity of the system to the behavior under observation. At some point, as the information value diminishes, one no longer wants to be "bothered" by the data. In many systems, due to the inherent limitations of the mechanisms of observation, the information becomes suspect below a certain level of reliability.

Fuzzy alpha-cut technique is based on the extension principle, which implies that functional relationships can be extended to involve fuzzy arguments and can be used to map the dependent variable as a fuzzy set. In simple arithmetic operations, this principle can be used analytically. However, in most practical modeling applications, relationships involve complex structures (e.g. partial differential equations) that make analytical application of the principle difficult. Therefore, interval arithmetic can be used to carry out the analysis [3]. Membership functions define the degree of participation of an observable element in the set, not the desirability or value of the information.

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The membership function is cut horizontally at a finite number of α-levels between 0 and 1. For each α-level of the parameter, the model is run to determine the minimum and maximum possible values of the output. This information is then directly used to construct the corresponding fuzziness (membership function) of the output which is used as a measure of uncertainty. If the output is monotonic with respect to the dependent fuzzy variable/s, the process is rather simple since only two simulations will be enough for each α-level (one for each boundary). Otherwise, optimization routines have to be carried out to determine the minimum and maximum values of the output for each α-level.

2.3 Transformation Method (TM)

The TM presented by [6] uses a fuzzy alpha-cut approach based on interval arithmetic: the uncertain response reconstructed from a set of deterministic responses, combining the extreme of each interval in every possible way unlike the FAC technique where only a particular level of membership (α-level) values for uncertain parameters are used for simulation. In this study reduced TM has been used which will be explained. Given an arithmetic function f that depends on n uncertain parameters $x_1, x_2, x_3, \dots x_n$ represented as fuzzy numbers, the function output $q = f(x_1, x_2, x_3, \dots x_n)$is also a fuzzy number. Using the α-level technique, each input parameter is decomposed into a set Pi of m + 1 intervals $X_i^{(j)}, j = 0, 1, \dots, m$ where

$$p_i = \{X_i^{(0)}, X_i^{(1)}, \dots, X_i^{(m)}\} \quad (1)$$

With

$$X_i^{(j)} = [a_i^{(j)}, b_i^{(j)}],\ a_i^{(j)} \le b_i^{(j)}, i = 1,2, \dots n \quad j = 1,2, \dots m \quad (2)$$

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Where $a_i^{(j)}$ and $b_i^{(j)}$ denote the lower and upper bound of the interval at the membership level μ_j. Instead of applying interval arithmetic like FAC method, intervals are now transformed into arrays $\mathrm{X}_\mathrm{i}^{(\mathrm{j})}$ of the following form:

$$\hat{\mathrm{X}}_\mathrm{i}^{(\mathrm{j})} = \left(\overbrace{\alpha_i^{(j)}, \beta_i^{(j)}\ \alpha_i^{(j)}, \beta_i^{(j)}, \dots, \alpha_i^{(j)}, \beta_i^{(j)}}^{2^{\mathrm{i}-1}\ \text{Pairs}}\right) \tag{3}$$

$$\alpha_\mathrm{i}^{(\mathrm{j})} = \left(\overbrace{\mathrm{a}_\mathrm{i}^{(\mathrm{j})}, \dots, \mathrm{a}_\mathrm{i}^{(\mathrm{j})}}^{2^{\mathrm{i}-1}\ \text{Pairs}}\right), \beta_\mathrm{i}^{(\mathrm{j})} = \left(\overbrace{\mathrm{b}_\mathrm{i}^{(\mathrm{j})}, \dots, \mathrm{b}_\mathrm{i}^{(\mathrm{j})}}^{2^{\mathrm{i}-1}\ \text{Pairs}}\right) \tag{4}$$

The evaluation of function f is now carried out by evaluating the expression separately at each of the positions of the arrays using the conventional arithmetic. Result obtained is deterministic in decomposed and transformed form which can be retransformed to get fuzzy valued result using recursive approximation.

