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A New Method for Ranking Fuzzy Numbers with Using TRD Distance Based on mean and standard deviation

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Abstract

In this paper, we want to propose a new method for ranking trapezoidal fuzzy numbers with using TRD distance based on mean and standard deviation. In this method, used mean and standard deviation in membership function and inverse function for ranking fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

Keywords: Mean, Standard Deviation , Trapezoidal Fuzzy Numbers, TRD distance, Ranking Method.

1. Introduction

In most of cases in our life, the data obtained for decision making are only approximately known. In1965, Zadeh [27] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [9]. Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. Most of the ranking procedures proposed so far in the literature cannot discriminate fuzzy quantities and some are counterintuitive. As fuzzy numbers are represented by possibility distributions, they may overlap with each other, and hence it is not possible to order them. Ranking fuzzy numbers were first proposed by Jain [11] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Since then,

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various procedures to rank fuzzy quantities are proposed by various researchers. Bortolan and Degani [6] reviewed some of these ranking methods [5, 10, 11, 16, 17] for ranking fuzzy subsets and fuzzy numbers. Chen [3] presented ranking fuzzy numbers with maximizing set and minimizing set. Dubois and Prade [10] presented the mean value of a fuzzy number. Chu and Tsao [6] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu [7] presented a centroid-index method for ranking fuzzy numbers. Liang et al. [13] and Wang and Lee [25] also used the centroid concept in developing their ranking index. Chen and Chen [4] presented a method for ranking generalized trapezoidal fuzzy numbers.

Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α–levels of trapezoidal fuzzy numbers. Chen and Chen [5] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads and Parandian [15] proposed a method for ranking numbers by using distance between convex combination of upper and lower central gravity of α-level and the origin of coordinate systems. Also Some of the interesting metric distance method Of Trapezoidal Fuzzy Number can be found in Chen [12]. Rezvani ([16],[22]) evaluated the system of ranking fuzzy numbers. Moreover, Rezvani [22] proposed a ranking trapezoidal fuzzy numbers based on apex ngles.

In this paper, we want to propose a new method for ranking trapezoidal fuzzy numbers with using TRD distance based on mean and standard deviation. In this method, used mean and standard deviation in membership function and inverse function for ranking fuzzy numbers . For the validation the results of the proposed approach are compared with different existing approaches.

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2. Preliminaries

Definition 1. Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions,

i) μ_A is a continuous mapping from R to the closed interval $[0,1]$,

ii) $\mu_A(x) = 0$, $-\infty < x \le a$,

iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a,b]$,

iv) $\mu_A(x) = w$, $b \le x \le c$,

v) $\mu_A(x) = R(x)$ is strictly increasing on $[c,d]$,

vi) $\mu_A(x) = 0$, $d \le x < \infty$

where $0 < w \le 1$ and a,b,c and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by

$$A = (a,b,c,d;w). \tag{1}$$

When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by

$$A = (a,b,c,d). \tag{2}$$

$A = (a,b,c,d;w)$ is a fuzzy set of the real line R whose membership function $\mu_A(x)$ is defined as

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$$\mu_A(x)=\begin{cases} w\,\dfrac{x-a}{b-a} & if \quad a\le x\le b \\ w & if \quad b\le x\le c \\ w\,\dfrac{d-x}{d-c} & if \quad c\le x\le d \\ 0 & otherwise \end{cases} \tag{3}$$

Now, let two monotonic functions be

$$L(x)=w\,\frac{x-a}{b-a}, \qquad R(x)=w\,\frac{d-x}{d-c} \tag{4}$$

Then the inverse functions of function L and R are L^{-1} and R^{-1} respectively.

