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Observer-based Adaptive Fuzzy Wavelet Network Controller with H_∞ Tracking Performance for a Class of Uncertain Nonlinear Systems

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Abstract

This paper proposes a new observer-based adaptive fuzzy wavelet network controller (AFWNC) with H_∞ tracking performance for uncertain nonlinear systems subject to external disturbance and unmeasured states. Proposed scheme consists of two terms: observer-based AFWNC term and robust H_∞-based control law. The first part that invokes adaptive fuzzy wavelet network (FWN) to represent the model of the unknown dynamics of the system is designed to deal with the unmeasured states and unknown dynamics of the system while the second part is designed to attenuate the effects of the modeling error and external disturbance. Orthogonal least square (OLS) algorithm is used to determine network size, number of fuzzy rules and efficient wavelets, automatically. Also, integral-type adaptive laws are developed to adjust the adaptation parameters of the controller, on-line. No prior knowledge about unknown dynamics of the plant and initial values of the adaptation parameters is required. Proposed control scheme guarantees that all signals of the closed loop system are bounded and H_∞ tracking performance is achieved. Simulation and comparison results are presented to demonstrate the superior performance of the proposed scheme.

Keywords: *Adaptive fuzzy wavelet networks, Observer based controller,* H_∞ *tracking performance, uncertain nonlinear systems.*

1. Introduction

Lately, wavelet theory has found many applications in the different research areas such as numerical analysis, signal processing and other engineering applications [1, 5, 12, 16]. Wavelets are basis functions with the time-frequency localization and multi-resolution approximation property; So wavelets owning to their mentioned inherent capabilities are combined with soft computing techniques and lead to a number of new techniques such as: wavelet neural networks (WNNs) [22] and fuzzy wavelet networks [10]. Fuzzy wavelet network not only preserves multi resolution analysis of the wavelet functions and learning ability of the neural networks but also has the inference property of fuzzy concepts to handle uncertain and unknown conditions. Each fuzzy rule corresponds to one sub-WNN which consists of wavelets with a specified dilation value and determines the contribution degree of its sub-WNN on the approximated output. So, based on these advantages, FWN has many considerable capabilities such as approximation accuracy and good generalization performance. Inspired from the mentioned capabilities, they have found wide applications in many research areas such as system identification, fault diagnosis, modeling and control [2, 8, 11, 19, 20, 25]. On the other hand, lately approximator-based adaptive control of uncertain nonlinear systems

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has attracted considerable attention since it provides an efficient approach for control of complex or uncertain nonlinear systems [4, 5, 20, 23, 24, 26, 28]. In these schemes, traditional approximators such as NNs, fuzzy systems (FSs) and orthogonal functions such as wavelet functions due to their inherent capabilities in function approximation have been widely used to represent the model of unknown nonlinear systems. Then, adaptive laws are derived based on backstepping algorithm or Lyapunov stability theorem such that the stability and performance of the closed loop system is satisfied.

However, traditional approximator-based adaptive control schemes provide efficient tools to handle uncertain nonlinear systems but they suffer from some difficulties like: (i) Lack of any proper method to determine the number of fuzzy rules, location and parameters of membership functions (or number of neurons, hidden layers and initial values of the NN parameters). (ii) On one hand, conventional approximators only present local approximation of the function and cannot capture all local and global behavior of it. (iii) Also conventional approximator-based adaptive control schemes require considerable number of fuzzy basis function (large number of neurons and hidden layers in the case of NN) to handle problems with large dimension.

To overcome the difficulties associated with the traditional approximator-based adaptive control schemes, FWN as a powerful approximator with high approximation accuracy and good generalization capability has been applied to derive control schemes for nonlinear systems [2, 6, 7, 11, 13, 21].

Despite the potential of each proposed control scheme with different structures, they have some difficulties. Lin [7] presented a nonsingular terminal sliding mode controller which uses FWN to approximate the unknown dynamic of the robotic systems and robust control term to eliminate the effects of the approximation error. It has no off-line learning phase, number of rules and fuzzy wavelet basis functions and network initialization are done via trial and error, which is time consuming process. An observer based adaptive fuzzy wave-net controller was designed for uncertain nonlinear systems in [13]. Both scale and wavelet functions were applied to approximate unknown function, also self-structuring algorithm was designed to determine number of scale and wavelet functions, on-line. In fact application of both scale and wavelet functions for approximation purpose makes it different from the proposed work. In [21] adaptive fuzzy wavelet network controller (AFWNC) was designed for affine nonlinear systems in a direct manner which employs off-line learning phase to construct FWN for approximating feedback linearization control input. Fuzzy wavelet network model was used to approximate the control input on-line and adaptive law was derived to tune free parameters of the controller. In [6], presented structure by Lin [7] was utilized to develop adaptive fuzzy wavelet network controller for chaos synchronization. This structure suffers from online computational complexity; moreover improper initialization of the network may lead to the undesirable response or may lead to a response with undesirable transient performance.

Considering the difficulties of the proposed schemes in the literatures, in this paper an observer-based adaptive fuzzy wavelet network controller with H_∞ tracking performance is proposed for uncertain nonlinear systems. Proposed controller uses observer-based adaptive FWN control term to handle nonlinear system with unknown dynamic and unmeasured states. Also, robust H_∞-based control law is designed to attenuate the effects of the modeling error and external disturbance. No prior knowledge about the unknown dynamics of the system and initial values of the adaptation parameters are required in the design step. Main preference of the proposed controller over the other reported adaptive FWN-based schemes is the use of both off-line and on-line parameter learning. Main reason for the use of off-line learning phase is that if the initial values of the FWN parameters for real-time applications are not properly set, the closed loop system may exhibit undesirable transient performance. Further, without the off-line learning phase, it is difficult to have a good choice of the network size and it can be either too risky. On one hand, trial and error approach to determine network size is high time-consuming process. In this regard, at the off-line

phase, two FWNs are constructed to represent the model of the unknown dynamics of the system and then they are applied for system identification. For this, wavelet lattice is constructed to pick up candidate wavelets and then OLS algorithm is applied to purify candidate wavelets to select efficient wavelets for network constructions. To handle any uncertain condition, adaptive laws are derived based on Lyapunov direct method to adjust the controller parameters, online. Also H_∞-based control law is used to attenuate the effects of the modeling error and external disturbance.

