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Improved sliding mode control of a class of nonlinear systems: Application to quadruple tanks system

Hossein Nejatbakhsh Esfahani*, Seyed Mohammad Reza Sajadi

The Center of Excellence for Mechatronics, School of Engineering Emerging Technologies, University of Tabriz, Tabriz, Iran

*Corresponding Author's E-mail: *hnejatbakhsh.esf@gmail.com*

Abstract

In this paper, a stable adaptive sliding mode based on tracking control is developed for a class of nonlinear Multi Input Multi Output (MIMO) systems with external disturbances. In order to reduce the chattering phenomenon without deteriorating the tracking performance, the discontinuous term in the conventional sliding mode technique is replaced by an adaptive Proportional derivative (PD) term. The effect of the approximation error which arises from the PD term is reduced by adding a robust term in the proposed control law. All parameter, adaptive laws and the robust control term, are derived based on the Lyapunov stability analysis. The overall adaptive sliding mode scheme guarantees the convergence to zero of tracking errors and the boundedness of all signals in the closed-loop system. The proposed approach is applied to a quadruple tanks system and achieves satisfactory simulation results.

Keywords: sliding mode control, adaptive control, robust control, quadruple tank

1. Introduction

Several dynamical systems present, in addition to the external disturbances, nonlinearities and parametric perturbations. Therefore, the use of robust control is desirable. During recent years, many researchers have been interested by Sliding Mode Control (SMC), a robust control strategy for non linear systems [1]–[3]. Known for its simple structure, simple

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implementation and its robustness to external disturbances, the SMC has been a topic of great interest in control theory and represented a great potential for practical applications. However, it suffers from a main disadvantage: the chattering phenomenon, which is the high frequency oscillation of the controller output. In the literature, several methods of chattering reduction have been reported [4]–[7]. In [8], the boundary layers approach can reduce this phenomenon. This method consists in replacing the discontinuous switching action by a continuous saturation function. This approach is generally appropriate for low disturbances and it requires an approximation of the term of discontinuity.

Furthermore, in [9] an asymptotic observer can eliminate the chattering phenomenon. The application of such observer is only possible when the system is nonlinear and uncertain. To attain the same objective, another common method based on the high order SMC can be elaborated [10]. However, this method requires a complex calculation. Another way to solve this chattering problem is based on combining intelligent controllers and SMC: a fuzzy neural network [11], [12], a fuzzy system to approximate the switching control term. The free parameters of the adaptive fuzzy controller can be tuned on-line based on the Lyapunov approach [13], [14]. In [15], the authors proposed a method to eliminate the chattering phenomenon by using an adaptive Proportional Integral controller for a SISO nonlinear system. The multi-input multi-output (MIMO) nonlinear systems are investigated in [1], [4].

In this paper, a sliding mode control is developed for a class of nonlinear MIMO system. Our objective is to reduce the chattering phenomenon, so we decided to combine the sliding mode approach with an adaptive proportional derivative controller (PD controller). The sections of this paper are organized as follows. In section 2, we present the class of nonlinear MIMO disturbed system. In section 3, we introduce the adaptive sliding mode control which combines the SMC mode with an adaptive proportional derivative controller and an integral surface in order to reduce the chattering phenomenon. The simulation results of quadruple

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tanks are given to show the effectiveness of the proposed control strategies. Finally, section 4 gives a conclusion on the main works developed in this paper.

2. Context and Formulation

Consider a class of nonlinear, Multi-Input Multi-Output (MIMO), disturbed system:

$$\begin{cases} y_1^{(n_1)} = f_1(x) + \sum_{j=1}^{p} g_{1j}(x)u_j + d_1 \\ y_2^{(n_2)} = f_2(x) + \sum_{j=1}^{p} g_{2j}(x)u_j + d_2 \\ \vdots \\ y_p^{(n_p)} = f_p(x) + \sum_{j=1}^{p} g_{pj}(x)u_j + d_p \end{cases} \tag{1}$$

Where

$x = \left[y_1, \dots, y_1^{(n_1-1)}, \dots, y_p, \dots y_p^{(n_p-1)}\right]^T = [x_1, x_2, \dots, x_n]^T$ is the state vector;

$U = \left[u_1 \dots u_p\right]^T$ is the input vector;

$Y = \left[y_1 \dots y_p\right]^T$ is the output vector;

$f_i(x)$ and $g_{ij}(x)$ are nonlinear functions;

$d = \left[d_1\ d_2 \dots d_p\right]^T$ is the vector of external disturbances such

$|d_i| \leq D_i;\ \forall i = 1 \dots p$.

