

Various Types of Parametric Methods of Power Spectral Density Estimation

Meysam Bayat and Hassan Abdollahi

Shahid Sattari Aeronautical University of Science & Technology

*Corresponding Author's E-mail: Meysam.bayat302@gmail.com

Phone Number: +98-21-66693400

Abstract

The basic problem of the single-channel speech enhancement methods lies in a rapid and precise method for estimating noise, on which the quality of enhancement method depends. The paper describes Types of Parametric Methods of Power Spectral Density Estimation in spectral domain. To reduce periodogram variance the proposed method use the procedure of Thresholding, the coefficients of a Periodogram, Then the smoothed estimate of power spectral density of noise is obtained using the inverse discrete Parametric Method.

Keywords: Speech Enhancement, Power Spectral Density, Periodogram, Parametric Method.

1- Introduction

Most of the speech enhancement methods use estimation of noise and interference characteristics. Thus the notion of power spectral density is introduced, which defines the density of total noise energy of a random signal in dependence on frequency. In statistical signal processing, the goal of spectral density estimation (SDE) is to estimate the spectral density of a random signal from a sequence of time samples of the signal. Intuitively speaking, the spectral density characterizes the frequency content of the signal. One purpose of estimating the spectral density is to detect any periodicities in the data, by observing peaks at the frequencies corresponding to these periodicities. SDE should be distinguished from the field of frequency estimation, which assumes that a signal is composed of a limited number of generating frequencies plus noise and seeks to find the location and intensity of the generated frequencies. SDE makes no assumption on the number of components and seeks to estimate the whole generating spectrum. Frequency domain speech enhancement systems typically consist of a spectral analysis/synthesis system, a spectral gain computation method, and a background noise power spectral density (PSD) estimation algorithm. While the former two are well understood [1]–[3] and easily implemented the noise estimator has frequently received less attention. The noise estimator is, however, a very important component of the overall system, especially if the algorithm should be capable of handling non-stationary noise. In fact the noise estimator has a major impact on the overall quality of the speech enhancement system. If the noise estimate is too low, unnatural residual noise will be perceived. If the estimate is too high, speech sounds will be muffled and intelligibility will be lost. The traditional SNR based voice activity detectors (VAD) are difficult to tune and their application to low SNR speech results often in clipped speech. Current research [4]–[6] aims therefore at incorporating soft-decision schemes which are also capable of updating the noise PSD during speech activity. Here Power Spectral Density (PSD) estimation is computed by using parametric and non-parametric methods. The reduction of noise ratio in PSD is considered as the parameter and it is estimated through crest factor. Finally the paper concludes with the need of best windowing method for PSD particularly in parametric techniques. Evaluation is handled both objectively and subjectively for Tamil speech datasets. In this paper, we present a novel noise estimation algorithm which is based

on an optimal signal PSD smoothing method and on minimum statistics. The PSD smoothing algorithm utilizes a first order recursive system with a time and frequency dependent smoothing parameter. The smoothing parameter is optimized for tracking non-stationary signals by minimizing a conditional mean square error criterion. Speech enhancement based on minimum statistics was proposed in [7] and modified in [8]. In contrast to other methods the minimum statistics algorithm does not use any explicit threshold to distinguish between speech activity and speech pause and is therefore more closely related to soft-decision methods than to the traditional voice activity detection methods. Similar to soft-decision methods it can also update the estimated noise PSD during speech activity. It was recently confirmed [9] that the minimum statistics algorithm [7] performs well in non-stationary noise. The remainder of this paper is organized as follows. After a brief introduction to noise estimation via minimum statistics in Section II, we will derive the optimum smoothing parameter and a heuristic error monitoring algorithm in Section III. In Section IV, we investigate the statistics of minimum (noise) power spectral density estimates. An algorithm for the compensation of the bias which is associated with minimum power spectral density estimates is developed in Section V. Section IV presents the algorithm for searching spectral minima. Special emphasis is placed on a novel extension which significantly improves the tracking of non-stationary noise. Finally, in Section VII we summarize experimental results in terms of measurements and listening tests.