To demonstrate the superiority of the proposed scheme, hybrid adaptive fuzzy control [18] and adaptive fuzzy neural control [27] schemes are applied for comparison. Simulation results verify that the tracking performance and transient response specifications are improved considerably, while the number of parameters and on-line computational complexity are reduced.

This paper is organized as follows. Problem statement is presented in section 2. Adaptive fuzzy wavelet network is described in section 3, it consists of two sub-sections which describe FWN structure and model construction. The design procedure of the proposed observer-based adaptive FWN controller with H_∞ tracking performance is explained in section 4. Simulation and comparison results that demonstrate the effectiveness of the proposed method are presented in section 5; finally, some concluding remarks are given in section 6.

2. Problem Statement

Consider a class of nonlinear systems which are described by the following form:

$$\begin{aligned} \dot{\underline{x}} &= A\underline{x} + B[f(\underline{x}) + g(\underline{x})u + d] \\ y &= C\underline{x} \end{aligned} \tag{1}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{n\times n}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{n\times 1}, \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}_{n\times 1}^{T} \tag{2}$$

and $f(\underline{x})$ and $g(\underline{x})$ are unknown nonlinear functions, $u \in R$ and $y \in R$ are the control input and system output, respectively. $\underline{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^T = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the state vector of the system which is not assumed to be available for measurment and d is the unknown but bounded external disturbance. It is assumed that nonlinear system (1) is controllable thus it requires that $g(\underline{x}) \neq 0$ for any $\underline{x}$ in the certain controllability region $U_c \subset R^n$, so $g(\underline{x})$ is nonlinear invertible function, without loss of generality it is assumed that $g(\underline{x}) > 0$.

The control objective is to design a control input such that the output of system (1) tracks a given desired trajectory $\underline{x}_d$ and closed loop system remains stable. Also, H_∞ tracking design technique is reduced to attenuate the effect of approximation error and external disturbance to a desired level..

Since in practice not all states of the system are measurable, nonlinear functions $f(\underline{x})$ and $g(\underline{x})$ may be unknown and effects of the external disturbance and undesirable effects are inevitable, so it is a hard task to implement conventional control schemes [14]. Hence, an observer-based adaptive FWN tracking controller with H_∞ tracking performance is developed to deal with nonlinear uncertain systems subject to unmeasured states and external disturbance. Proposed scheme invokes adaptive

FWN to approximate the unknown dynamics of the plant (i.e., $f(\underline{x}), g(\underline{x})$) and designs FWN-based observer to estimate the states of the system. Then, controller is designed considering that the FWN-model represents true plant. To reduce the influence of the approximation error and external disturbance, the robust H_∞-based control input is employed. Then, adaptive laws are designed to adjust the controller free parameters such that all signals of the closed loop system become bounded and the H_∞-based tracking performance is achieved.

3. Structure of Adaptive Fuzzy Wavelet Network

As mentioned above, the first task to implement the proposed scheme is to construct two FWNs for approximating the unknown nonlinear functions (i.e., $f(\underline{x}), g(\underline{x})$). For this, brief description about FWN structure is presented and then model construction is given.

3.1. Fuzzy Wavelet Network Structure

Fuzzy wavelet network for approximating the typical unknown nonlinear function $f(\underline{x})$ can be described by a set of fuzzy IF-THEN rules to perform a mapping from an input vector $\underline{x} = [x_1, x_2, \ldots, x_n]^T \in R^n$ to an output variable f [10]. The ith fuzzy IF-THEN rule has the following form:

$$R_f^i : If\ x_1\ is\ A_1^i\ and\ x_2\ is\ A_2^i, \ldots, and\ x_n\ is\ A_n^i\ Then\ \hat{f}_i = \theta_i f_{w_i} \tag{3}$$

where $f_{w_i} = \sum_{k=1}^{T_i} w_{M_i,\underline{t}^k} \psi_{M_i,\underline{t}^k}^{(k)}(\underline{x})$, R_f^i is the ith rule $(1 \le i \le c_f)$ and c_f indicate the number of fuzzy rules that are used to approximate unknown function $f(\underline{x})$. Also x_j $(1 \le j \le n)$ stands for the jth input variable of the network, T_i is the total number of selected wavelets for the ith rule and θ_i is the adjustable parameter which should be adapted for the ith rule to approximate unknown function $f(\underline{x})$. It should be mentioned that $\hat{f}_i$ is the output of the local model for the ith rule which generally is equal to the linear combination of a finite set of wavelets $\psi_{M_i,\underline{t}^k}^k(\underline{x})$ with the same dilation value $M_i \in Z$. Also, $t_i \in R$ is the symbol for translation parameter and $\underline{t}^k = [t_1^k, t_2^k, \ldots, t_n^k]$ where t_j^k is the translation value for corresponding wavelet k. Finally, A_i^j demonstrates the fuzzy set characterized by the following Gaussian-type membership function and $A_i^j(x_j)$ is the membership grade of x_j in A_i^j.

$$A_j^i(x_j) = \exp[-(((x_j - p_{j1}^i)/p_{j2}^i)^2)^{p_{j3}^i/2}] \tag{4}$$

where $p_{j1}^i, p_{j2}^i \in R$, $0 < p_{j3}^i \le 5$. p_{j1}^i represents the center of membership function, p_{j2}^i and p_{j3}^i determine the width and the shape of the membership function, respectively. Also in (4), wavelets $\psi_{M_i,\underline{t}}^k(\underline{x})$ are expressed by the tensor product of 1-D mother wavelet functions as follows:

$$\psi_{M_i,\underline{t}}^{(k)}(\underline{x}) = 2^{M_i/2}\psi^{(k)}(2^{M_i}\underline{x} - \underline{t}^k) = \prod_{j=1}^{n} 2^{M_i/2}\psi^{(k)}(2^{M_i}x_j - t_j^k) \tag{5}$$

using the singleton fuzzifier, product inference and center average defuzzifier, the output of the FWN approximator, $\hat{f}(\underline{x})$, can be expressed as:

$$\hat{f}(\underline{x}) = \sum_{i=1}^{c_y} \hat{\mu}_i(\underline{x})\hat{f}_i = \sum_{i=1}^{c_y} \hat{\mu}_i(\underline{x})\theta_i f_{w_i} = \frac{\sum_{i=1}^{c_y} \theta_i \left(\prod_{j=1}^{n} A_j^i(x_j)\right)\left(\sum_{k=1}^{T_i} w_{M_i,\underline{t}^k} \psi_{M_i,\underline{t}^k}^{(k)}(\underline{x})\right)}{\sum_{i=1}^{c_y}\left(\prod_{j=1}^{n} A_j^i(x_j)\right)} = \underline{\theta}^T \underline{\zeta}(\underline{x}) \tag{6}$$

where $\mu_i(\underline{x}) = \prod_{j=1}^{n} A_j^i(\underline{x})$ is the firing strength of the ith rule and $\hat{\mu}_i(\underline{x}) = \mu_i(\underline{x}) / \sum_{i=1}^{c_y} \mu_i(\underline{x})$ which satisfies $0 \le \hat{\mu}_i(\underline{x}) \le 1$ and $\sum_{i=1}^{c_y} \hat{\mu}_i(\underline{x}) = 1$. In general, $\hat{\mu}_i(\underline{x})$ determines the contribution degree of each output of the wavelet based model with resolution level M_i. Also, $\underline{\theta} = [\theta_1, \theta_2, \ldots, \theta_{c_y}]^T$ is an adjustable parameter vector and $\underline{\zeta} = [\zeta_1, \zeta_2, \ldots, \zeta_{c_y}]^T$ is the fuzzy wavelet basis vector.

3.2. Model Construction

For the modeling purpose, during off-line learning phase, two FWNs are constructed to approximate unknown nonlinear functions $f(\underline{x})$ and $g(\underline{x})$. Off-line learning phase is important because it determines the proper and accurate model for real time applications. On one hand, since the off-line training phase is not time-critical, it allows the flexibility to study and compare the effectiveness and performance of different choices for the size of the network. Now the procedure of FWN construction for approximating unknown nonlinear function $f(\underline{x})$ is briefly reviewed. Same procedure is done to model unknown function $g(\underline{x})$.

At the first step, candidate wavelets are selected by constructing wavelet lattice and are purified using the OLS algorithm [17, 21]. Orthogonal least square algorithm selects efficient wavelets, determines network size, initial weights, number of fuzzy rules and sub-WNNs based on the dilation parameters of selected efficient wavelets. In fact, OLS algorithm is employed to optimize the architecture of the network. At the end of OLS algorithm, total number of selected wavelets for each rule at dilation value M_i, number of fuzzy rules and initial weights are determined and network is initialized. At the second step, parameters of the network should be learned. Two efficient off-line learning algorithms including extended Kalman filter (EKF) and recursive least square (RLS) are used to train network. All nonlinear parameters involving p_{jr}^i and t_j^k where $1 \le j \le n$, $1 \le i \le c_f$, $r = 1,2,3$ and $k = T_1, T_2, \ldots, T_{cf}$ are learned via EKF algorithm while linear parameters, w_{M_i,t^k}, are tuned using RLS estimator. Details description of EKF and RLS algorithms can be found in [9, 15].

In summery, at the end of off-line learning phase, two FWNs which are denoted by $\hat{f}(\underline{x}|\underline{\theta}_f)$ and $\hat{g}(\underline{x}|\underline{\theta}_g)$ are constructed to model the unknown dynamics of the nonlinear systems and are used for system identification during real time operation. In order to maintain consistent performance of the FWN-based control scheme in situations for which there are large uncertainties or unknown

variations in the plant parameters and structures, the structure should be adaptive. Propose scheme assumes that the presented model is linear with respect to its parameters and only parameter vector $\underline{\theta}$ is adjustable during real time operation. Hence, proper adaptive laws should be developed to tune adjustable parameter vector, on-line.

Now, it is assumed that $\hat{f}(\underline{x}|\underline{\theta}_f)$ and $\hat{g}(\underline{x}|\underline{\theta}_g)$ represent the approximated models of the uncertain nonlinear functions $f(\underline{x})$ and $g(\underline{x})$, respectively, which are linear models with respect to their adjustable parameter vectors $\underline{\theta}_f$, $\underline{\theta}_g$. According to the output of the FWN approximator in (6), the model of the unknown system (1) which are described by $\hat{f}(\underline{x}|\underline{\theta}_f)$ and $\hat{g}(\underline{x}|\underline{\theta}_g)$ can be expressed as:

$$\begin{aligned} \hat{f}(\underline{x}|\underline{\theta}_f) &= \underline{\theta}_f^T \underline{\zeta}_f(\underline{x}) = \underline{\zeta}_f^T(\underline{x})\underline{\theta}_f \\ \hat{g}(\underline{x}|\underline{\theta}_g) &= \underline{\theta}_g^T \underline{\zeta}_g(\underline{x}) = \underline{\zeta}_g^T(\underline{x})\underline{\theta}_g \end{aligned} \tag{7}$$

where $\underline{\theta}_f = [\theta_1, \theta_2, \ldots, \theta_{c_f}]^T$ and $\underline{\theta}_g = [\theta_1, \theta_2, \ldots, \theta_{c_g}]^T$ are adaptation parameters of the FWNs and $\underline{\zeta}_f(\underline{x}) = [\zeta_1, \zeta_2, \ldots, \zeta_{c_f}]^T$, $\underline{\zeta}_g(\underline{x}) = [\zeta_1, \zeta_2, \ldots, \zeta_{c_g}]^T$ are fuzzy wavelet basis vectors corresponding to each function which are considered as an output of each sub-WNN. Also c_f and c_g denote the number of fuzzy rules that are used to approximate unknown functions $f(\underline{x})$ and $g(\underline{x})$, respectively. Next section is devoted to design proposed control scheme.