We define:

$G(x) = [g_{ij}]$, an invertible matrix, $\forall x\ 1 \leq i, j \leq p F(x) = \left[f_1(x) \dots f_p(x)\right]^T$ and $y^{(n)} = \left[y_1^{(n_1)}, \dots, y_p^{(n_p)}\right]^T$.

Let us consider desired trajectories $y_{d_i}(t), \forall i = 1 \dots p$ that are known bounded functions of time with bounded known derivatives and are assumed to be ri-time differentiable. We define the tracking errors:

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$$e_i = y_i - y_{d_i}, \forall i = 1 \dots p \tag{2}$$

The system (1) can be written in a compact form as:

$$y^{(n)} = F(x) + G(x)U + d \tag{3}$$

Our objective is to develop a control low allowing the output of the system y to follow a given signal y_{d_i}despite the external disturbances. Based on the classical sliding mode, we noticed that the presence of the signum function in the control law leads to the chattering phenomenon which can excite the high frequency dynamics. In order to reduce this phenomenon and to achieve the control objective, an adaptive PD term is used in the control law with an integral surface.

3. Adaptive Sliding Mode Control

To develop the adaptive sliding mode approach for the MIMO system, two steps are required. First, the choice of sliding surface and second the calculation of the control law.

A. The sliding surface

The integral sliding surface is defined by the following expression:

$$s_i = e_i^{(r-1)} + \sum_{j=2}^{r-1} \alpha_{r-j}^i e^{(r-j)} + k_i \int_0^t e_i(\tau) d\tau \tag{4}$$

where

$\alpha_{(r-1)}^i = 1, i = 1 \dots p$

r represents the relative degree of the system. In fact, the sliding variable has a relative degree equal to one compared to the control law. This implies that the control law

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appears explicitly in the derivative of the sliding surface. Denote the sliding surface vector:

$$S = \left[s_1, \dots, s_p\right]^T \tag{5}$$

The parameters $\alpha^i_{r-2}, \dots, k_i$ are chosen such that all roots of $h_i(p) = p^{(r-1)} + \alpha^i_{(r-2)} p^{(r-2)} + \dots + \alpha^i_1 p + \alpha^i_0 + k_i$ are in the left half plane.

B. The Control Law

The control law is designed as:

$$U = u_{eq} + u_{sw} \tag{6}$$

The equivalent control law u_{eq} is determined by$\dot{S} = 0$. The time derivative of S is given by:

$$\dot{S} = F(x) + G(x)U - Y_d^{(r)} - \sum_{j=2}^{r} \gamma_{j-2} E^{(j-1)} + d \tag{7}$$

where

$$Y_d^{(r)} = \left[Y_{d_1}^{(r)} \dots Y_{d_p}^{(r)}\right]^T \tag{8}$$

$$\gamma_j = diag\left[\alpha_{1j} \dots \alpha_{pj}\right] \tag{9}$$

$$\gamma_0 = diag\left[k_1 \dots k_p\right] \tag{10}$$

$$E^{(j)} = \left[e_1^{(j)} \dots e_p^{(j)}\right]^T \tag{11}$$

The equivalent control term is defined by the following expression:

$$u_{eq} = G^{-1}(x)\left[-F(x) + Y_d^{(r)} - \sum_{j=2}^{r} \gamma_{j-2}\, E^{(j-1)}\right] \tag{12}$$

The switching control term is defined by:

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$$u_{sw} = G^{-1}\begin{pmatrix} -\eta sgn(s_1) \\ \cdots \\ -\eta sgn(s_p) \end{pmatrix}, \eta > 0 \tag{13}$$

sgn is the signum function. The presence of the signum function in the term u_{sw} in the classical sliding mode technique leads to the chattering phenomenon, which can excite the high frequency dynamics. To avoid this problem and to achieve the previous control objectives, an adaptive PD term is used. The expression of the proportional derivative term is written as follows:

$$u_{PD} = \begin{bmatrix} k_{p_1} s_1(t) + k_{d_1} \frac{d}{dt} s_1(t) \\ \vdots \\ k_{p_p} s_1(t) + k_{d_p} \frac{d}{dt} s_p(t) \end{bmatrix} \tag{14}$$

Where

k_{p_j} and k_{d_j}, $j = 1 \cdots p$ are the control gains adjusted online from an adaptive law. The adaptive PD term derived from (14) can be rewritten as:

$$u_{PD} = \hat{\rho}(S|\theta_\rho) = \left[\hat{\rho}_1(s_1|\theta_{\rho_1}) \cdots \hat{\rho}_p(s_p|\theta_{\rho_p})\right]^T \tag{15}$$

$$u_{PD} = \theta_\rho^T \emptyset(s) = \left[\theta_{\rho_j}^T \emptyset(s_1) \cdots \theta_{\rho_p}^T \emptyset(s_p)\right]^T \tag{16}$$

Where

θ_{ρ_j} is the adjustable parameters vector given by $\theta_{\rho_j} = \left[k_{p_j} \;\; k_{d_j}\right]^T$ and $\emptyset^T(s_j) = \left[s_j(t) \frac{d}{dt} s_j(t)\right]$ are the regressive vectors, $j = 1 \dots p, \theta_\rho = \left[\theta_{p_1}^T \cdots \theta_{p_p}^T\right]^T, \emptyset(s) = diag\left[\emptyset^T(s_1) \cdots \emptyset^T(s_p)\right]$.

Let us define the following variables:

$$\theta_{\rho_j}^* = argmin\left(\sup\left|\hat{\rho}\left(S\middle|\theta_{\rho_j}\right) - \eta sgn(s_i)\right|\right), \eta > 0$$

$$\theta_{\rho_j} \in \Omega_{\rho_j} \quad s \in \Re^n \tag{17}$$

where Ω_{ρ_j} denotes the set of suitable bound on $\theta_{\rho_j}^*$ and $\eta sgn(s_i)$ is the discontinuous term of the classical sliding mode control. The parameter approximation error:

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$$\tilde{\theta}_{\rho_j} = \theta^*_{\rho_j} - \theta_{\rho_j} \quad (18)$$

The minimum approximation error is given by the following expression:

$$\omega_{j_{PD}} = \hat{\rho}_j\left(S\middle|\theta^*_{\rho_j}\right) - \eta sgn(s_j) \quad (19)$$

$$\omega_{PD} = \left[\omega_{1_{PD}} \cdots \omega_{p_{PD}}\right] \quad (20)$$

To achieve the control objective, we need to establish a control law that forces the trajectories of system status to reach and remain on the sliding surface despite the presence of external disturbances. We suggest adding terms of robustness $u_1$and u_2 in order to cancel the effect of the error of approximation. The control law is written as follows:

$$U = G^{-1}(x)\left[-F(x) + Y_d^{(r)} - \sum_{j=2}^{r} \gamma_{j-2}\, E^{(j-1)} - u_{PD} + u_1 + u_2\right] \quad (21)$$

where

$$\begin{cases} u_{PD} = \theta_\rho^T \emptyset(s) \\ u_1 = \widehat{\omega}_{1_{PD}} \\ u_2 = \widehat{\omega}_{2_{PD}} \end{cases} \quad (22)$$

and $\widehat{\omega}_{1,2_{PD}}$ is the estimated of $\omega_{1,2_{PD}}$ to be determined yet. The parameter vector θ_ρ is adjusted online by the following adaptive laws:

$$\dot{\theta}_\rho = -\delta_\rho S \emptyset(S) \quad (23)$$

$$\dot{\hat{\omega}}_{PD} = -\delta_{PD} S \quad (24)$$

where $\delta_\rho > 0$and $\delta_{PD} > 0$ are the adaptation gains. The main result of the improved sliding mode control proposed is summarized in the following theorem:

Theorem 2.1: Consider the class of MIMO nonlinear systems (3), if the control law (21) is applied, where the terms u_{PD}, u_1 and u_2 are respectively given by (16) and (22).

