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Oxygen Excess Ratio control of PEM Fuel Cell System Based on a Fractional Order model Approximation

M. Shahiri [1], A. Ranjbar N. [1*], M. Karami [1] and R. Ghaderi [2]

[1]Department of Computer and Electrical Engineering, Babol University of Technology, Babol, Iran

[2]Department of Electrical and Computer Engineering, Shahid Beheshti University, Tehran, Iran

*Corresponding Author's E-mail: *a.ranjbar@nit.ac.ir*

Abstract

In order to effectively control takes place, an accurate model is required. High order and complicated system needs for being reduced during analysis and control design. A nonlinear model of PEMFC was primarily linearized to an 8th order. In this paper the fractional calculus is gained to reduce the linear model of FEMFC into a lower order fractional dynamic. In this approach two: Charef's approximation method and a heuristic fractional order approximation algorithm are used. Thereafter a controller will be designed for the lower order approximated system. The quality of the designed controller will be assessed through simulation. The controller will be applied to the linearized and also the original nonlinear model in presence of load disturbance. The same procedure will be gained when the system is reduced through the Hankel matrix. The outcome verifies performance of the fractional order control using a Heuristic algorithm, with respect to the control in the original model, in terms of reducing the negative peak and also the speed of the response.

Keywords: Oxygen Excess Ratio, Charef's approximation, Fractional Order model, PEMFC, Control.

1. Introduction

A most effective Fuel Cells (FCs) is the so-called polymer electrolyte membrane or proton exchange membrane (PEM) FC which convert hydrogen fuel to electricity

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without combustion process. PEM generates electrical energy from an electrochemical reaction from a hydrogen-rich fuel gas and an oxidant (air or oxygen) exhausts water and heat. Membrane separates the anode and the cathode in a FC. During electrons emitted from the anode through an external circuit as a load until they reach the cathode. Very soon electrons combine with the protons (also generated in the anode, but diffuse through the membrane to the cathode) to produce oxygen and water (Fig . 1). A single cell of polymer electrolyte membrane fuel cell (PEMFC) produces a voltage of ~0.7 V at a nominal current density of 1 A/cm^2 [1]. The overall voltage generation will be increased by stacking FCs in series, in which forms a stack of PEMFC.

During a sudden change of the load control of transient behavior of PEMFC system is an important issue. A fast and efficient control of air flow is necessary to avoid the oxygen starvation to prolong the stack life [2]. Adjusting the oxygen excess ratio (i.e. λ_{O_2}) which is the input oxygen flow divided by the reacted oxygen flow in the cathode, optimize the conversion of energy in the fuel cell. This action maximizes net power of the system, operating under different load conditions. If partial pressure of oxygen in the air stream in the cathode side drops down from a certain critical level, a complicated phenomenon called oxygen starvation occurs [3].This phenomenon causes a sudden decrease in the fuel cell output voltage and causes a hot spot. This may even burn the surface of the membrane in some severe situations [4]. A sufficient mass flow of oxygen through the stack complies the load demand. λ_{O_2} control not only minimizes the fuel consumption but also the starvation will be avoided. An optimal mass flow of oxygen is achieved by maintaining the oxygen excess ratio to its optimal value. Stoichiometric ratio or the oxygen excess ratio is seen in [5].

Over the last decade, several models of fuel cell were developed [6-8]. However an accurate model is complicated which makes computationally intensive to be used for

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real-time applications. Contradiction between measurements from real systems and numerical simulations are usually due to uncertainties and model inaccuracy.

In this wok, a Pukrushpan's control-oriented FC system dynamic [6, 9] is used to control the transient behavior. The derived linear small-signal state-space model of the hydrogen PEMFC is of order of nine. Among conventional order reduction method e.g. in [10], a fractional order model approximation is gained. Two Charef's approximation method and Heuristic fractional order approximation are used to reconstruct a fractional order model. Thereafter a controller is designed for the fractional order model to investigate its performance with respect to the original system. Ultimately the outcome will be compared with respect to the conventional Hankel singular values order reduction algorithm.

The rest of this paper is organized as follows. PEM fuel cell model is interpreted in Section 2. Section 3 addresses a brief explanation of the fractional approximation of integer order system. PEM Model Approximation with fractional order system is addressed in Section 4. Section 5 discusses controller designation of the original and that of the reduced order system together with simulation results. Finally a conclusion closes the work in Section 6.

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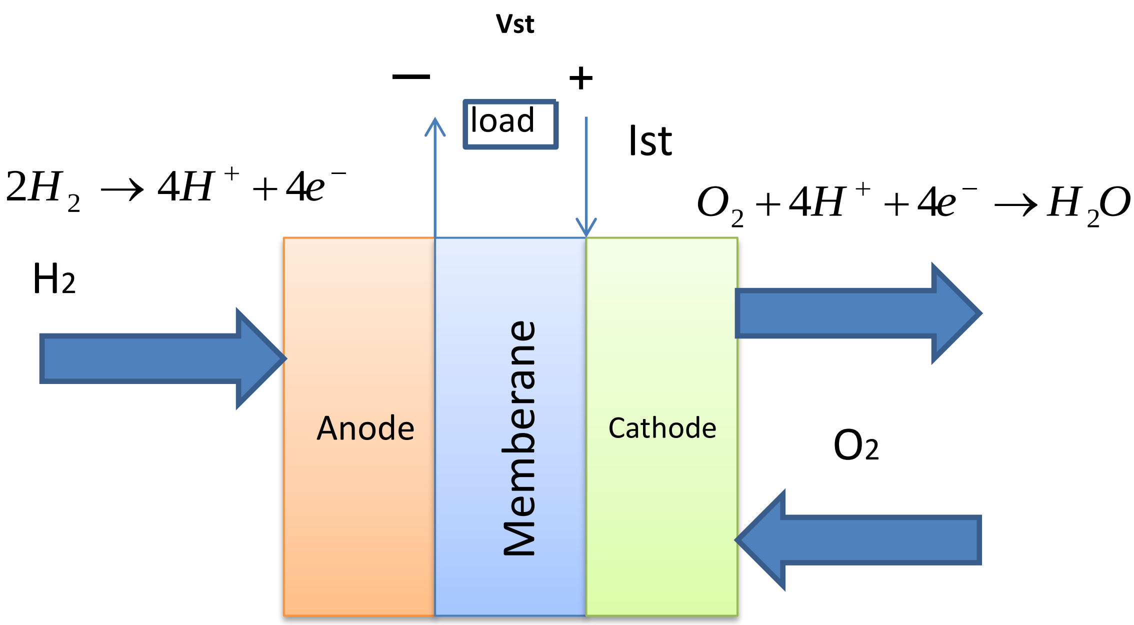