4. Observer-Based Adaptive Fuzzy Wavelet Network Control with H∞ Tracking Performance

In this section, fuzzy wavelet network-based adaptive observer is designed as follow:

$$\begin{aligned} \dot{\hat{\underline{x}}} &= A\hat{\underline{x}} + B[\hat{f}(\hat{\underline{x}}|\underline{\theta}_f) + \hat{g}(\hat{\underline{x}}|\underline{\theta}_g)u + u_a] + L(y - \hat{y}) \\ \hat{y} &= C\hat{\underline{x}} \end{aligned} \tag{8}$$

where L is the observer gain, u is adaptive control law and u_a represents the H∞-based control term which is used to attenuate the effect of modeling error and external disturbance.
Let define the state estimation error as $\underline{e}_x = \underline{x} - \hat{\underline{x}}$; Using (1) and (8), the dynamic of the state estimation error is obtained as:

$$\dot{\underline{e}}_x = (A - LC)\underline{e}_x + B[f(\underline{x}) - \hat{f}(\hat{\underline{x}}|\underline{\theta}_f) + (g(\underline{x}) - \hat{g}(\hat{\underline{x}}|\underline{\theta}_g))u - u_a + d] \tag{9}$$

Let define the parameters estimation error as $\tilde{\underline{\theta}}_f = \underline{\theta}_f - \underline{\theta}_f^*$ and $\tilde{\underline{\theta}}_g = \underline{\theta}_g - \underline{\theta}_g^*$, then (9) can be written as:

$$\dot{\underline{e}}_x = (A - LC)\underline{e}_x + B[\underline{\zeta}_f^T(\hat{\underline{x}})\tilde{\underline{\theta}}_f + \underline{\zeta}_g^T(\underline{x})\tilde{\underline{\theta}}_g u - u_a + \omega + d] \tag{10}$$

where $\omega = f(\underline{x}) - f(\hat{\underline{x}}|\underline{\theta}_f^*) + (g(\underline{x}) - g(\hat{\underline{x}}|\underline{\theta}_g^*))u$, while $\underline{\theta}_f^*$ and $\underline{\theta}_g^*$ are optimal parameter estimations that are defined as follows:

$$\underline{\theta}_f^* = \arg\min_{\underline{\theta}_f \in \Omega_f} \left[\sup_{\underline{x} \in R^n} \left\| \hat{f}(\hat{\underline{x}} | \underline{\theta}_f) - f(\underline{x}) \right\| \right]$$

$$\underline{\theta}_g^* = \arg\min_{\underline{\theta}_g \in \Omega_g} \left[\sup_{\underline{x} \in R^n} \left\| \hat{g}(\hat{\underline{x}} | \underline{\theta}_g) - g(\underline{x}) \right\| \right]$$

where Ω_f and Ω_g are constraint sets for $\underline{\theta}_f$ and $\underline{\theta}_g$, respectively.

Let choose the adaptive control law u and H∞-based control term u_a as follows:

$$u = \frac{1}{\hat{g}(\hat{\underline{x}} | \underline{\theta}_g)} [-\hat{f}(\hat{\underline{x}} | \underline{\theta}_f) + y_d^n + \underline{k}^T \underline{e}_d] \tag{11}$$

$$u_a(\underline{x}) = -\frac{1}{r} \underline{B}^T P \underline{e} \tag{12}$$

where r is the scalar positive constant and $\underline{e}_d = \underline{x}_d - \hat{\underline{x}}$ denotes the tracking error. Also, adaptive laws to adjust the free parameters $\underline{\theta}_f$ and $\underline{\theta}_g$ are designed as follows:

$$\dot{\underline{\theta}}_f = -\gamma_1 \underline{\zeta}_f(\hat{\underline{x}}) B^T P \underline{e}_x \tag{13}$$

$$\dot{\underline{\theta}}_g = \begin{cases} -\gamma_2 \underline{\zeta}_g(\hat{\underline{x}}) B^T P \underline{e}_x u & if\ (\underline{\theta}_g \in \mathrm{B}^o)\ or \\ & (\underline{\theta}_g \in \overline{\mathrm{B}}\ and\ B^T P \underline{e}_x u \geq 0 \\ 0 & otherwise \end{cases} \tag{14}$$

where γ_1 and γ_2 are positive adaptation rates and $P_{n\times n}$ is a positive semidefinite matrix satisfying the following Riccati-like equation:

$$(A - LC)^T P + P(A - LC) + Q - \frac{2}{r} PBB^T P + \frac{1}{\rho^2} PBB^T P = 0 \tag{15}$$

where $Q_{n\times n}$ is a positive definite matrix.

Theorem1. Consider the control problem of nonlinear system (1), if the control inputs (11) and (12) are applied, where $\hat{f}(\hat{\underline{x}} | \underline{\theta}_f)$ and $\hat{g}(\hat{\underline{x}} | \underline{\theta}_g)$ are modeled by (7) and the adjustable free parameters $\underline{\theta}_f$ and $\underline{\theta}_g$ are adjusted by the adaptive laws (13) and (14), then the proposed control scheme guarantees that: (i) All signals of the closed loop system are bounded and (ii) The following H_∞ tracking performance is achieved for a prescribe attenuation level ρ [3]:

$$\int_0^T \underline{e}^T Q \underline{e} dt \leq \underline{e}^T(0) P \underline{e}(0) + \frac{1}{r} \tilde{\underline{\theta}}_f^T(0) \tilde{\underline{\theta}}_f + \frac{1}{r} \tilde{\underline{\theta}}_g^T(0) \tilde{\underline{\theta}}_g + \rho^2 \int_0^T (\omega + d)^T (\omega + d) dt \qquad \forall T \in [0, \infty) \tag{16}$$

Proof. Let define the following Lyapunov function:

$$V = \frac{1}{2} \underline{e}_x^T P \underline{e}_x + \frac{1}{2\gamma_1} \tilde{\underline{\theta}}_f^T \tilde{\underline{\theta}}_f + \frac{1}{2\gamma_2} \tilde{\underline{\theta}}_g^T \tilde{\underline{\theta}}_g \tag{17}$$