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The parameters θ_ρ and $\widehat{\omega}_{1,2_{PD}}$ are respectively adjusted on-line by applying the adaptation laws (23) and (24) then, the proposed control scheme guarantees the following properties:

(i) The signals of the closed-loop system are bounded;

(ii) The tracking errors converge to zero;

Proof. Let us consider the following Lyapunov function:

$$V = \frac{1}{2}S^T S + \frac{1}{2\delta_\rho}\tilde{\theta}_\rho^T\tilde{\theta}_\rho + \frac{1}{\delta_{PD}}(\widetilde{\omega}_{PD}^T\widetilde{\omega}_{PD}) \quad (25)$$

We define:

$$\widetilde{\omega}_{PD} = \omega_{PD} - \widehat{\omega}_{PD} \quad (26)$$

$$\dot{S} = u_1 + u_2 - u_{PD} + d \quad (27)$$

knowing that:

$$\begin{cases} \dot{\tilde{\theta}}_\rho = -\dot{\theta}_\rho \\ \dot{\widetilde{\omega}}_{PD} = -\dot{\widehat{\omega}}_{PD} \end{cases} \quad (28)$$

The time derivative of V is given by:

$$\dot{V} = S^T\dot{S} + \frac{1}{\delta_\rho}\tilde{\theta}_\rho^T\dot{\tilde{\theta}}_\rho + \frac{1}{\delta_{PD}}(\widetilde{\omega}_{PD}^T\dot{\widetilde{\omega}}_{PD}) \quad (29)$$

By substituting (27), (28) in (29), we obtain:

$$\dot{V} = S^T(u_1 + u_2 - u_{PD} + d) - \frac{1}{\delta_\rho}\tilde{\theta}_\rho^T\dot{\theta}_\rho - \frac{1}{\delta_{PD}}(\widetilde{\omega}_{PD}^T\dot{\widehat{\omega}}_{PD})$$

$$\dot{V} = S^T u_1 + S^T u_2 - S^T\hat{\rho}(S|\theta_\rho) + S^T\hat{\rho}(S|\theta_\rho^*) - S^T\hat{\rho}(S|\theta_\rho^*) - \frac{1}{\delta_\rho}\tilde{\theta}_\rho^T\dot{\theta}_\rho - \frac{1}{\delta_{PD}}(\widetilde{\omega}_{PD}^T\dot{\widehat{\omega}}_{PD}) + S^T d$$

$$\dot{V} = S^T u_1 + S^T u_2 - S\tilde{\theta}_\rho^T\emptyset(S) - S^T\hat{\rho}(S|\theta_\rho^*) - \frac{1}{\delta_\rho}\tilde{\theta}_\rho^T\dot{\theta}_\rho - \frac{1}{\delta_{PD}}(\widetilde{\omega}_{PD}^T\dot{\widehat{\omega}}_{PD}) + S^T d \quad (30)$$

$$\dot{V} = \tilde{\theta}_\rho^T\left(S\emptyset(s) - \frac{1}{\delta_\rho}\dot{\theta}_\rho\right) + S\widehat{\omega}_{1_{PD}} + S\widehat{\omega}_{2_{PD}} - S^T\hat{\rho}(S|\theta_\rho^*) - \frac{1}{\delta_{PD}}(\widetilde{\omega}_{PD}^T\dot{\widehat{\omega}}_{PD}) + S^T d$$

$$\dot{V} = \tilde{\theta}_\rho^T\left(S\emptyset(s) - \frac{1}{\delta_\rho}\dot{\theta}_\rho\right) + S(\widehat{\omega}_{1_{PD}} + \widehat{\omega}_{2_{PD}} - (\omega_{PD} - u_{PD})) - \frac{1}{\delta_{PD}}(\widetilde{\omega}_{PD}^T\dot{\widehat{\omega}}_{PD}) + S^T d$$

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By substituting (23), (24) in (30), we get:

$$\dot{V} \leq -\eta S^T sign(S) + S^T d \quad (31)$$

$$\dot{V} \leq -\eta S^T sign(S) + |S^T| D \quad (32)$$

$$\dot{V} \leq -|S|(\eta - D) \quad (33)$$

$$\dot{V} \leq -\eta |S| < 0, \forall\, \eta > D \quad (34)$$

Finally, the expression (34) proves that the semi global asymptotic stability and the robustness of the closed loop system are guaranteed. Moreover, S will converge to zero.