Figure 1: Schematic of the fuel cell supply action

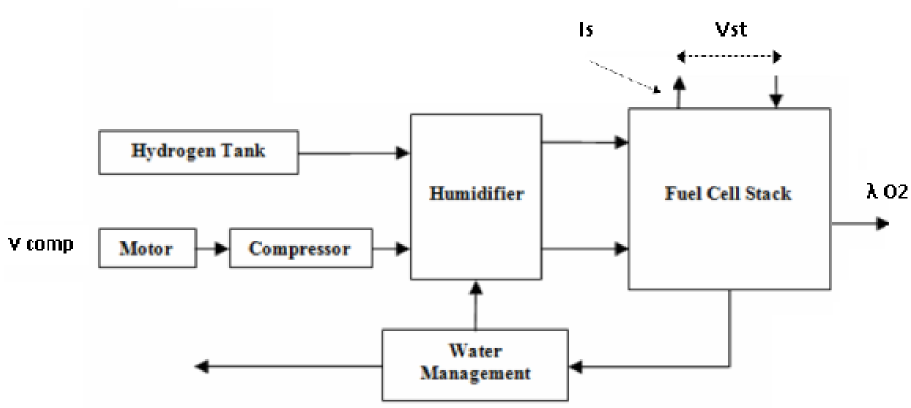

Figure 2: Main component of PEM fuel cell system

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2. PEM Fuel Cell Model

A nonlinear dynamic of FC system in state space form will be presented here. Compressed hydrogen is assumed available. Oxygen is also provided using air containing neutral Nitrogen. A governing schematic diagram of PEM FC is shown in Fig. 2.

2.1. A state space representation

A state space representation of the model which is proposed by Pukrushpan et al. [6, 9] incorporates the following nine states:

1. Mass of oxygen in the cathode, m_{O_2} (kg).
2. Mass of nitrogen in the cathode, m_{N_2} (kg).
3. Mass of hydrogen in the anode, m_{H_2} (kg).
4. Mass of water in the cathode, $m_{w,Ca}$ (kg).
5. Mass of water in the anode, $m_{w,An}$ (kg).
6. Mass of air in the supply manifold, m_{SM} (kg).
7. Speed of the compressor, ω_{Cp} (rad/s).
8. Return manifold pressure, p_{RM} (Pa).
9. Supply manifold pressure, p_{SM} (Pa).

The mass conservation is used to derive equations for oxygen, nitrogen and water mass inside the cathode volume. A state description of non-linear model of fuel cell [1, 11] is as follows:

$$\frac{dm_{o_2}}{dt} = W_{o_2,in} - W_{o_2,out} - W_{o_2,rec} \tag{1}$$

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$$\frac{dm_{N_2}}{dt} = W_{N_2,in} - W_{N_2,out} - W_{N_2,rec} \tag{2}$$

$$\frac{dm_{H_2}}{dt} = W_{H_2,in} - W_{H_2,out} \tag{3}$$

$$\frac{dm_{W,Ca}}{dt} = W_{v,Ca,in} - W_{v,Ca,out} + W_{v,Ca,gen} + W_{v,m} \tag{4}$$

$$\frac{dm_{W,An}}{dt} = W_{v,An,in} - W_{v,An,out} - W_{v,m} \tag{5}$$

$$\frac{dm_{SM}}{dt} = W_{Cp} - W_{Sm,out} \tag{6}$$

Equation of speed of the compressor is defined by the power conservation principle as:

$$J_{Cp}\frac{d\omega_{Cp}}{dt} = (\tau_{CM} - \tau_{Cp}) \tag{7}$$

Meanwhile the governing equations for the supply manifold pressure and the return manifold pressure are respectively defined using the energy conservation principle and thermodynamic relationships, as:

$$\frac{dp_{RM}}{dt} = \frac{R_a T_{RM}}{V_{RM}}(W_{Ca,out} - W_{RM,out}) \tag{8}$$

$$\frac{dp_{SM}}{dt} = \frac{\gamma R_a}{V_{SM}}(W_{Cp}T_{Cp} - W_{SM,out}T_{SM}) \tag{9}$$

2.2. Linearization of the Model

A nonlinear dynamic of FC model incorporates nine states and one control input together with one disturbance input [4].

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$$\dot{x}_{NonLinear} = f\,(x_{NonLinear}, u, w) \qquad \textit{state Equations}$$
$$u = v_{comp} \ \textit{Control Input},$$
$$w = I_{st} \qquad \textit{disturbance Input}$$
$$y^T = [W_{cp}, V_{st}, P_{sm}]$$
$$Z^T = [P_{net}, \lambda_{O_2}]$$
$$x_{NonLinear} = [m_{O_2}\ m_{N_2}\ m_{H_2}\ m_{w,Ca}\ m_{w,An}\ m_{SM}\ \omega_{Cp}\ p_{RM}\ p_{SM}]$$

In the model compressor motor voltage v_{comp}, is the control input u; the current I_{st} drawn from the FCS is treated as measureable disturbance input w. The net power of FCS P_{net} and excess oxygen ratio λ_{O_2} are performance variables produced by the FCS. Depending on control objectives different output ,$[W_{cp}, V_{st}, P_{sm}]$ may be achieved. A MATLABTm Simulink® nonlinear Model is used for control studies as shown in Fig.3 [1].

In this paper a PI plus dynamic feed-forward controllers are designed to effectively rejects the disturbance of λ_{O_2} during the transient behavior of the load current variation I_{st} [1]. It is primarily necessary to linearize the plant about a nominal operating point.

During the linearization procedure the mass of the water in the cathode, as an unobservable state, is removed from the model which is briefly discuss in [1].