The time derivative of V is calculated as:

$$\begin{aligned}\dot{V} &= \frac{1}{2}[\underline{e}_x^T(A-LC)^T P\underline{e}_x + \underline{\tilde{\theta}}_f^T \underline{\zeta}_f(\underline{\hat{x}})B^T P\underline{e}_x + u^T \underline{\tilde{\theta}}_g^T \underline{\zeta}_g(\underline{\hat{x}})B^T P\underline{e}_x \\ &+ (\omega+d)^T B^T P\underline{e}_x - \frac{1}{r}\underline{e}_x^T PBB^T P\underline{e}_x + \underline{e}_x^T P(A-LC)\underline{e}_x + \underline{e}_x^T PB\underline{\zeta}_f^T(\underline{\hat{x}})\underline{\tilde{\theta}}_f \\ &+ \underline{e}_x^T PB\underline{\zeta}_g^T(\underline{\hat{x}})\underline{\tilde{\theta}}_g u + \underline{e}^T PB(\omega+d) - \frac{1}{r}\underline{e}_x^T PBB^T P\underline{e}_x] + \frac{1}{\gamma_1}\underline{\dot{\hat{\theta}}}_f^T \underline{\tilde{\theta}}_f + \frac{1}{\gamma_2}\underline{\dot{\hat{\theta}}}_g \underline{\tilde{\theta}}_g \\ &= \frac{1}{2}\underline{e}_x^T((A-LC)^T P + P(A-LC) - \frac{2}{r}PBB^T P)\underline{e}_x + (\underline{e}_x^T PB\underline{\zeta}_f^T(\underline{\hat{x}}) + \frac{1}{\gamma_1}\underline{\dot{\hat{\theta}}}_f^T)\underline{\tilde{\theta}}_f \\ &+ (\underline{e}_x^T PB\underline{\zeta}_g^T(\underline{\hat{x}})u + \frac{1}{\gamma_2}\underline{\dot{\hat{\theta}}}_g^T)\underline{\tilde{\theta}}_g + \frac{1}{2}((\omega+d)^T B^T P\underline{e}_x + \underline{e}_x^T PB(\omega+d))\end{aligned} \tag{18}$$

by applying the adaptive laws in (13) and (14) and using the Riccati-like equation (15), we have:

$$\dot{V} = -\frac{1}{2}\underline{e}_x^T Q\underline{e}_x - \frac{1}{2\rho^2}\underline{e}_x^T PBB^T P\underline{e}_x + \frac{1}{2}((\omega+d)^T B^T P\underline{e}_x + \underline{e}_x^T PB(\omega+d)) \tag{19}$$

while (19) can be rewritten in the following form:

$$\begin{aligned}\dot{V} &= -\frac{1}{2}\underline{e}_x^T Q\underline{e}_x - \frac{1}{2}(\frac{1}{\rho}B^T P\underline{e}_x - \rho(\omega+d))^T(\frac{1}{\rho}B^T P\underline{e}_x - \rho(\omega+d)) \\ &+ \frac{1}{2}\rho^2(\omega+d)^T(\omega+d) \le -\frac{1}{2}\underline{e}_x^T Q\underline{e}_x + \frac{1}{2}\rho^2(\omega+d)^T(\omega+d)\end{aligned} \tag{20}$$

If $(\omega+d)\in L_2$, then all signals of the closed loop system are bounded, i.e., $\underline{e}, \underline{\tilde{\theta}}_f, \underline{\tilde{\theta}}_g \in L_\infty$ and the tracking error will converge to a small neighborhood of the origin; the size of this neighborhood depends on the approximation error and can be made small by choosing proper design parameters ρ, Q. Integrating both sides of (20) form $t=0$ to $t=T$ yields:

$$V(T) - V(0) \le -\frac{1}{2}\int_0^T \underline{e}_x^T Q\underline{e}_x\, dt + \frac{1}{2}\rho^2\int_0^T(\omega+d)^T(\omega+d)\,dt \tag{21}$$

Since $V(T)\ge 0$ so the following inequality can be concluded from (21):

$$\frac{1}{2}\int_0^T \underline{e}_x^T Q\underline{e}_x\, dt \le V(0) + \frac{1}{2}\rho^2\int_0^T(\omega+d)^T(\omega+d)\,dt \tag{22}$$

Thus according to the considered Lyapunov function in (20), following inequality is obtained:

$$\begin{aligned}\frac{1}{2}\int_0^T \underline{e}_x^T Q\underline{e}_x dt &\le \frac{1}{2}\underline{e}_x^T(0)P\underline{e}_x^T(0) + \frac{1}{2\gamma_1}\underline{\tilde{\theta}}_f^T(0)\underline{\tilde{\theta}}_f(0) \\ &+ \frac{1}{2\gamma_2}\underline{\tilde{\theta}}_g^T(0)\underline{\tilde{\theta}}_g(0) + \frac{1}{2}\rho^2\int_0^T(\omega+d)^T(\omega+d)\,dt\end{aligned} \tag{23}$$

which implies that the H_∞ tracking performance in (16) is satisfied. ■

Remark 1: Adaptation for $\underline{\theta}_g$ in (14) is constrained within the convex set B to satisfy:

$$\Omega_g = \left\{\underline{\theta}_g \,\middle|\, \hat{g}(\underline{\hat{x}}|\underline{\theta}_g) \ge \varepsilon > 0,\ \forall \underline{x}\in U_c\right\} \tag{24}$$

in this adaptive law, whenever $\underline{\theta}_g \in \overline{\Omega}_g$ we have $\dot{\underline{\theta}}_g^T \nabla_{\underline{\theta}_g} \hat{g} = -\gamma_2 \underline{e}^T PB \underline{\zeta}_g(\hat{\underline{x}}) \underline{\zeta}_g^T(\hat{\underline{x}}) u \le 0$. This condition implies the vector $\dot{\underline{\theta}}_g$ points within $90°$ of $-\nabla_{\underline{\theta}_g} \hat{g}$, so it points inside Ω_g or along the tangent plane of $\overline{\Omega}_g$ at point $\underline{\theta}_g$. Hence, $\underline{\theta}_g$ will never leave Ω_g, i.e., $\underline{\theta}_g \in \Omega_g \ \forall t \ge 0$.