C. Simulation Results

In order to illustrate the previous concepts, let us consider a liquid level control of quadruple tanks system shown in Fig. 1.

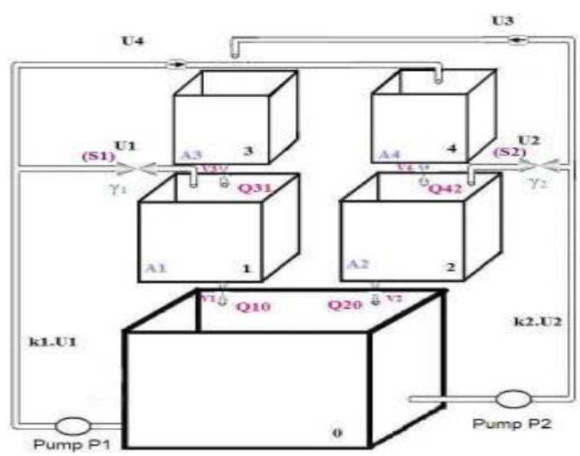

Fig.1: Schematic diagram of the quadruple tank system

The quadruple tank system consists of two double tanks (1, 2, 3 and 4) and a reservoir with different sections connected to each other by cylindrical tubes. Tanks 1and 2 are mounted below the other two tanks for receiving water flow by gravity. The

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reservoir supplies the four tanks through two pumps P1 and P2 with variable speed. Discharge from pump P1 is split between tank 1 and tank 4. Similarly, pump P2 splits its discharge between tank 2 and tank 3. Split of flow from P1 and P2 can be varied by manual adjustment of valves S1 and S2. Tank 1 and tank 2 also receive gravity flow from tank 3 and tank 4, respectively. Opening of these valves (V1, V2, V3 and V4), and the flow split valves (S1 and S2) can be manually adjusted to substantially alter the characteristics of the system. Achieving desired levels in tank 1 and tank 2 is the control objective. The dynamical equation for each tank is used to write the mathematical model of the system. In fact, we can adopt the Johansson model equations by neglecting the pressure drop in the piping compared to the pressure drop in the valves:

- The process has two inputs, flow from P1and P2. These are set by signal inputs u1 and u2;
- There are four levels (h1, h2, h3 and h4) that are measured, transmitted and are available on-line;
- Ai the cross-section of tank i, i = (1, 2, 3, 4) ;
- Qij the variation of the flow between the tank i and j, i = (1, 2, 3, 4) et j = (0, 1, 2);
- ai open cross-section of the outlet line valve Vi;
- kiui the water flow rate in pump i;
- γi the flow area of the valve Si ;
- g gravitational constant.

The general model of the quadruple tanks system can be written as follows:

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$$\begin{cases} A_1\frac{dh_1}{dt} = \delta_1 k_1 u_1 + Q_{31} - Q_{10} \\ A_2\frac{dh_2}{dt} = \delta_2 k_2 u_2 + Q_{42} - Q_{20} \\ A_3\frac{dh_3}{dt} = (1-\delta_2) k_2 u_2 - Q_{31} \\ A_4\frac{dh_4}{dt} = (1-\delta_1) k_1 u_1 + Q_{42} \end{cases} \tag{35}$$