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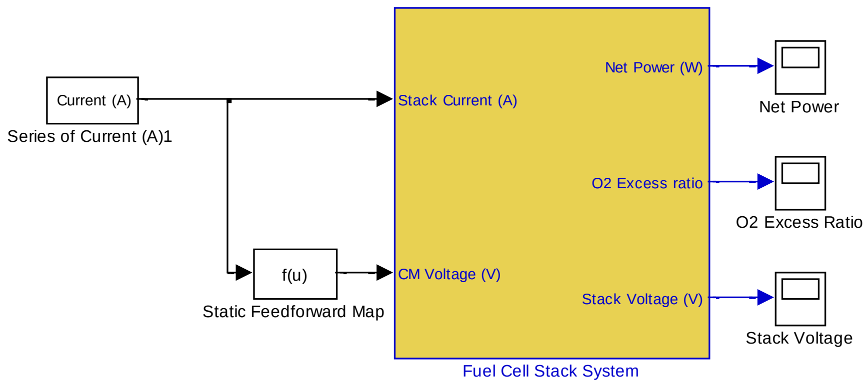

Figure 3: A MATLABTm Simulink® model of the PEM FC model.

The nonlinear model is linearized which is developed in [4]. A nominal operating point is chosen where the system net power is assumed 40 kW and oxygen excess ratio will be adjusted on 2. The corresponding inputs to this operating point are 191 Amps of the stack current and 164 V as the compressor motor voltage [4]. The linear model includes the static feed-forward controller (sFF) which is given by:

$$\begin{aligned} \delta\dot{x} &= A\delta x + B_u\delta u + B_w\delta w \\ \delta z &= C_z\delta x + D_{zu}\delta u + D_{zw}\delta w \\ \delta y &= C_y\delta x + D_{yu}\delta u + D_{yw}\delta w \end{aligned} \quad (10)$$

The derived matrices of coefficients using static feed-forward controller (SFF) around selected operating point (Ist=191,Vcomp=164) are as follows [4]:

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$$A = \begin{bmatrix} 6.30908 & 0 & -10.9544 & 0 & 83.74458 & 0 & 0 & 24.05866 \\ 0 & -161.083 & 0 & 0 & 51.52923 & 0 & -18.0261 & 0 \\ -18.7858 & 0 & -46.3136 & 0 & 275.6592 & 0 & 0 & 158.3741 \\ 0 & 0 & 0 & -17.3506 & 193.9373 & 0 & 0 & 0 \\ 1.299576 & 0 & 2.969317 & 0.3977 & -38.7024 & 0.105748 & 0 & 0 \\ 16.64244 & 0 & 38.02522 & 5.066579 & -479.384 & 0 & 0 & 0 \\ 0 & -450.386 & 0 & 0 & 142.2084 & 0 & -80.9472 & 0 \\ 2.02257 & 0 & 4.621237 & 0 & 0 & 0 & 0 & -51.2108 \end{bmatrix}$$

$$B_u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3.942897 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad B_w = \begin{bmatrix} -0.03159 \\ -0.00398 \\ 0 \\ 2.681436 \\ 0 \\ 0 \\ -0.05242 \\ 0 \end{bmatrix}$$

$$C_z = \begin{bmatrix} -2.48373 & -1.9773 & 0.109013 & -0.21897 & 0 & 0 & 0 & 0 \\ -0.63477 & 0 & -1.45035 & 0 & 13.84308 & 0 & 0 & 0 \end{bmatrix}$$

$$C_y = \begin{bmatrix} 0 & 0 & 0 & 5.066579 & -116.446 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 12.96989 & 10.32532 & -0.56926 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{zu} = \begin{bmatrix} 0.168979 \\ 0 \end{bmatrix} \quad D_{zw} = \begin{bmatrix} 0.104116 \\ -0.01041 \end{bmatrix} \quad D_{yu} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad D_{yw} = \begin{bmatrix} 0 \\ 0 \\ -0.29656 \end{bmatrix}$$

After linearization the linear subsystems will be approximated with fractional order model. In the following section a fractional order based reduction technique will be proposed.

3. Fractional Order Approximation Of Integer Order System

A non-integer order derivative is currently used in many fields of science and engineering. Several contributions have been developed for the modelling of physical phenomena for

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which the use of the classic derivative requires a very high order model. However a fractional order approximation requires for less terms in the transfer function. In continuing two distinct methods will be interpreted to reduce the order of the system in a fractional one.

3.1. Approximation to a First Order Fractional Model

Among several different fractional order approximation methods like Crone, Carlson, Matsuda,… approximation [12] , Charef's approximation method will be described for its compatibly to our propose. Charef's approximation method [13] has been introduced to represent dynamic behavior of fractal systems which is called Fractional Power Pole (FPP)[14]. A fractal system representation in the transfer function form (11) will be approximated by a singularity function of a series of poles and zeros. Number and partition within the frequency bandwidth $[0,\omega_{max}]$ depends on a previously defined approximation error.

$$F(s)=\frac{1}{(1+\frac{s}{P_T})^{\alpha}} \qquad (11)$$

P_T is the transitional frequency, and α is a real number between 0 and 1. An approximation of $F(s)$, on a frequency range $[0,\omega_{max}]$, can be obtained using an integer transfer function[15] of:

$$F_{est}(s)=\frac{\prod_{i=0}^{N-1}\left(1+\frac{s}{\omega_{Z_i}}\right)}{\prod_{i=0}^{N}\left(1+\frac{s}{\omega_{P_i}}\right)} \qquad (12)$$

Frequencies ω_{Z_i} and ω_{P_i} are given by a simple geometrical calculation based on the maximum error $\varepsilon>0$ (in db.) between the approximation staircase line given by $F_{est}(s)$ and

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the exact line of slope $-20\alpha dB/dec$ given by $F(s)$ (Fig. 4). The following equations summarize the computations stages. Given constants a and b by:

$$a = \frac{\omega_{Z_i}}{\omega_{P_i}} = 10^{\frac{\varepsilon}{10(1-\alpha)}},\ b = \frac{\omega_{P_{i+1}}}{\omega_{Z_i}} = 10^{\frac{\varepsilon}{10\alpha}} \quad (13)$$

These are governed to a non integer order α by:

$$\alpha = \frac{\log(a)}{\log(ab)} \quad (14)$$

The first pole is:

$$\omega_{P_0} = p_T\, 10^{\frac{\varepsilon}{20\alpha}} \quad (15)$$

Such other singularities are found as:

$$\begin{aligned} \omega_{p_i} &= (ab)^i\, \omega_{p_0} \quad i = 1,2,\ldots,N \\ \omega_{Z_i} &= (ab)^i\, a\omega_{p_0} \quad i = 1,2,\ldots,N-1. \end{aligned} \quad (16)$$

N the number of singularities also constitutes the integer model order, is given by:

$$N = integer\left[\log\left(\frac{\omega_{max}}{p_0}\right) \Big/ \log(ab) \right] + 1 \quad (17)$$

integer [.] stands for the integer part. In follows, the procedure is given to approximate the parameters of the corresponding fractional models.