Remark 2: In the case that $\underline{\theta}_g \in \overline{\mathrm{B}}$ and $\underline{e}_x^T u < 0$, the term $\underline{e}_x^T PB \underline{\zeta}_g^T(\hat{\underline{x}}) u \tilde{\underline{\theta}}_g$ in (31) cannot be eliminated since $\dot{\underline{\theta}}_g = 0$. Because of $\underline{\theta}_g^* \in \mathrm{B}$, so $\tilde{\underline{\theta}}_g \nabla_{\underline{\theta}_g} \hat{g} \ge 0$, (i.e., $\tilde{\underline{\theta}}_g \underline{\zeta}_g^T(\hat{\underline{x}}) \ge 0$) when $\underline{\theta}_g \in \overline{\mathrm{B}}$. Since $\underline{e}_x^T u < 0$ from the adaptive law, the term $\underline{e}_x^T PB \underline{\zeta}_g^T(\hat{\underline{x}}) u \tilde{\underline{\theta}}_g$ is nonpositive and mentioned results are obtained.

Remark 3: Since finite number of basis functions are used to approximate the unknown dynamic of the system, so approximation error is inevitable and the tracking error is influenced by it; hence without loss of generality, robust H_∞ control algorithm is employed to attenuate the effect of the approximation error ω on the tracking error to a prescribed level, and this attenuation level can be made arbitrary small.

6. Simulation Results

In order to show the capability and effectiveness of the proposed approach, it is applied to the nonlinear inverted pendulum system. To examine the performance of the proposed control scheme on the inverted pendulum system, simulations are performed for two control objectives. The first objective is to balance the inverted pendulum in the vertical position and the second is to let the system output tracks the desired trajectory. Moreover, to exhibit the merit of the proposed adaptive control method than other schemes, two different controllers, which were developed in similar control structure, are adopted from the literature and applied for comparison. In the following, a brief description of the inverted pendulum system is presented.

The dynamic equation of the inverted pendulum system is expressed as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(\underline{x}) + g(\underline{x}) u \end{cases} \tag{25}$$

where $\underline{x} = [x_1, x_2]$, $x_1 = \theta$ is the angular position of the pendulum with respect to the vertical axis, and $x_2 = \dot{\theta}$ is the angular velocity of the pole and $f(\underline{x})$, $g(\underline{x})$ are defined as follows:

$$f(\underline{x}) = \frac{g\sin(x_1) - \dfrac{m_c l x_2^2 \sin x_1 \cos x_2}{m_c + m}}{L\left(\dfrac{4}{3} - \dfrac{m_c \cos^2(x_1)}{m_c + m}\right)} \qquad g(\underline{x}) = \frac{\dfrac{\cos x_1}{m_c + m}}{L\left(\dfrac{4}{3} - \dfrac{m_c \cos^2(x_1)}{m_c + m}\right)}$$

where g, l, m_c and m represent the acceleration due to gravity, half length of the pole, mass of cart and mass of pole, respectively. Also, u denote to the control input.

As mentioned before, the control objective includes two parts. 1) Regulating the pole in the vertical position, 2) Tracking the desired trajectory.

In the following, a brief description of FWNs construction is given. In this paper, "Mexican Hat" wavelet function is used to construct FWNs for approximating unknown nonlinear dynamics of the

system. The dilation parameter of this wavelet function ranges from -5 to 4. In design procedure, two steps are used to select important wavelets as follows:

- Selection of candidate wavelets by constructing wavelet lattice.
- Using OLS algorithm for purifying candidate wavelets to choose important and efficient wavelets for determining network structure [17]. The input-output pairs are collected by exciting the system with an input signal and measuring the corresponding outputs. In an online identification scheme, as an input signal excite the system and the identification model simultaneously, then modeling error is used by an adaptive algorithm for adjusting the model parameters.

To constructing the FWN for approximating the nonlinear function $f(\underline{x})$, the selected efficient wavelets are divided into c_f group according to their dilation values. Each group corresponds to one fuzzy rule which includes one sub-WNN. Eight efficient wavelets which correspond to five fuzzy rules or sub-WNNs are represented to approximate the unknown nonlinear function $f(\underline{x})$. Also, same procedure is done for approximating function $g(\underline{x})$ and five efficient wavelets which correspond to five fuzzy rules or sub-WNNs are obtained to model the unknown nonlinear function $g(\underline{x})$. Structure of each FWN model is given in Tables 1 and 2.

Table 1: Structure of FWN model for approximating unknown nonlinear function $f(\underline{x})$

Number of sub-WNNs	5				
Total number of efficient wavelets by OLS	8				
Number of fuzzy rules	5				
Rules (R^i)	R^1	R^2	R^3	R^4	R^5
Number of selected Wavelets for each rule (T_{i_f})	1	1	2	1	3
Dilation parameter of each rule (M_{i_f})	-1	-2	1	0	2
Number of neurons for each rule	1	1	2	1	3

Table 2: structure of FWN model for approximating unknown nonlinear function $g(\underline{x})$

Number of sub-WNNs	5				
Total number of efficient wavelets by OLS	5				
Number of fuzzy rules	5				
Rules (R^i)	R^1	R^2	R^3	R^4	R^5
Number of selected Wavelets for each rule (T_{i_g})	1	1	1	1	1
Dilation parameter of each rule (M_{i_g})	-1	-2	0	-3	1
Number of neuron for each rule	1	1	1	1	1

In the following, the proposed controller is applied to balance the pole in the vertical position, (part A). The simulation studies include two stages: (a) for different initial conditions of state variables $\underline{x}_0$ and (b) for the condition where the mass and length of the pole are considered to be time variant. In order to verify the effectiveness of the proposed control strategy under a wide operating range, every simulation is done for two different initial conditions including $\underline{x}_0 = [\pi/6, 0]$, $\underline{x}_0 = [\pi/7, 0]$.

In order to verify the effectiveness of the proposed controller, two different control methods are studied and applied for the purpose of comparison. Selected methods are as follows:

Method 1. Hybrid adaptive fuzzy controller [18]: The hybrid adaptive fuzzy control system (HAFCS) in [18] is a combination of a direct adaptive fuzzy controller and an adaptive fuzzy identification. Proposed controller uses both tracking error and modeling error in the adaptation process. The number of fuzzy rules, number and location of membership functions must be tuned by trial and error.

On the contrary, in the proposed scheme, all of the networks parameters are obtained and adjusted automatically, so there is no need to predefine fuzzy rules and other parameters. Moreover in the proposed scheme, fuzzy rules are fully utilized to capture various essential components of the system due to the MRA of wavelets, while in [18] just local model of system is presented and fuzzy rules are increased, considerably.