Applying Torricelli's law, we have:$Q_{10} = a_1\sqrt{2gh_1}$, $Q_{20} = a_2\sqrt{2gh_2}$, $Q_{31} = a_3\sqrt{2gh_3}$and$Q_{42} = a_4\sqrt{2gh_4}$. The dynamic model of the system can be written:

$$\begin{cases} A_1\frac{dh_1}{dt} = \delta_1 k_1 u_1 + a_3\sqrt{2gh_3} - a_1\sqrt{2gh_1} \\ A_2\frac{dh_2}{dt} = \delta_2 k_2 u_2 + a_4\sqrt{2gh_4} - a_4\sqrt{2gh_4} \\ A_3\frac{dh_3}{dt} = (1-\delta_2) k_2 u_2 - a_3\sqrt{2gh_3} \\ A_4\frac{dh_4}{dt} = (1-\delta_1) k_1 u_1 + a_4\sqrt{2gh_4} \end{cases} \tag{36}$$

The model (36) can also be written as:

$$\begin{cases} \dot{h}_1 = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\delta_1 k_1}{A_1}u_1 \\ \dot{h}_2 = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\delta_2 k_2}{A_2}u_2 \\ \dot{h}_3 = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\delta_2)k_2}{A_3}u_2 + d_2^1 \\ \dot{h}_4 = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\delta_1)k_1}{A_4}u_1 + d_2^2 \end{cases} \tag{37}$$

We apply the concept of feedback linearization of above model equation for output $y_1 = h_1 = x_1^1$and$y_2 = h_2 = x_1^2$:

$$\dot{y}_1 = \dot{x}_1^1 = \frac{(-a_1\sqrt{2gh_1} + a_3\sqrt{2gh_3} + \delta_1 k_1 u_1)}{A_1} = x_2^1 \tag{38}$$

$$\dot{x}_2^1 = \ddot{h}_1 = \left(\frac{a_1^2}{A_1^2} - \frac{a_1 a_3 g\sqrt{h_3}}{A_1^2\sqrt{h_1}} - \frac{a_3^2 g}{A_1 A_3}\right) + \left(-\frac{a_1 g \delta_1 k_1}{A_1^2\sqrt{2gh_1}}\right)u_1 + \left(-\frac{a_3 g k_2}{A_1 A_3\sqrt{2gh_3}}(1-\delta_2)\right)u_2 + d_2^1 \tag{39}$$

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$$x_2^2 = \dot{h}_2 = \left(\frac{-a_2\sqrt{2gh_2}+a_4\sqrt{2gh_4}+\delta_2 k_2 u_2}{A_3}\right) \tag{40}$$

$$\dot{x}_2^2 = \ddot{h}_2 = \left(\frac{a_2^2 g}{A_2^2} - \frac{a_2 a_4 g\sqrt{h_4}}{A_2^2\sqrt{h_2}} - \frac{a_4^2 g}{A_2 A_4}\right) + \left(\frac{a_4 g(1-\delta_1)k_1}{A_2 A_4\sqrt{2gh_4}}\right)u_1 + \left(\frac{-a_2 g\delta_2 k_2}{A_2^2\sqrt{2gh_2}}\right)u_2 + d_2^2 \tag{41}$$

The dynamic model of the system can be written in a compact form as:

$$\dot{x} = Ax + B(F + Gu) + d \tag{42}$$

$\dot{x} = \left[\dot{x}_1^1\ \dot{x}_2^1\ \dot{x}_1^2\ \dot{x}_2^2\right]^T, x = [x_1^1\ x_2^1\ x_1^2\ x_2^2]^T, u = [u_1\ \ u_2\ ]^T, d = [0\ d_2^1\ 0\ d_2^2]^T,$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, F = \begin{pmatrix} \frac{a_1^2 g}{A_1^2} - \frac{a_1 a_3 g\sqrt{h_3}}{A_1^2\sqrt{h_1}} - \frac{a_3^2 g}{A_1 A_3} \\ \frac{a_2^2}{A_2^2} - \frac{a_2 a_4 g\sqrt{h_4}}{A_2^2\sqrt{h_2}} - \frac{a_4^2 g}{A_2 A_4} \end{pmatrix}, G = \begin{pmatrix} \frac{-a_1 g\delta_1 k_1}{A_1^2\sqrt{2gh_1}} & \frac{a_3 g(1-\delta_2)k_2}{A_1 A_3\sqrt{2gh_3}} \\ \frac{a_4 g(1-\delta_1)k_1}{A_2 A_4\sqrt{2gh_4}} & \frac{-a_2 g\delta_2 k_2}{A_2^2\sqrt{2gh_2}} \end{pmatrix}$$