$$L^{-1}(x)=\frac{x}{w}(b-a)+a, R^{-1}(x)=d-\frac{x}{w}(d-c) \tag{5}$$

Definition 2. The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number A

$$I(A)=\int_0^1 [CL_A(\alpha)+(1-C)R_A(\alpha)]d\alpha \tag{6}$$

And

$$D(A)=\int_0^1 [R_A(\alpha)-L_A(\alpha)]f(\alpha)d\alpha \tag{7}$$

Here $0\le C\le 1$ denotes an ”optimism/pessimism” coefficient in conducting operations on fuzzy numbers. The function $f(\alpha)$ is nonnegative and increasing function on [0, 1] with $f(0)=0, f(1)=1$ and $\int_0^1 f(\alpha)d\alpha=\frac{1}{2}$. The function $f(\alpha)$ is also called weighting function. In actual applications, function $f(\alpha)$ can be chosen according to the actual situation. In this article, in practical case, we assume that $f(\alpha)=\alpha$.

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Definition 3. *TRD* distance between the exponential fuzzy number *A* as following

$$TRD(A)=\sqrt{[I(A)]^2+[D(A)]^2} \tag{8}$$

Definition 4. A trapezoidal fuzzy number $A=(a,b,c,d)$ can be approximated as a symmetry fuzzy number $S[\mu,\sigma]$, μ denotes the mean of *A*, σ denotes the standard deviation of A, and the membership function of A is defined as follows :

$$f_A(x)=\begin{cases}\dfrac{x-(\mu-\sigma)}{\sigma} & if \quad \mu-\sigma\le x\le\mu\\ \dfrac{(\mu+\sigma)-x}{\sigma} & if \quad \mu\le x\le\mu+\sigma\end{cases} \tag{9}$$

Where μ and σ are calculated as follows :

$$\sigma=w\,\frac{2(d-a)+c-b}{4} \tag{10}$$

$$\mu=w\,\frac{a+b+c+d}{4} \tag{11}$$

The inverse functions g^L and g^R of f^L and f^R respectively, are shown as follows :

$$g^L(x)=(\mu-\sigma)+\sigma x \tag{12}$$

$$g^R(x)=(\mu+\sigma)-\sigma x \tag{13}$$

3. Proposed Approach

We know of definition 4. that

$$f_A(x)=\begin{cases}\dfrac{x-(\mu-\sigma)}{\sigma} & if \quad \mu-\sigma\le x\le\mu\\ \dfrac{(\mu+\sigma)-x}{\sigma} & if \quad \mu\le x\le\mu+\sigma\end{cases}$$

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So

$$f^{L} = \frac{x-(\mu-\sigma)}{\sigma}$$

And

$$f^{R} = \frac{(\mu+\sigma)-x}{\sigma}$$

Theorem 1. The values constitute of weighted averaged representative and weighted width, of the exponential fuzzy number A are following

$$I(A) = \frac{C}{\sigma}[1-2\mu] + \frac{1}{\sigma}[\mu+\sigma-\frac{1}{2}] \tag{14}$$

And

$$D(A) = \frac{3\mu-2}{3\sigma} \tag{15}$$

Proof:

$$I(A) = \int_0^1 [CL_A(\alpha)+(1-C)R_A(\alpha)]d\alpha = \int_0^1 [C(\frac{\alpha-(\mu-\sigma)}{\sigma})+(1-C)(\frac{(\mu+\sigma)-\alpha}{\sigma})]d\alpha$$

$$= C[\frac{1}{2\sigma}+\frac{\sigma-\mu}{\sigma}]+(1-C)[\frac{\mu+\sigma}{\sigma}-\frac{1}{2\sigma}] = \frac{C}{\sigma}[1-2\mu]+\frac{1}{\sigma}[\mu+\sigma-\frac{1}{2}].$$

And

$$D(A) = \int_0^1 [R_A(\alpha)-L_A(\alpha)]f(\alpha)d\alpha = \int_0^1 [\frac{(\mu+\sigma)-\alpha}{\sigma}-\frac{\alpha-(\mu-\sigma)}{\sigma}]f(\alpha)d\alpha$$

$$= \int_0^1 [\frac{(\mu+\sigma)-\alpha}{\sigma}-\frac{\alpha-(\mu-\sigma)}{\sigma}]\alpha d\alpha = [\frac{\mu+\sigma}{2\sigma}-\frac{1}{3\sigma}]-[\frac{1}{3\sigma}+\frac{\sigma-\mu}{2\sigma}] = \frac{3\mu-2}{3\sigma}$$