2- Power Spectral Density features and their estimators

The simple and easy non-parametric methods do not assume a fixed structure of a model. It can expand to accommodate the complexity of data. It is based on fewer assumptions like wide sense stationary hence their applicability is much wider than parametric methods. As discussed earlier, we would like to estimate the power spectral density (PSD) of the signal $y(t)$, which is obtained by filtering white noise $e(t)$ of power σ^2 through the rational stable and causal filter with the transfer function $H(\omega) = B(\omega)/A(\omega)$, where

$$\begin{aligned} A(\omega) &= 1 + a_1 e^{-i\omega} + \dots + a_n e^{-in\omega} \\ B(\omega) &= 1 + a_1 e^{-i\omega} + \dots + a_n e^{-in\omega} \end{aligned} \quad (1)$$

In the time domain, the above filtering can be represented as

$$y(t) + \sum_{i=1}^m a_i y(t-i) = \sum_{j=0}^m b_j e(t-j), (b_0 = 1) \quad (2)$$

We further divide this problem into three categories based on the values of m and n :

- (i) If both m and n are non-zero, then the signal is said to be Auto-regressive moving average (ARMA) and is denoted by ARMA(n, m).
- (ii) If $m=0$, then the signal is an auto-regressive (AR) signal and is denoted by AR(n); and finally,
- (iii) If $n = 0$, the signal is a moving average (MA) signal and is denoted by MA(m).

3- COVARIANCE STRUCTURE OF ARMA PROCESSES

In this section, we provide an expression for the covariance of a general ARMA process. Multiplying both sides in (2) by $y^*(t-k)$ and taking expectation yields

$$r(k) + \sum_{i=1}^n a_i r(k-i) = \sum_{j=0}^m b_j E\{e(t-j)y^*(t-k)\} \tag{3}$$

To simplify (3) further, we use the fact that $H(q) = B(q)/A(q)$ is causal and stable, that is, we can write:

$$H(q) = \frac{B(q)}{A(q)} = \sum_{l=0}^{\infty} h_l q^{-l}, (h_0 = 1) \tag{4}$$

and thus:

$$y(t) = H(q)e(t) = \sum_{l=0}^{\infty} h_l e(t-l) \tag{5}$$

Therefore, the term $E\{e(t-j)y^*(t-k)\}$ becomes

$$\begin{aligned} E\{e(t-j)y^*(t-k)\} &= E\left\{e(t-j) \sum_{l=0}^{\infty} h_l^* e^*(t-k-l)\right\} \tag{6} \\ &= \sigma^2 \sum_{l=0}^{\infty} h_l^* \delta_{j,k+l} = \sigma^2 h_{j-k}^* \end{aligned}$$

Substituting (6) into (3), we get

$$r(k) + \sum_{i=1}^n a_i r(k-i) = \sigma^2 \sum_{j=1}^m b_j h_{j-k}^* \tag{7}$$

Note that since $H(q)$ is causal, $h_l = 0$ for $l < 0$. This means that for $k \geq m + 1$, equation (7) can be further simplified to

$$r(k) + \sum_{i=1}^n a_i r(k-i) = 0, \quad k > m \tag{8}$$

This equation plays an important role for many estimation techniques, as we will see.

4- AR SIGNALS: YULE-WALKER METHOD

In this section, we derive the so-called Yule-Walker method for estimating the PSD of AR signals. For AR signals, $m=0$ and $B(q) = 1$. From equation (7) we have

$$r(0) + \sum_{i=1}^n a_i r(-i) = \sigma^2 \sum_{j=0}^0 b_j h_{j-k}^* = \sigma^2 \tag{9}$$

for $k=0$. Also for $k=1, \dots, n$, we can use (8) repeatedly. Combining these two and representing the result in a matrix form, we have the following system of linear equations

$$\begin{bmatrix} r(0) & r(-1) & \dots & r(-n) \\ r(1) & r(0) & \dots & r(-n+1) \\ \vdots & \vdots & \ddots & \vdots \\ r(n) & r(n-1) & \dots & r(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sigma^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{10}$$