The parameter values of the quadruple tanks process are presented in the following table: [16]

Table 1: Values of the Parameters of the Quadruple Tanks

The cross-section of tank i: Ai, i = (1, 2, 3, 4)	$138.9\,cm^2$
Open cross-section of the outlet line valve Vi, i = (1, 2, 3, 4)	$0.5026\,cm^2$
Gravitational constant: g	$981\,cm/s^2$
The flow area of the valve S1: γ_1	0.42
The flow area of the valve S2: γ_2	0.34
k_1	27.43
k_2	19.55

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The objective of the control scheme is to adjust the outputs $y_1 = h_1$and$y_2 = h_2$to a desired value H . The simulation results are given by figures 2-7.

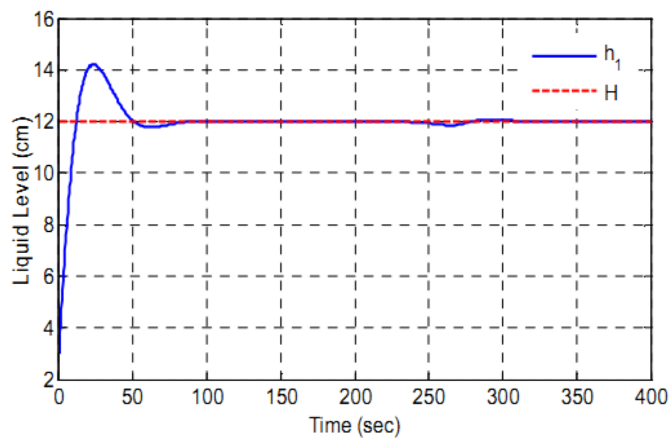

Fig.2: The trajectories of liquid level h_1 and desired liquid level H

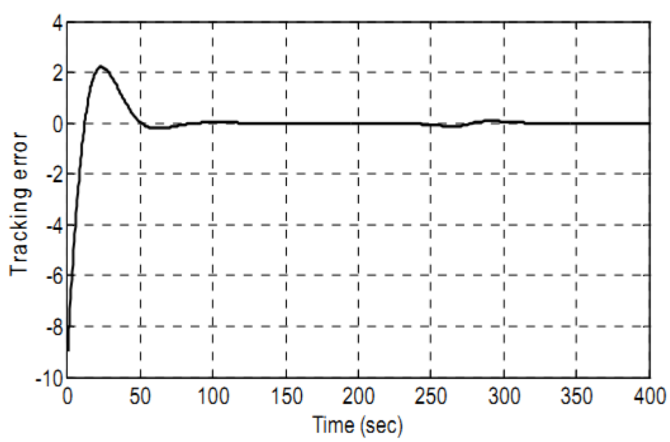

Fig.3: Evolution of the tracking error e_1

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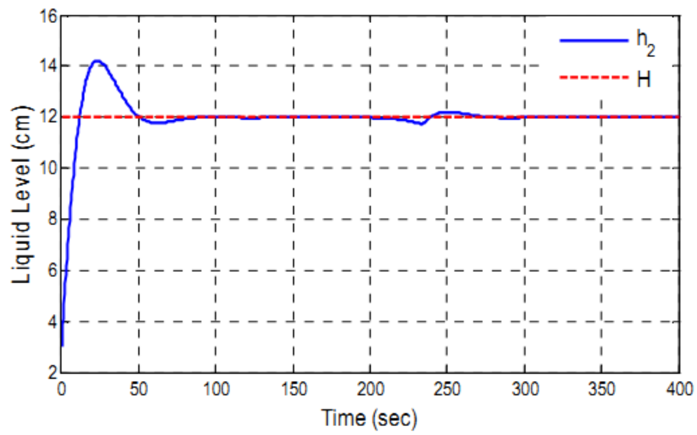