Theorem 2. *TRD* distance between the exponential fuzzy number A as following

$$TRD(A) = \sqrt{[\frac{C}{\sigma}[1-2\mu]+\frac{1}{\sigma}[\mu+\sigma-\frac{1}{2}]]^2+[\frac{3\mu-2}{3\sigma}]^2} \tag{16}$$

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Definition 5. Let $A_1 = (a_1, b_1, c_1, d_1; w_1)$ and $A_2 = (a_2, b_2, c_2, d_2; w_2)$ be two fuzzy numbers, then

i) If $TRD(A_1) < TRD(A_2)$, then $A_1 < A_2$,

ii) If $TRD(A_1) > TRD(A_2)$, then $A_1 > A_2$,

iii) If $TRD(A_1) \approx TRD(A_2)$, then $A_1 \approx A_2$.

3.1. Method to find the values of $TRD(A)$ and $TRD(B)$

Let $A = (a_1, b_1, c_1, d_1; w_1)$ and $B = (a_2, b_2, c_2, d_2; w_2)$ be two fuzzy numbers, then use the following steps to find the values of $TRD(A)$ and $TRD(B)$

∗ Step 1

Find σ_A , σ_B and μ_A , μ_B of formula (10), (11)

∗ Step 2

Find I_A and I_B

∗ Step 3

Find D_A and D_B

∗ Step 4

Calculation $TRD(A)$ and $TRD(B)$ and use definition 5., for ranking this method.

4. Results

Example 1. Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy number, then

∗ Step 1

$$\sigma_A = 0.12 , \ \sigma_B = 0.12$$

And

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$$\mu_A = 0.17\text{, } \mu_B = 0.17$$

∗ Step 2

$$I_A = 0.08C - 1.75\text{ , } I_B = 0.08C - 1.75$$

∗ Step 3

$$D_A = -4.14\text{ , } D_B = -4.14$$

∗ Step 4

$$TRD(A) = \sqrt{[0.08C - 1.75]^2 + [-4.14]^2}$$

For

$C = 0 \Rightarrow TRD(A) = 4.5$,

$C = 0.5 \Rightarrow TRD(A) = 4.48$,

$C = 1 \Rightarrow TRD(A) = 4.46$,

$$TRD(B) = \sqrt{[0.08C - 1.75]^2 + [-4.14]^2}$$

$C = 0 \Rightarrow TRD(B) = 4.5$,

$C = 0.5 \Rightarrow TRD(B) = 4.48$,

$C = 1 \Rightarrow TRD(B) = 4.46$,

Then , $TRD(A)$ □ $TRD(B) \Rightarrow A$ □ B .

Example 2. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

∗ Step 1

$$\sigma_A = 0.25\text{, } \sigma_B = 0.2$$

And

$$\mu_A = 0.3\text{, } \mu_B = 0.3$$

∗ Step 2

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$$I_A = 1.6C + 0.2 \ , \ I_B = 1.6C + 0.25$$

$*$ Step 3

$$D_A = -1.47 \ , \ D_B = -1.83$$

$*$ Step 4

$$TRD(A) = \sqrt{[1.6C + 0.2]^2 + [-1.47]^2}$$

For

$C = 0 \Rightarrow TRD(A) = 1.48$,

$C = 0.5 \Rightarrow TRD(A) = 1.78$,

$C = 1 \Rightarrow TRD(A) = 2.32$,

$$TRD(B) = \sqrt{[1.6C + 0.25]^2 + [-1.83]^2}$$

$C = 0 \Rightarrow TRD(B) = 1.85$,

$C = 0.5 \Rightarrow TRD(B) = 2.11$,

$C = 1 \Rightarrow TRD(B) = 2.6$,

Then , $TRD(A) < TRD(B) \Rightarrow A < B$.