The above equations are called the Yule-Walker or Normal equations. If $\{r(k)\}_{k=0}^n$ were known, we could use all but the first row of (10) to find the coefficients a_1, \dots, a_n and substituting the resulting coefficients into the first row to find σ^2 . More precisely, we have

$$\underbrace{\begin{bmatrix} r(1) \\ \vdots \\ r(n) \end{bmatrix}}_{\equiv r_n} + \underbrace{\begin{bmatrix} r(0) & \cdots & r(-n+1) \\ \vdots & \ddots & \vdots \\ r(n-1) & \cdots & r(0) \end{bmatrix}}_{\equiv R_n} \underbrace{\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}}_{\equiv \theta} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \tag{11}$$

and thus $\theta = -R_n^{-1}r_n$ and once θ is found, σ^2 can be found by using the first row of (10). The Yule-Walker method for estimating the spectrum of an AR signal is based on the above Yule-Walker equations. More precisely, we first obtain sample covariance's $\{\hat{r}(k)\}_{k=0}^n$ from the observed sequence, using the standard biased estimator

$$\hat{r}(k) = \frac{1}{N} \sum_{t=k+1}^n y(t) y^*(t-k), \quad 0 \leq k \leq N-1 \tag{12}$$

and then, we use (10) and (11) to obtain θ and σ^2 respectively.

5- ORDER-RECURSIVE SOLUTIONS TO THE YULE-WALKER EQUATIONS

In applications, where the degree n of the AR process is known a priori, then the Yule-Walker method can be used to find the spectrum of the signal. However, when the degree of the AR process is not known beforehand, the spectrum of the signal is computed for different but predefined values of n . That is, the spectral is estimated assuming all values of $n \in \{1, 2, \dots, n_{max}\}$ and the "best" estimate is chosen according to some criteria as the ultimate spectral estimator. Since n_{max} is large in some applications, the computational complexity of doing so can be unacceptably high. More precisely, the computational complexity is:

$$\sum_{n=1}^{n_{max}} O(n^3) = O(n_{max}^4) \tag{13}$$

where we assume that the computational complexity of the Yule-Walker method for obtaining the spectrum of an AR(n) process is $O(n^3)$ due to the requirement for inversion of the $n \times n$ matrix R_n in (11). Many algorithms have been proposed to reduce the complexity in such scenarios. In this section, we introduce the Levinson-Durbin algorithm (LDA), which uses the specific structure of the covariance matrices R_n to compute the AR parameters recursively. Before that, we need the following preliminaries. First, to explicitly show the dependence of the AR parameters to the AR degree n , we write (10) as

$$R_{n+1} \begin{bmatrix} 1 \\ \theta_n \end{bmatrix} = \begin{bmatrix} \sigma_n^2 \\ 0 \end{bmatrix} \tag{14}$$

Let ρ_k denote $r(k)$. Since $r(-k) = r^*(k)$, we have:

$$R_{n+1} = \begin{bmatrix} \rho_0 & \rho_{-1} & \cdots & \rho_{-n} \\ \rho_1 & \rho_0 & \cdots & \rho_{-n+1} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_n & \rho_{n-1} & \cdots & \rho_0 \end{bmatrix} = \begin{bmatrix} \rho_0 & \rho_1^* & \cdots & \rho_n^* \\ \rho_1 & \rho_0 & \cdots & \rho_{n-1}^* \\ \vdots & \vdots & \ddots & \vdots \\ \rho_n & \rho_{n-1} & \cdots & \rho_0 \end{bmatrix} \tag{15}$$

and thus R_{n+1} is Hermitian. Also we can see that R_{n+1} is Toeplitz. This enables us to use an important property for Hermitian Toeplitz matrix R , namely

$$y = Rx \Rightarrow \hat{y} = R\hat{x} \tag{16}$$

where for a vector $\hat{x} = [x_n^* \dots x_1^*]$, we define $x = [x_1 \dots x_n] \sim \tilde{x}$, $[x * n \dots x * 1]$. This result follows readily from the following calculation:

$$\begin{aligned} \tilde{y}_i &= y_{n-i+1}^* = ((R_x)_{n-i+1})^* = \sum_{k=1}^n R_{n-i+1,k}^* x_k^* \\ &= \sum_{k=1}^n \rho_{n-i+1-k}^* x_k^* = \sum_{k=1}^n \rho_{i-k} x_{n-k+1}^* = (R\tilde{x})_i \end{aligned} \tag{17}$$

6- LEVINSON-DURBIN ALGORITHM

The basic idea of LDA is to solve (14) recursively in n , starting from the trivial solution for $n=1$:

$$\theta_1 = -\frac{\rho}{\rho_0}, \quad \sigma_1^2 = \rho_0 - \frac{|\rho_1|^2}{\rho_0} \tag{18}$$