2.4 Monte-Carlo Simulation (MCS)

The MCS technique requires the use of parameter distribution for the input variable and results are given as a distribution, rather than a point value [7]. In this approach, the measured quantities are varied randomly in ways that represent the experimental uncertainty and variability, and the calculations leading to the final answer are repeated with these generated variables. This is done repeatedly, and the variance and covariance in the resulting final answers are calculated.

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3. Case study

A two-dimensional solute transport, with a continuous point source of pollution in a uniform flow field was studied. For this purpose, numerical solution for contaminant transport model for saturated pores media has been used. Such solution generally requires extreme simplifications, but the results can be used for approximate solutions. They are also very useful to illustrate the sensitivity of different parameters in overall uncertainty. A numerical model consisting of 40x30 nodal grids with a uniform grid spacing of 50 m in both directions was used to simulate the two-dimension solute transport using equation [8].

$$C_{i,j}^{n+1} = C_{i,j}^{n} + \Delta t\left(\left(\frac{\alpha_L V}{\Delta x^2} + \frac{V}{\Delta x}\right)\right) C_{i-1,j}^{n} - \left(2\frac{\alpha_L V}{\Delta x^2} + 2\frac{\alpha_T V}{\Delta y^2} + \frac{V}{\Delta x}\right) C_{i,j}^{n} + \left(\frac{\alpha_L V}{\Delta x^2} C_{i+1,j}^{n} + \frac{\alpha_T V}{\Delta y^2} C_{i,j-1}^{n} + \frac{\alpha_T V}{\Delta y^2} C_{i,j+1}^{n}\right) + \frac{M_{i,j}\Delta t}{\Delta x \Delta y \varepsilon b} \quad (5)$$

Where $C_{i,j}^{n}$ is the concentration of dissolved chemical (mg/L), V is seepage velocity in given an arithmetic function f that depends on n uncertain parameters $x_1, x_2, \dots x_n$ represented as fuzzy numbers, αL and αT are the longitudinal and transverse dispersion coefficients (m), respectively, b is thickness of aquifer (m), Ɛ is effective porosity, t is time increment (day), Δx and Δy are grid spacing in x and y direction respectively (m). Zero concentration boundaries were placed at the left, upper and lower model boundaries with a constant source placed at 750 m from surface. Characteristics of the uncertain parameters and other data used in the simulation are shown in Table 1 and Table 2 respectively.

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Table 1: Triangular fuzzy numbers for uncertain simulation

	Low 0 M-value	Medium 1 M-value	High 0 M-value
V(m/day)	0.3	0.6	0.1
α_L(m)	100	200	300
α_T(m)	20	4-0	60

Table 2: Other crisp input data used in simulation

Parameters	Value
Thickness of flow, b	50 m
Source strength, M	120kg/day
Effective porosity, p	0.17
Grid distance (Δx)	50 m
Grid distance (Δy)	50 m
Time increment	1day

The membership functions for input parameters that were used for the fuzzy techniques are shown on Figure 1.