Method 2. Adaptive fuzzy neural controller [27]: The proposed adaptive fuzzy neural controller (AFNC) in [27] is presented based on the generalized fuzzy neural network (GFN). This network is a multilayer feed-forward network based on fuzzy inference system, so unlike FWN, this network just represent localized approximation of the system.

Also, adaptive learning algorithm is used to adjust all of premise and consequent parameters of the network. Then network is used to model the inverse dynamics of the system.

In the following, the proposed controller and two adopted controllers are applied to the considered case study and the obtained results are given and discussed.

Part A. Balancing the pole in the vertical position

In this part, proposed controller is applied to regulate the inverted pendulum in the vertical position for different initial conditions. The pendulum parameters were chosen as $g = 9.81 m/s^2$, $l = 0.5m$, $m_c = 1kg$ and $m = 0.1kg$. The output response of the proposed controller and other applied controllers for two different initial conditions are shown in Fig. 1.

The obtained results clearly demonstrate that the angle of the pole is balanced in the vertical position for various initial conditions and output response has good transient characteristics. This figure shows that all control methods provide a good performance for balancing the inverted pendulum system but the proposed method improves the transient response characteristics and has a better performance in terms of overshoot, settling time and response speed, compared to the other applied methods.

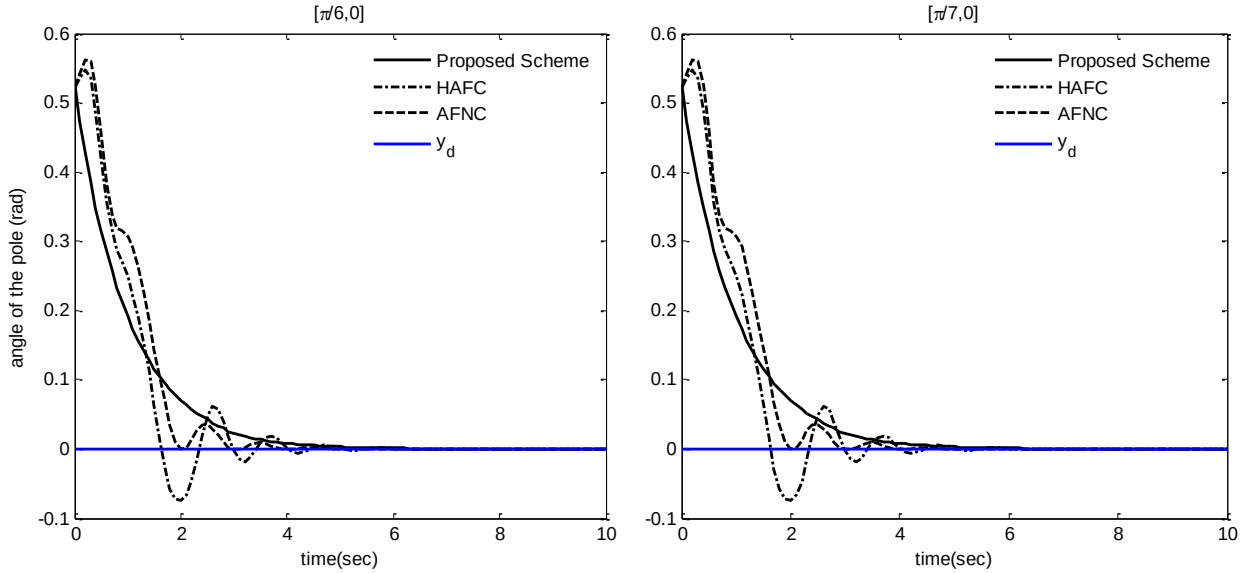

Figure 1: Output response of the applied methods for different initial conditions (Part A).

To have a better perceptiveness about the performance of the proposed method with respect to the others, two performance indices ISE (integrated square error) and IAE (integrated absolute error) for all methods are calculated and reported in Table 3. Presented results in Table 3 show that the values of the two considered indices of the proposed approach are much smaller than other approaches (HAFC, and AFNC).

Table 3: ISE and IAE criterions of all schemes for part A.

Methods	Part A	ISE	IAE
Proposed scheme	Case 1	0.0084	0.0340
HAFC [18]		0.0149	0.0497
AFNC [27]		0.0198	0.0587

Part B. Tracking the desired trajectory

In this part, efficiency and robustness of the proposed controller for the tracking problem is investigated. The control objective is to force the pole to track the desired trajectory $y_d(t) = 0.2(\cos t + \cos(2t))$ for different initial conditions.

The pendulum parameters are chosen as part A. The obtained results of the proposed controller and two other methods for tracking the desired trajectory is given in Fig. 2.

This figure reveals that the proposed integral type adaptive laws can adjust the controller parameters for different initial conditions and good tracking performance is obtained.