3.2. Charef's approximation procedure

Let assume G(s) as an integer order model of a large dimension N with real poles and zeros of the following form:

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$$G(s) = k \frac{\prod_{i=0}^{m}\left(1+\frac{s}{\omega_{Z_i}}\right)}{\prod_{i=0}^{n}\left(1+\frac{s}{\omega_{P_i}}\right)} \qquad (18)$$

K is a static gain, $-\omega_{Z_i}$ and $-\omega_{P_i}$ are poles and zeros of $G(s)$ respectively. It is an aim to replace the maximum number of poles and zeros by the first order model (11). The rest of the approximation is as follows:

Construct two vectors containing absolute values of zeros and poles of $G(s)$ whose sorted in ascending order.

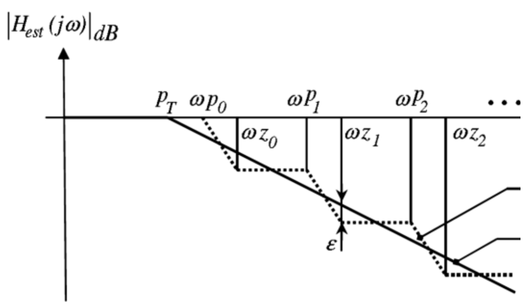

Figure 4: A Graphical description of Charef's approximation technique

Then concatenate these two vectors in the same vector S which must also be sorted in an increasing order.

$$\begin{cases} zero = \left[\omega_{Z_1}, \omega_{Z_2}, \ldots, \omega_{Z_M}\right] \\ pole = \left[\omega_{P_1}, \omega_{P_2}, \ldots, \omega_{P_M}\right] \\ A = [zero, pole]. \end{cases} \qquad (19)$$

The minimum number of singularities extract from the vector A named Nmin. Vector A is sorted increasingly with poles and zeros vector in one another begins and ends with poles.

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Now configure first selection of poles and zeros which are recursively distributed according to (16).

For each triple combination of $(\omega_{P_i}, \omega_{P_{i+1}}, \omega_{Z_i})$ use (13) to calculate non integer order α_i by:

$$\alpha_i = \frac{\log\left(\omega_{Z_i} / \omega_{P_i}\right)}{\log\left(\omega_{P_{i+1}} / \omega_{P_i}\right)} \quad i = 1, 2, ..., N_z \tag{20}$$

N_z is number of achieved zeros in the combination. In an ideal case, where poles and zeros are recursively distributed, these non integer orders will be found the same. Otherwise, the mean value is chosen by:

$$\alpha = \frac{\sum_{i=1}^{N_z} \alpha_i}{N_z} \tag{21}$$

For each pair $(\omega_{P_{i+1}}, \omega_{Z_i})$ parameter P_{T_i} the fractional model can be calculated using (15):

$$P_{T_i} = \omega_{p_0} 10^{-\frac{\log\left(\frac{\omega_{p_i}+1}{\omega_{z_i}}\right)}{2}} \quad i = 1, 2, ..., N_z \tag{22}$$

Finally P_T is replaced by the mean value as:

$$P_T = \frac{\sum_{i=1}^{N_z} P_{T_i}}{N_z} \tag{23}$$

In this regard $G(s)$ can be approximated by a fractional model of:

$$G_{est}(s) = \frac{1}{(1+\frac{s}{P_T})^{\alpha}} G_R(s) \tag{24}$$

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3.3. Heuristic approximation procedure

In this procedure first the numbers of singularities of fractional model are heuristically supposed through the frequency diagram of the original integer order model which is:

$$G_{frac_{est}}(s) = \frac{\sum_i (1+s/P_{Ti})^{\alpha_i}}{\sum_j (1+s/P_{Tj})^{-\beta_j}} \tag{25}$$

$$\alpha_i, \beta_j \in \mathbb{R}^+ \qquad P_{Ti}, P_{Tj} \in \mathbb{R}$$

Then best value of unknown parameters are optimized using a previously defined cost function on the bode diagram. This may be achieved using traditional optimization method like GA, PSO, … . The cost function will be defined by minimizing the difference between the gain (in dB) and phase (degree) diagram of the fractional order estimated model and that of the original integer order system.

4. Approximation Of Linearized PEM Model With Fractional Order System

The prescribed scheme will be performed on a PEMFC model. A linearized PEM FC system described in section 2 will be treated as single input- two output (SIMO) or single input - single output together with a disturbance input (SISO). Figs. 5 and 6 show linearized subsystems of SIMO and SISO respectively. On those compressor voltage and the load current are assumed as control and disturbance inputs respectively. In order to assess linear transfer functions of each subsystems a linearization Matrix defined in section 2 will be used. In continuing reduction will be interpreted using fractional order model order reduction technique.

A linearized transfer function of subsystem G_{12} (compressor voltage to λ_{o2}) is as follows [1]:

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$$G_{12} = \frac{21.73s^4 + 1574s^3 + 7911s^2 + 1.316e004s + 7153}{s^6 + 159.9s^5 + 7539s^4 + 1.166e5s^3 + 5.252e5s^2 + 8.9e5s + 5.086e5}$$

It is primarily required to sort zeros and poles of transfer function in increasing order to perform the fractional reduction technique.

$$Zero = [1.3472 \quad 1.4774 \quad 2.4622 \quad 67.1728]$$
$$\downarrow \qquad \downarrow$$
$$Pole = [1.4038 \;\; 1.6473 \;\; 2.9151 \;\; 18.2582 \;\; 46.1768 \;\; 89.4853]$$

Then each zeros and poles are sorted in one another. In the case each term ruins the order it is omitted from the arranging rule where here is shown by red pen. These omitted dynamics of course conure a residual function of $G_R(s)$.