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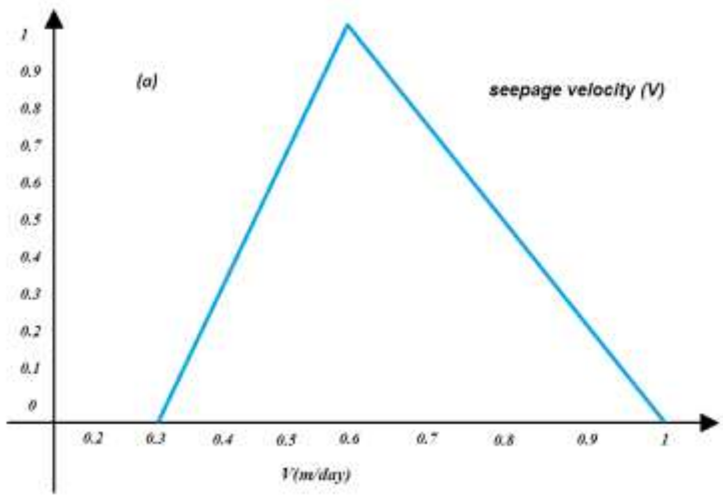
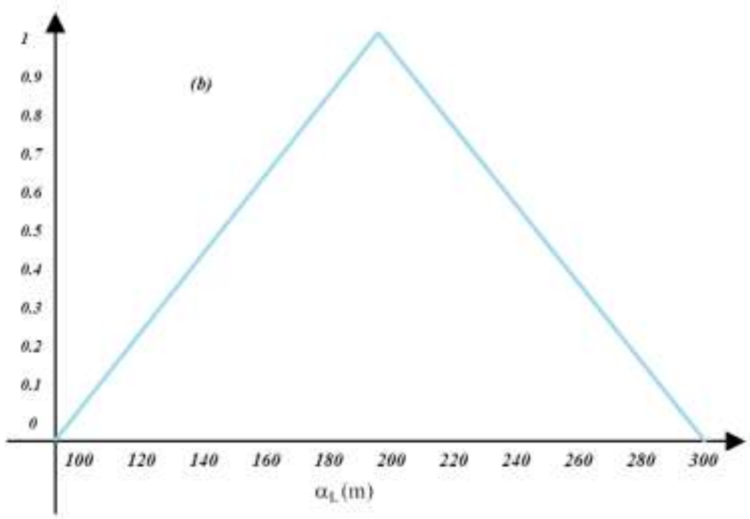


Figure 1: Membership functions of input parameters for 2D solute transport (a) seepage velocity (V)



(b) Longitudinal dispersivity (αL)

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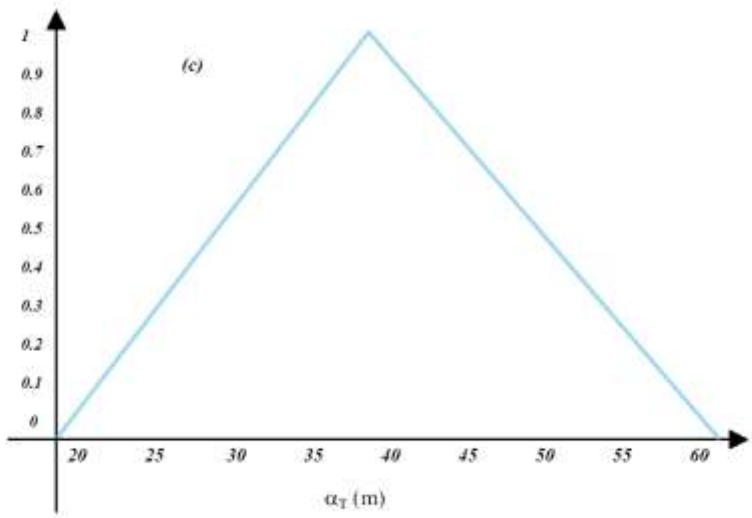

(c) Transverse dispersivity (αT)

4. Results and Discussion

Different fuzzy and Monte-Carlo analysis were carried out and comparative measures of uncertainty were devised for comparison of these methods. For analysis, the probability density function (for the MCS technique) and the membership function (for the Fuzzy techniques) of the output (concentration) were analyzed at a given point (600 m from the pollution point source). Similarly to evaluate the spatial distribution of uncertainty, the ratio of the standard deviation to the mean concentration of the solute at each grid cell in case of MCS has been compared with the ratio of the 0.1-level support to the value of the concentration for which the membership function is equal to 1 in case of FAC technique and overall influence in case of TM. The results of different methods and effect of different parameters on overall uncertainty using TM are shown in Table 3 and Table 4 respectively.