Example 3. Let $A = (0.1, 0.2, 0.4, .5; 1)$ and $B = (1,1,1,1;1)$ be two generalized trapezoidal fuzzy number, then

$*$ Step 1

$$\sigma_A = 0.25 , \ \sigma_B = 0$$

And

$$\mu_A = 0.3 , \ \mu_B = 1$$

$*$ Step 2

$$I_A = 1.6C + 0.2 \ , \ I_B = \infty$$

$*$ Step 3

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$$D_A = -1.47 \ , \ D_B = \infty$$

* Step 4

$$TRD(A) = \sqrt{[1.6C + 0.2]^2 + [-1.47]^2}$$

For

$C = 0 \Rightarrow TRD(A) = 1.48$,

$C = 0.5 \Rightarrow TRD(A) = 1.78$,

$C = 1 \Rightarrow TRD(A) = 2.32$,

$$TRD(B) = \infty$$

$C = 0 \Rightarrow TRD(B) = \infty$,

$C = 0.5 \Rightarrow TRD(B) = \infty$,

$C = 1 \Rightarrow TRD(B) = \infty$,

Then , $TRD(A) < TRD(B) \Rightarrow A < B$.

Example 4. Let $A = (-0.5, -0.3, -0.3, -0.1; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

* Step 1

$$\sigma_A = 0.1, \ \sigma_B = 0.2$$

And

$$\mu_A = -0.3, \ \mu_B = 0.3$$

* Step 2

$$I_A = 16C - 7 \ , \ I_B = 1.6C + 0.25$$

* Step 3

$$D_A = -9.7 \ , \ D_B = -1.83$$

* Step 4

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$$TRD(A)=\sqrt{[16C-7]^2+[-9.7]^2}$$

For

$C=0 \Rightarrow TRD(A)=11.9$,

$C=0.5 \Rightarrow TRD(A)=9.7$,

$C=1 \Rightarrow TRD(A)=13.23$,

$$TRD(B)=\sqrt{[1.6C+0.25]^2+[-1.83]^2}$$

$C=0 \Rightarrow TRD(B)=1.85$,

$C=0.5 \Rightarrow TRD(B)=2.11$,

$C=1 \Rightarrow TRD(B)=2.6$,

Then , $TRD(A)>TRD(B) \Rightarrow A>B$.

Example 5. Let $A=(0.3,0.5,0.5,1;1)$ and $B=(0.1,0.6,0.6,0.8;1)$ be two generalized trapezoidal fuzzy number, then

∗ Step 1

$$\sigma_A=0.35,\ \sigma_B=0.17$$

And

$$\mu_A=0.57,\ \mu_B=0.52$$

∗ Step 2

$$I_A=-0.4C+1.2\ ,\ I_B=-0.23C+1.12$$

∗ Step 3

$$D_A=-0.28\ ,\ D_B=-0.86$$

∗ Step 4

$$TRD(A)=\sqrt{[-0.4C+1.2]^2+[-0.28]^2}$$

For

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$C=0 \Rightarrow TRD(A)=1.23$,

$C=0.5 \Rightarrow TRD(A)=1.04$,

$C=1 \Rightarrow TRD(A)=0.85$,

$$TRD(B)=\sqrt{[-0.23C+1.12]^2+[-0.86]^2}$$

$C=0 \Rightarrow TRD(B)=1.41$,

$C=0.5 \Rightarrow TRD(B)=1.32$,

$C=1 \Rightarrow TRD(B)=1.24$,

Then , $TRD(A)<TRD(B) \Rightarrow A<B$.