$$A = [1.40 \quad 1.47 \quad 1.64 \quad 2.46 \quad 2.91 \quad 67.17 \quad 89.48]$$

Term α is achieved using equation (21) and (22) whilst P_T is found using (24) and (25) which is as follows:

$$\alpha = 0.6467$$
$$P_T = 1.2786$$

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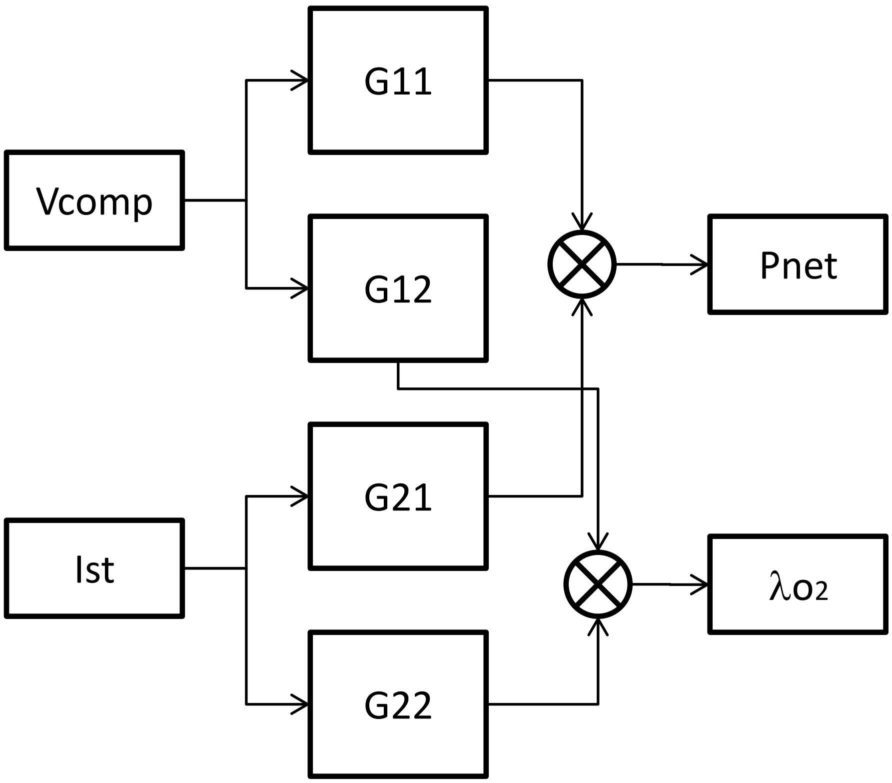

Figure 5: A single input- two output block diagram (SIMO)

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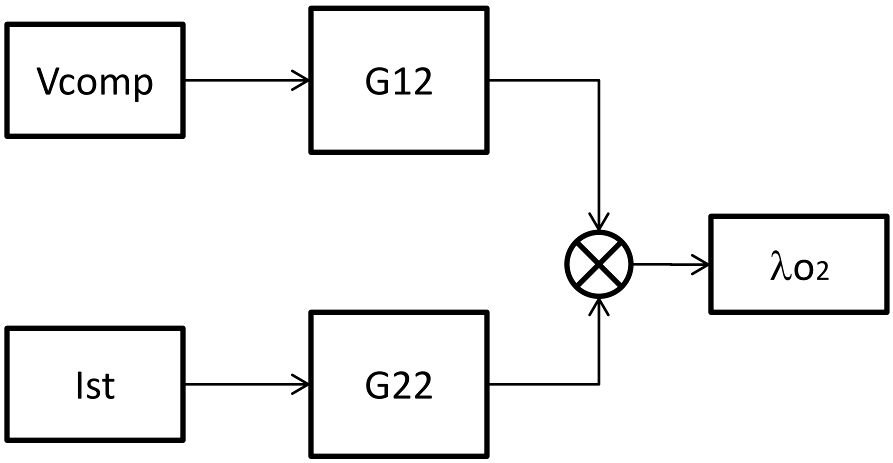

Figure 6: A single input- single output block diagram (SISO)

Finally a fractionally estimated model is reconstructed as:

$$G_{12_{Frac}} = (1+s/1.2786)^{(-0.6467)}$$

$$G_{R_{12}} = \frac{(s+1.3472)}{(s+18.2582)(s+46.1768)}$$

$$G_{12_{est}} = \frac{(s+1.3472)}{(s+18.2582)(s+46.1768)} G_{Frac_{12}}$$

The order is seen reduced to 2.64 where concerning frequency response of the original transfer function G12, and that of fractionally estimated is seen in Fig. 7. Meanwhile a heuristic search algorithms (GA or PSO) counterpart estimates the model as:

$$G_{12est} = \frac{1}{(s/32+1)^{1.85}}$$

This function is gaining lower order whilst performing more accuracy as seen in Fig. 7. Similarly the procedure will be performed on G_{22} as:

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$$G_{22} = \frac{0.01041s^6 - 1.644s^5 - 76.83s^4 - 1178s^3 - 5243s^2 - 8745s - 4912}{s^6 + 159.9s^5 + 7539s^4 + 1.166e5s^3 + 5.252e5s^2 + 8.9e5s + 5.086e5}$$

Containing:

$$Zero = [1.4014 \;\; 1.5723 \;\; 2.9208 \;\; 18.1266 \;\; 45.9941 \;\; 87.9449]$$

$$\downarrow \qquad\qquad\qquad \downarrow \qquad \downarrow$$

$$Pole = [1.4038 \;\; 1.6473 \;\; 2.9151 \;\; 18.2582 \;\; 46.1768 \;\; 89.4853]$$

These arranges the Vector S by:

$$A = [1.4031.572 \; 1.647 \;\; 2.920 \; 18.25 \; 45.99 \; 46.17 \; 87.94 \; 89.48]$$

Forming:

$$\alpha = 0.7291$$
$$P_T = 1.1814$$

Finally a fractional order transfer function is estimated as:

$$G_{Frac_{22}} = (1 + s/1.1814)^{(-0.7291)}$$
$$G_{R_{22}} = \frac{(s+1.4014)(s+18.1266)}{(s+2.9151)}$$
$$G_{22_{est}} = G_{R_{22}} G_{Frac_{22}}$$

Unlike to the previous model, the last reduced order function is seen improper. However this may be transformed into a proper one. In the following Bode diagram (in Fig. 8) of the linearized original system and that of fractionally reduced one is depicted. Furthermore estimated function of G_{22est} using the Heuristic algorithm is found as: $G_{22est} = \frac{(s/1.75+1)^{0.335}}{(s/1.95+1)^{0.325}(s/60+1)^{0.01}}$ which can be seen in Fig. 8.