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TABLE 3: OVER ALL UNCERTAINTY OF DIFFERENT METHODS

Parameters	Uncertainty
MCS	0.1073
FAC	0.0917
TM	0.0917

TABLE 4: EFFECT OF UNCERTAINTY OF DIFFERENT PARAMETERS ON OVERALL UNCERTAINTY(TM).

Parameters	%Uncertainty
V	0.4526
αL	0.1425
αT	0.4049

Figure 2 shows the normalized probability distribution of the concentration obtained from the MCS and the fuzzy number representing the concentration obtained from the TM in the same set of axes. The width of the output membership function is the indication of the sensitivity of the model to uncertain parameters. In Figure 3, the cumulative distribution function and the normalized-integrated fuzzy number are plotted. All three methods has shown comparable results, however there is clear indication of more consistency in case of FAC and TM. The output from fuzzy methods agreed well with that from the Monte Carlo method (Fig 2 & 3), however there is obvious lack of consistency in case of MCS. The other drawback of the Monte-Carlo approach for the present application is its time-consuming character.

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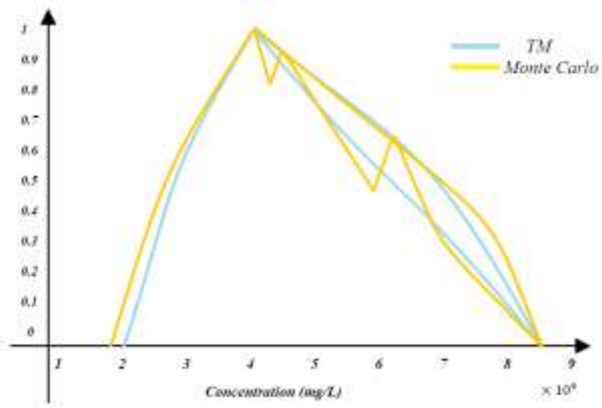

FIGURE 2: NORMALIZED PDF AND FUZZY MEMBERSHIP FUNCTION OF THE OUTPUT AT THE SELECTED POINT OF ANALYSIS.

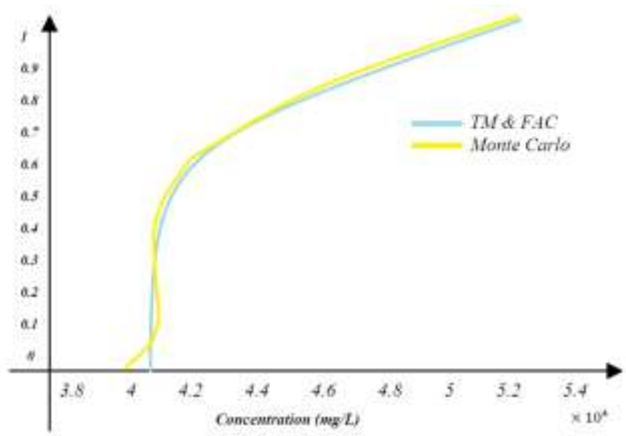

FIGURE 3: CDF AND INTEGRATED MEMBERSHIP FUNCTION

Conclusion

With regards to standard fuzzy methods, the serious drawback is the uncertainty of result for same problem depending on the form of solution procedure applied and widening of the fuzzy value set which is due to multi-occurrence of variables in function expression. TM is not dependent on solution procedure and can also prevent widening of the fuzzy value set.

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However this was first shown in vertex method [9] which also used the interval analysis but only suitable for uniform solution space and TM can be applied for both uniform and non-uniform solution space. In case of FAC technique, it requires less model runs compare to TM but it seriously lack the detail analysis of uncertainty, like sensitivity of different uncertain parameters, uncertainty at different membership levels. Also for non-monotonic problems, it lacks a clear procedure. We can safely conclude that fuzzy transformation method presents a strong alternative to the probabilistic and general fuzzy approach. A faster and accurate result in case of monotonic function and near proper result in case of non-monotonic can be achieved.

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