Example 6 Let $A=(0,0.4,0.6,0.8;1)$ and $B=(0.2,0.5,0.5,0.9;1)$ and $C=(0.1,0.6,0.7,0.8;1)$ be two generalized trapezoidal fuzzy number, then

∗ Step 1

$$\sigma_A=0.45,\ \sigma_B=0.35,\ \sigma_C=0.37$$

And

$$\mu_A=0.45,\ \mu_B=0.52,\ \mu_C=0.55$$

∗ Step 2

$$I_A=0.22C+0.9\ ,\ I_B=-0.11C+1.06,\ I_C=-0.27C+1.13$$

∗ Step 3

$$D_A=-0.48\ ,\ D_B=-0.42,\ D_C=-0.31$$

∗ Step 4

$$TRD(A)=\sqrt{[0.22C+0.9]^2+[-0.48]^2}$$

For

$C=0 \Rightarrow TRD(A)=1.02$,

$C=0.5 \Rightarrow TRD(A)=1.12$,

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$C = 1 \Rightarrow TRD(A) = 1.22$,

$$TRD(B) = \sqrt{[-0.11C + 1.06]^2 + [-0.42]^2}$$

$C = 0 \Rightarrow TRD(B) = 1.14$,

$C = 0.5 \Rightarrow TRD(B) = 1.089$,

$C = 1 \Rightarrow TRD(B) = 1.04$,

$$TRD(C) = \sqrt{[-0.27C + 1.13]^2 + [-0.31]^2}$$

$C = 0 \Rightarrow TRD(C) = 1.17$,

$C = 0.5 \Rightarrow TRD(C) = 1.086$,

$C = 1 \Rightarrow TRD(C) = 0.91$,

Then , $TRD(A) > TRD(B) > TRD(C) \Rightarrow A > B > C$.

Example 7. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (-2, 0, 0, 2; 1)$ be two generalized trapezoidal fuzzy number, then

∗ Step 1

$$\sigma_A = 0.25,\ \sigma_B = 2$$

And

$$\mu_A = 0.3,\ \mu_B = 0$$

∗ Step 2

$$I_A = 1.6C + 0.2\ ,\ I_B = 0.5C + 0.75$$

∗ Step 3

$$D_A = -1.47\ ,\ D_B = -0.33$$

∗ Step 4

$$TRD(A) = \sqrt{[1.6C + 0.2]^2 + [-1.47]^2}$$

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For

$C = 0 \Rightarrow TRD(A) = 1.48$,

$C = 0.5 \Rightarrow TRD(A) = 1.78$,

$C = 1 \Rightarrow TRD(A) = 2.32$,

$$TRD(B) = \sqrt{[0.5C + 0.75]^2 + [-0.33]^2}$$

$C = 0 \Rightarrow TRD(B) = 0.82$,

$C = 0.5 \Rightarrow TRD(B) = 1.05$,

$C = 1 \Rightarrow TRD(B) = 1.29$,

Then , $TRD(A) > TRD(B) \Rightarrow A > B$.

It is clear from Table 1. the results of the proposed approach are same as obtained by using the existing approach.

Table 1: A comparison of the ranking results for different approaches

Approaches	Example 1	Example 2	Example 3	Example 4	Example 5	Example 6	Example 7
Cheng [8]	A<B	A~B	Error	A~B	A>B	A<B<C	**Error**
Chu [9]	A<B	A<B	A<B	A<B	A>B	A<B<C	**Error**
Chen [4]	A<B	A<B	A<B	A<B	A>B	A<C<B	**A>B**
Abbasbandy [1]	Error	A~B	A<B	A~B	A<B	A<B<C	**A>B**
Chen [7]	A<B	A<B	A<B	A<B	A>B	A<B<C	**A>B**
Kumar [11]	A>B	A~B	A<B	A<B	A>B	A<B<C	**A>B**
Singh [15]	A<B	A<B	A<B	A<B	A>B	A<B<C	**A>B**
Rezvani [22]	A<B	A>B	A<B	A~B	A>B	A<C<B	**A<B**
Proposed approach	A~B	A<B	A<B	A>B	A<B	A>B>C	**A>B**

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Conclusion

In this paper, we want to propose a new method for ranking trapezoidal fuzzy numbers with using TRD distance based on mean and standard deviation. The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

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