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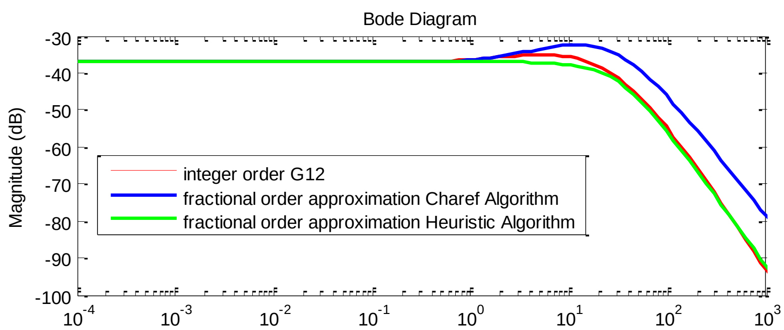

Figure 7: Diagram of the original G_{12} and the fractionally estimated equivalent

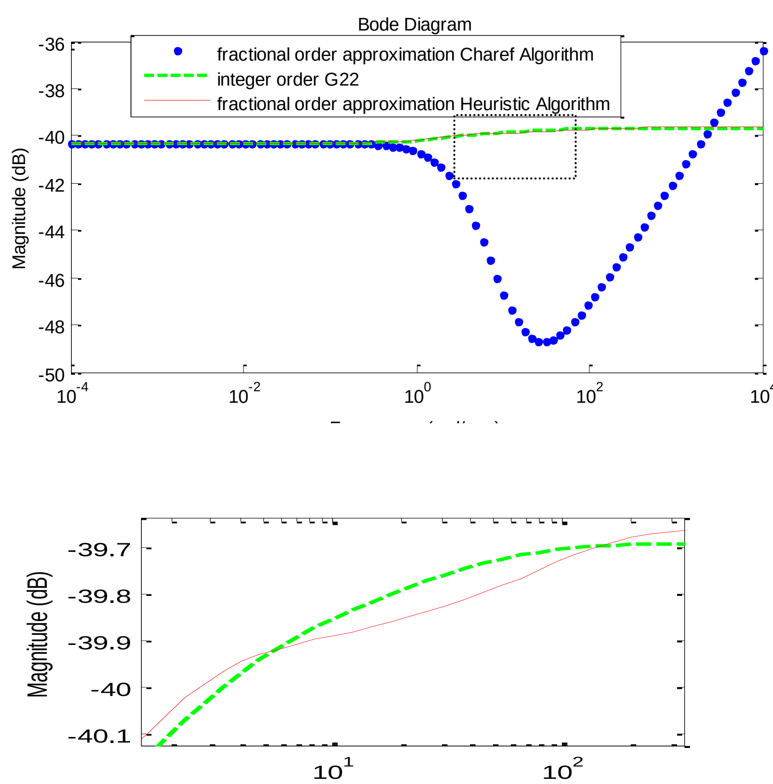

Figure 8: Bode diagram of the original G_{22} and fractionally estimated

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5. Controller Design And Simulation Result

In this section a controller will be designed for the original and reduced order models. The controller will be of a lower order and will be applied on both the original linearized and actual nonlinear models.

5.1. Dynamic Feed forward controller

Due to the topology in Fig. 6 it is seen that the control variable, $u = V_{comp}$, and the disturbance, $w = I_{st}$, directly affects λ_{o_2}. For good disturbance rejection, the control variable, u, can use a lead filter of the measured disturbance, w. This lead filter is based on the inversion of the open loop dynamics from V_{comp} to λ_{o_2} [1]. Then λ_{o_2} is defined as:

$$\Delta\lambda_{o_2} = G_{12} V_{comp} + G_{22} I_{st} \tag{26}$$

Using the linear model in [1], the system can be rearranged in the form of G_{12} and G_{22}. A feed forward controller of $\Delta u = K_{dynamic} \Delta w$ as shown in Fig. 9 is applied in the model. The transfer function from $w = I_{st}$ to λ_{o_2} can be written as:

$$T = \frac{\Delta\lambda_{o2}}{\Delta I_{st}} = G_{12} + G_{22} K_{dynamic} \tag{27}$$

For complete disturbance rejection $T = 0$ and $K_{dynamic}$ cancels the response of λ_{o_2} due to Ist:

$$K_{dynamic} = -G_{12}^{-1} G_{22} \tag{28}$$

In the case feed forward controller is improper, a high-frequency components of $K_{dynamic}$ will be removed using a low-pass filter of:

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$$K_{\text{Proper}} = -\frac{1}{\sum_{i=1}^{N}(1+\frac{s}{\alpha_i})} G_{12}^{-1} G_{22} \tag{29}$$