Fig.4: The trajectories of liquid level h_2 and desired liquid level H

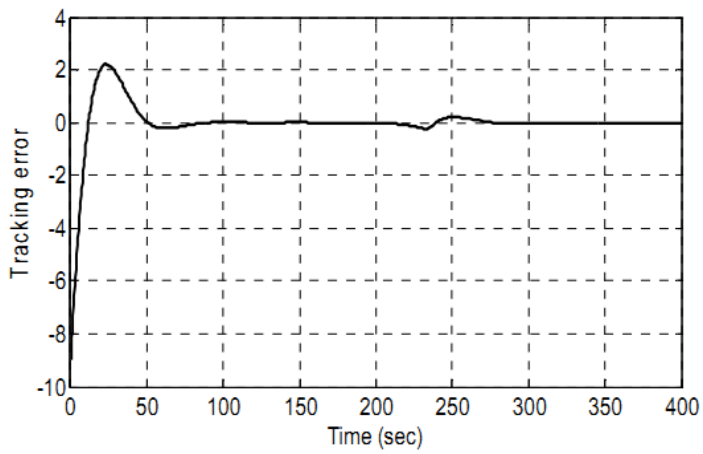

Fig.5: Evolution of the tracking error e_2

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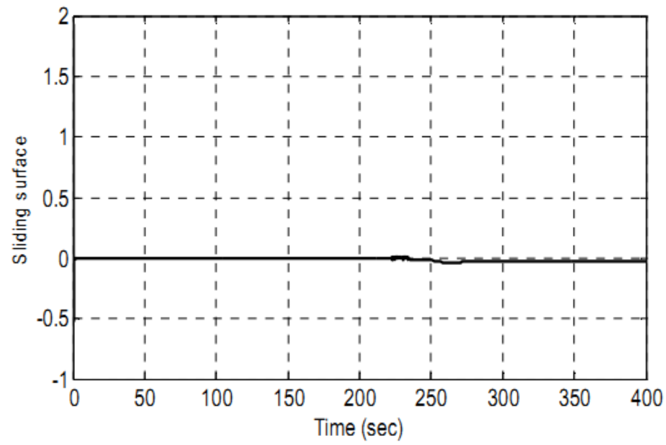

Fig.6: Evolution of the sliding surface s_1

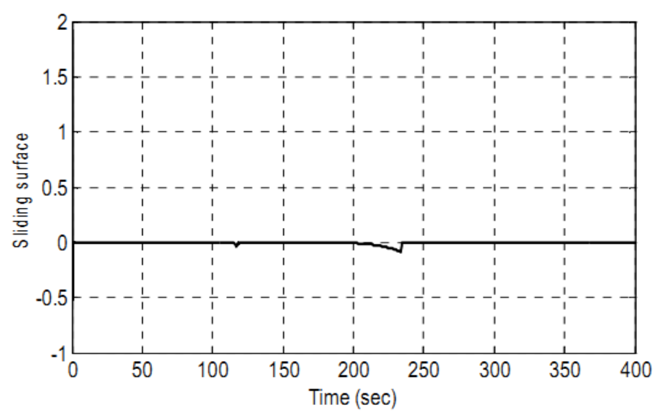

Fig.7: Evolution of the sliding surface s_2

The simulation results confirm that the adaptive proportional derived controller has good performance of trajectory tracking. We notice from Fig. 2 and 4 that the outputs of the system follow the reference signal H, the attenuation of the rapprochement phase and the reduction of the chattering phenomenon. The evolution of the sliding surfaces s1 and s2 trajectories are shown in Fig. 6 and 7. The convergence to zero of the system proves that the attractiveness of the sliding surface is guaranteed.

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Conclusion

In this paper, sliding mode controller is developed for a class of nonlinear multi-input multi-output disrupted systems. In order to overcome the chattering problem and to ensure the tracking of desired trajectories, we proposed to combine an adaptive PD controller into a sliding mode. Based on the Lyapunov stability approach, the proposed adaptive sliding mode control scheme has guaranteed the global stability and the robustness of the closed loop system with respect to disturbance. The simulation results of the quadruple tanks system shows the effectiveness of the proposed control method and good performances comparing to the other recent methods of SMC proposed in the literature.

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