Now assume that the solution at stage n is known. Using this known solution, we would like to find the solution at stage $n+1$. In other words, knowing θ_n and σ_n^2 from (14), we would like to solve:

$$R_{n+2} \begin{bmatrix} 1 \\ \theta_{n+1} \end{bmatrix} = \begin{bmatrix} \sigma_{n+1}^2 \\ 0 \end{bmatrix} \tag{19}$$

for θ_{n+1} and σ_{n+1}^2 . In order to do so, we use the nested structure of R_{n+2} . First note that

$$\begin{aligned} R_{n+1} &= \begin{bmatrix} \rho_0 & \rho_{-1} & \cdots & \rho_{-n} & \rho_{-n-1} \\ \rho_1 & \rho_0 & \cdots & \rho_{-n+1} & \rho_{-n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho_n & \rho_{n-1} & \cdots & \rho_0 & \rho_{-1} \\ \rho_{n+1} & \rho_n & \cdots & \rho_1 & \rho_0 \end{bmatrix} = \begin{bmatrix} \rho_0 & \rho_{-1}^* & \cdots & \rho_n^* & \rho_{n-1}^* \\ \rho_1 & \rho_0 & \cdots & \rho_{n-1}^* & \rho_n^* \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho_n & \rho_{n-1} & \cdots & \rho_0 & \rho_{-1}^* \\ \rho_{n+1} & \rho_n & \cdots & \rho_1 & \rho_0 \end{bmatrix} \\ &= \begin{bmatrix} R_{n+1} & \rho_{n+1}^* \\ & \hat{r}_n \\ \rho_{n+1} & \hat{r}_n^* & \rho_0 \end{bmatrix} \end{aligned} \tag{20}$$

where $r_n = [\rho_1 \dots \rho_n]^T$. Multiplying R_{n+2} with the vector $S_n \cong [1 \ \theta_n \ 0]^T$ and using the fact that:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Ax_1 + Bx_2 \\ Cx_1 + Dx_2 \end{bmatrix} \tag{21}$$

We can write:

$$R_{n+2} S_n = \begin{bmatrix} R_{n+1} & \rho_{n+1}^* \\ & \hat{r}_n \\ \rho_{n+1} & \hat{r}_n^* & \rho_0 \end{bmatrix} \begin{bmatrix} 1 \\ \theta_n \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_n^2 \\ 0 \\ \alpha_n \end{bmatrix} \tag{22}$$

Where $\alpha_n = \rho_{n+1} + \tilde{r}_n^* \theta_n$. As we see, if α_n were zero, then (22) would be the counterpart of (14) when n is increased by one. To make that happen, we define $k_{n+1} = -\alpha_n / \sigma_n^2$. Now using the fundamental result for Hermitian Toeplitz matrices given in (16), we see that

$$R_{n+2} (S_n + k_{n+1} \tilde{S}_n) = \begin{bmatrix} \sigma_n^2 \\ 0 \\ \alpha_n \end{bmatrix} + k_{n+1} \begin{bmatrix} \alpha_n^* \\ 0 \\ \sigma_n^2 \end{bmatrix} = \begin{bmatrix} \sigma_n^2 + k_{n+1} \alpha_n^* \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

Therefore, by comparing (23) with:

$$R_{n+2} \begin{bmatrix} 1 \\ \theta_{n+1} \end{bmatrix} = \begin{bmatrix} \sigma_{n+1}^2 \\ 0 \end{bmatrix} \tag{24}$$

We reach to the conclusion that,

$$\theta_{n+1} = \begin{bmatrix} \theta_n \\ 0 \end{bmatrix} + K_{n+1} \begin{bmatrix} \tilde{\theta}_n \\ 1 \end{bmatrix} \tag{25}$$

And

$$\sigma_{n+1}^2 = \sigma_n^2 (1 - |k_{n+1}|^2) \tag{26}$$

The computational complexity of finding the solution at stage n is $O(n)$, due to the matrix multiplication required for computation of α_n . Hence, the total computational complexity of LDA algorithm can be written as:

$$\sum_{n=1}^{n_{\max}} O(n) = O(n_{\max}^2) \tag{27}$$

Thus LDA algorithm can decrease the computational complexity of finding the spectrum by two orders of magnitude.