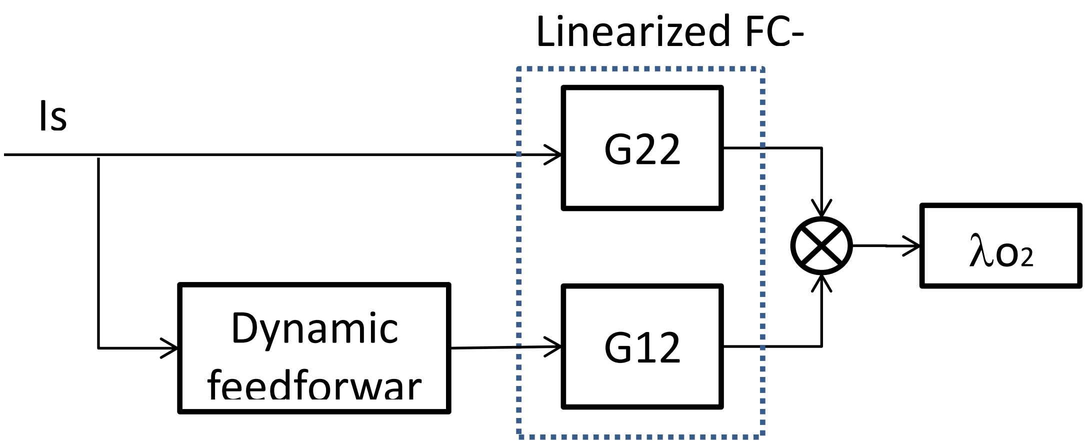

Figure 9: Schematic of applied feed forward controller

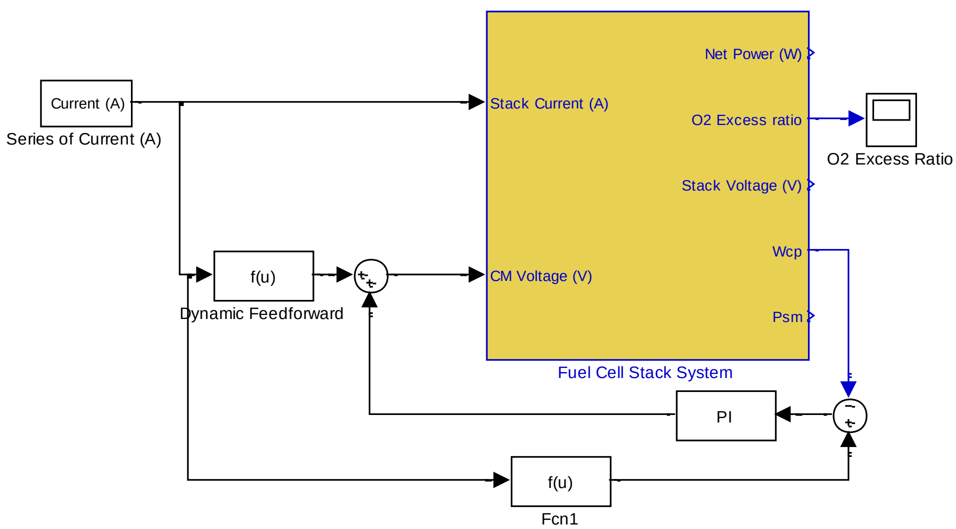

Figure 10: MATLAB™ Simulink model of PEM fuel cell model together with DFF+PI

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Amount of α_i and the number of N depend on order of $K_{dynamic}$. Closing a feedback may reduce sensitivity of the controller against unknown disturbance, modeling error, and parameter variation. Accordingly a PI controller [1] in combination with $K_{dynamic}$ is used to adjust λ_{o_2} The following scenario will be considered for $K_{dynamic}$ controller:

1. For the original linearized system.
2. For the reduced order by the Charef algorithm.
3. For the reduced by the Heuristic and artificial algorithm.
4. For the 3rd order reduced system using the Hankel singular value.

Finally performance of designed controllers will be verified on the full order linearized and that of the nonlinear system.

Fig. 11 shows the applied disturbance input (changes in the load) which led to different operating points.

5.2. Simulation result

Case1: Dynamic feed forward controller on the original linearized subsystem

Consider G_{12} and G_{22} as defined in section V and according to (27). A dynamic feed forward controller is designed as:

$$K_{DFF} = -\frac{1}{\sum_{i=1}^{N}(1+\frac{s}{\alpha_i})} G_{12}^{-1} G_{22} \;, \alpha = (80,120,120)$$

$$K_{DFF} = \frac{551.9283(s+87.94)(s+45.99)(s+18.13)}{(s+120)^2(s+80)(s+67.17)(s+2.462)} \cdot \frac{(s+2.921)(s+1.572)(s+1.401)}{(s+1.477)(s+1.347)}$$

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The outcome of the controller on the linearized system (Fig. 6) and on the nonlinear system (Fig. 10) are shown in Figs 12 and 13. Time of convergence of λ_{o_2} is seen about 0.04 sec.

Case 2: Dynamic feed forward controller based on reduced fractional order linearized system (Charef 's Algorithm)

Similarly consider $G_{12_{est}}$ and $G_{22_{est}}$ as derived in section V:

$$K_{DFF} = \frac{5.0372(s+46.18)(s+18.26)(s+18.13)(s+1.401)}{(s+200)^2(s+2.915)(s+1.347)}.$$

$$.G^{-1}_{12_{Frac}}.G_{22_{Frac}}$$

$$G_{12_{Frac}} = (1+\frac{s}{1.1814})^{(-0.7291)}$$

$$G_{22_{Frac}} = (1+\frac{s}{1.2786})^{(-0.6467)}$$

Result of using these controllers in Figs. 12 and 13 confirm that the speed is reduced. This is because those frequency response are not complete matched. To improve the Heuristic algorithm is used to find a fractionally equivalent.

Case 3: Dynamic feed forward controller based on fractional order reduced linearized system (Heurestic Algorithm)

Similarly consider $G_{12_{est}}$ and $G_{22_{est}}$ as derived in section V:

$$K_{DFF} = \frac{1}{(1+s/2000)}.\frac{(s/1.75+1)^{0.335}}{(s/1.95+1)^{0.325}(s/60+1)^{0.01}}.(s/32+1)^{1.85}$$

This controller provide faster response by factor of two as seen in Figs 12 and 13. Furthermore the controller reduces undesired negative peak where may ruin the FC

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membrane. This advantage is one of the most important objectives in PEM FC control to prolong the stack life.

Case 4: Dynamic feed forward controller based on Hankel Singular Values order reduction algorithm (integer 3rd order reduced linearized system

$$G_{12(reduced)} = \frac{0.003771s^2 + 20.56s + 38.47}{s^3 + 80.63s^2 + 1256s + 2715}$$

$$G_{22(reduced)} = \frac{-0.01041s^3 - 1.128s^2 - 20.99s - 29.69}{s^3 + 110.3s^2 + 2080s + 3074}$$

$$K_{DFF} = \frac{2760566.5632(s+60.68)(s+85.06)(s+21.77)}{(s+100)^3(s+86.72)(s+21.97)(s+1.872)} \frac{(s+17.39)(s+2.573)(s+1.54)}{(s+1.614)(s+5451)}$$