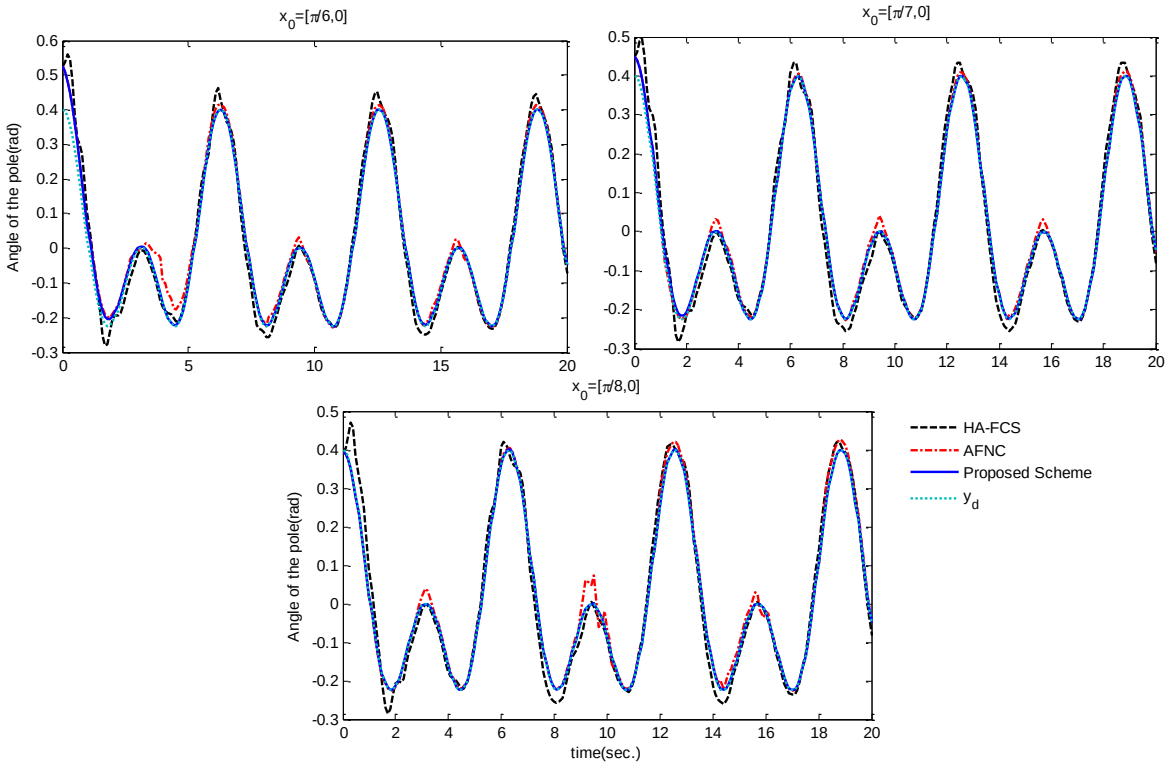

Figure 2: Output response of the applied controllers for different initial conditions.

Also comparison of the obtained results by all applied controllers in two parts, verifies that the proposed control scheme is able to track desired trajectory for different initial conditions. In addition, simulated results verify that the proposed control system performs better than the other controllers in terms of number of fuzzy rules and hidden neurons, appropriate control effort and transient response characteristics such as undershoot, settling time and speed response for the current problem.

Moreover the closed loop trajectories $(x_1(t), x_2(t))$ for tracking the desired trajectory $y_d(t) = 0.2(\sin t + \sin 2t)$ for initial conditions $x_0 = [\pi/4, 0.1]$ are depicted in Fig. 3. As can be seen, the close loop system trajectory from desired initial conditions moves toward equilibrium point (origin) in finite time for the desired output.

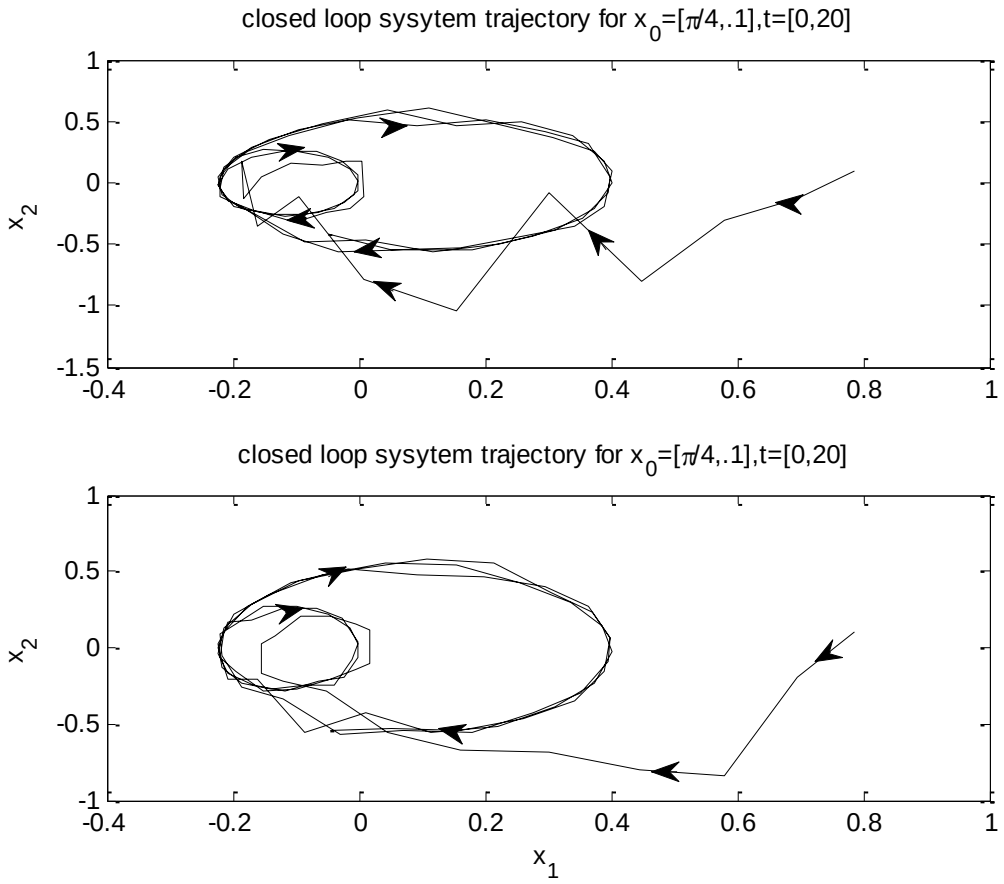

Figure 3: Closed loop trajectory for $\underline{x}_0 = [\pi/4, 0.1]$ and $t = [0,20]$

Conclusion

This paper proposes a new observer-based adaptive fuzzy wavelet network controller with H_∞ tracking performance for a class of nonlinear systems subjected to external disturbance. Proposed controller composed of two terms: adaptive fuzzy wavelet network control term and robust H_∞-based control law. Proposed controller utilizes both off-line and on-line learning phase. During off-line learning phase, two FWNs are constructed to represent the model of the unknown dynamics of the system. The set of fuzzy rules construct FWN, each rule corresponds to one sub-WNN and one adaptation parameter that needs to be adapted, on-line. Integral type adaptive laws are designed to adjust free parameters of the controller, on-line. To attenuate, the effect of the approximation error and external disturbance, H_∞-based control term is applied. Moreover, no prior knowledge about the mathematical model of the system is required. Finally, some simulation and comparison results verify the superior performance and efficiency of the proposed scheme.

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