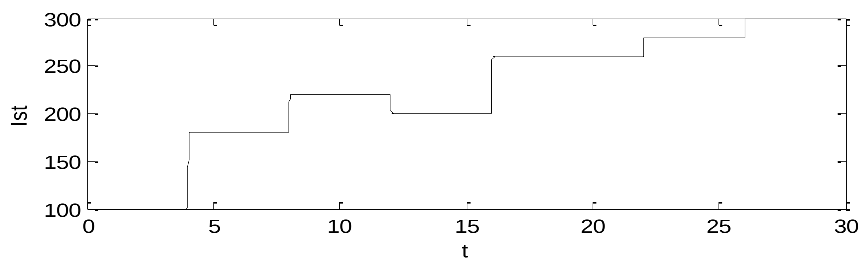
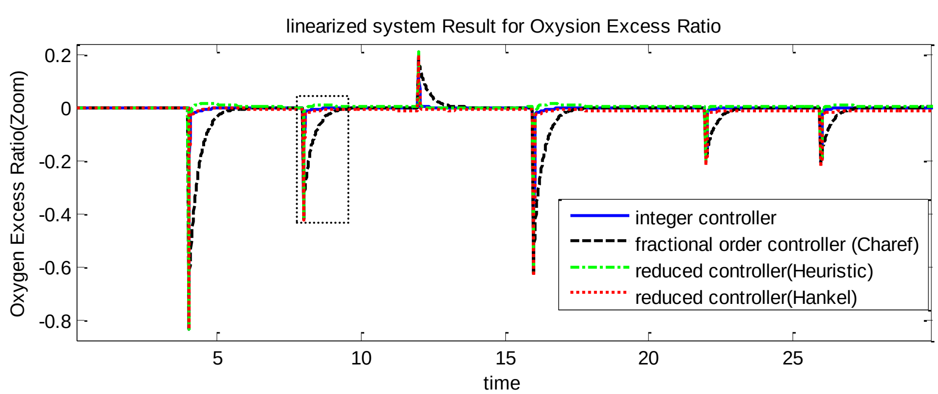


Figure 11: Disturbance input $w = I_{st}$.



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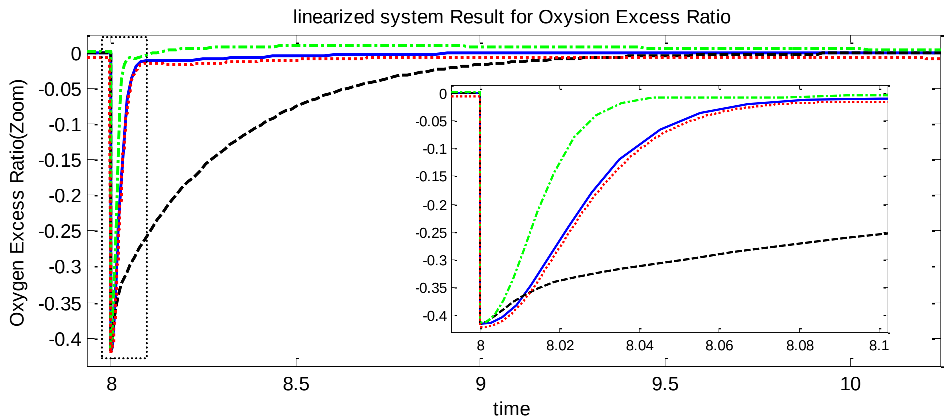
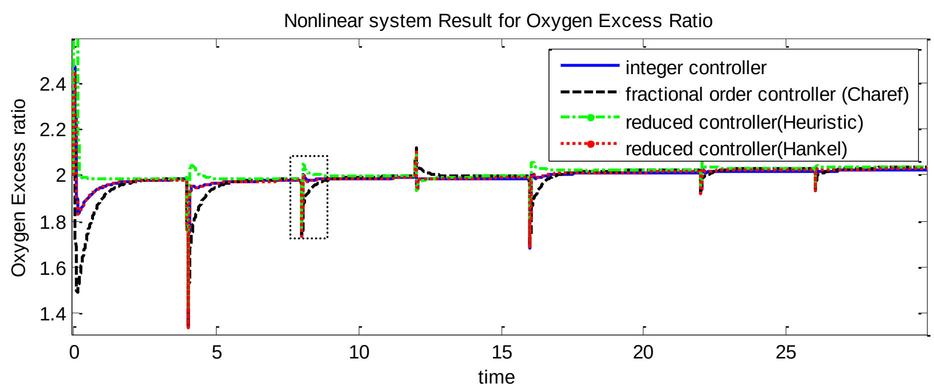
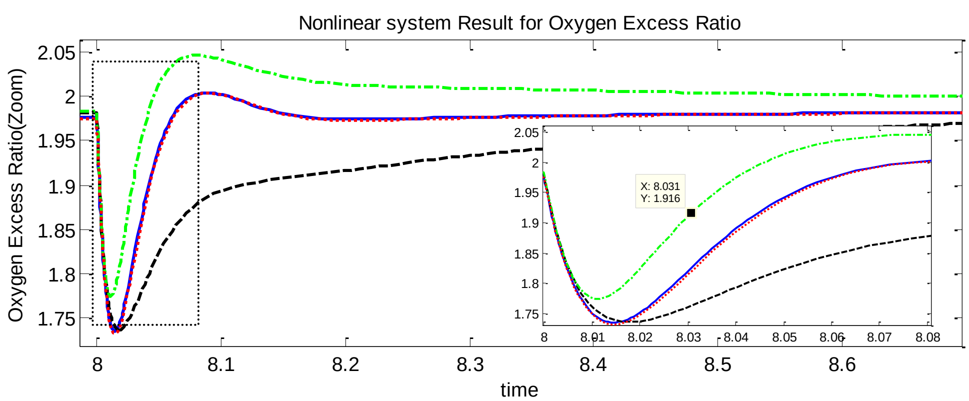


Figure 12: input disturbance(I_{st}) and oxygen excess ratio of the linearized model





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This controller provides similar accuracy and the time response as the controller of the original system(Figs. 12 and 13).

Conclusion

In this work the fractional order system approximation techniques were gained to provide equivalent lower order linear models with respect to linearized model in [1]. The designed controller improves transient dynamics of PEMFC system in terms of oxygen excess ratio adjustment. This controller also prolong the stack life due to combat the sudden change in the load demand. Performance of the designed controller investigated by means of simulation approach when it is tried on the original nonlinear and linearized systems. Specially the Heuristic designed controller provides robustness against the load disturbance as well as improves the speed response by factor of two. However the controller increases the undesired overshoot where confirming more attention during the designation procedure.

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