7- MA SIGNALS

Based on the definition, an MA signal is obtained by filtering white noise through an all-zero filter. Due to lack of poles in the corresponding filter transfer function, the MA model cannot represent the spectrum with narrow peaks well, unless the degree is chosen sufficiently large. This puts somehow a heavy restriction on the use of MA model for engineering applications, since most of the time we are interested in estimating the spectrum that has narrow peaks. In this section, we will introduce a method for spectral estimation using MA model. We first note that for MA signals, we have:

$$y(t) = \sum_{i=0}^m b_j e(t-j) \tag{28}$$

and hence

$$r(k) = 0, \quad \text{for } k > m \tag{29}$$

Since the covariance function $r(k)$ consists of finite number of lags, the power spectral density of an MA signal also includes finite number of components and is given by

$$\phi(\omega) = \sum_{k=-m}^m r(k) e^{-ik\omega} \tag{30}$$

Hence a simple estimate of the PSD is given by inserting sample covariance $\{\hat{r}(k)\}_{k=0}^m$ into (30), resulting in

$$\hat{\phi}(\omega) = \sum_{k=-m}^m \hat{r}(k) e^{-ik\omega} \tag{31}$$

This estimator has exactly the same form as that of the Blackman-Tukey estimator with a rectangular window of length $2m+1$, see nonparametric methods for spectral estimation. Since the Blackman-Tukey estimator has been covered in the previous lecture, we leave the details to the readers.

8- ARMA SIGNALS

Spectra with both narrow peaks and flat nulls cannot be represented with only the AR model or the MA model (at least with the finite length). There are also other examples where modeling according to only AR or MA models fails to represent the spectra well. In those situations, the ARMA model is a good alternative. There is a computational problem with the ARMA model though, namely finding the optimum filter coefficients is a hard task computationally. There are algorithms that find the solution iteratively. However, finding the global optimum is not guaranteed with those algorithms. There are, on the other hand, simpler algorithms for finding the filter coefficients. These algorithms are computationally efficient but the statistical accuracy may be poor in some situations. In this section, we briefly describe one of the latter algorithms known as the modified Yule-Walker (MYW) method.

The MYW method consists of two stages. In the first stage, the AR parameters $\{a_i\}_{i=1}^n$ are computed using (7). More precisely, writing (7) for $k=m+1, m+2, \dots, m+M$, (M to be defined later) in the matrix form and replacing the true covariance $\{r(k)\}$ by their corresponding sample estimates $\{\hat{r}(k)\}$ we get:

$$\begin{bmatrix} \hat{r}(m) & \hat{r}(m-1) & \dots & \hat{r}(m+1-n) \\ \hat{r}(m+1) & \hat{r}(m) & \dots & \hat{r}(m+2-n) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{r}(m+M-1) & \dots & \dots & \hat{r}(m+M-n) \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \vdots \\ \hat{a}_n \end{bmatrix} = - \begin{bmatrix} \hat{r}(m+1) \\ \hat{r}(m+2) \\ \vdots \\ \hat{r}(m+M) \end{bmatrix} \tag{32}$$

In order to solve this equation, we need M to be at least n . If we set $M=n$ in (32) we get

$$\begin{bmatrix} \hat{r}(m) & \dots & \hat{r}(m+1-n) \\ \vdots & \ddots & \vdots \\ \hat{r}(m+n-1) & \dots & \hat{r}(m) \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \vdots \\ \hat{a}_n \end{bmatrix} = - \begin{bmatrix} \hat{r}(m+1) \\ \vdots \\ \hat{r}(m+n) \end{bmatrix} \tag{33}$$

This equation is known as the MYW system of equations. The square matrix in (33) has been shown to be non-singular under mild conditions and hence the solution exists. If we choose $M > n$ in (32), we will get an over determined system of equations. Since we replace the true covariance with the corresponding sample estimates, the resulting over determined system of equations does not necessarily have a solution. However, we can solve for AR parameters by using for instance the least square or total least square methods. We leave the discussion about whether increasing M helps the accuracy of AR estimates to the readers. Once the MYW estimates of AR parameters $\{a_i\}$ are found, we move to the next stage which is finding the spectra of the MA part. Using the MA model in (30), the spectra of the MA part can be written as

$$\sigma^2 |B(\omega)|^2 = \sum \gamma_k r e^{-i\omega k} \tag{34}$$

Where

$$\gamma_k = E\{[B(q)e(t)][B(q)e^*(t-k)]\} \tag{35}$$

denotes the covariance of the MA part (q represents the shift operator; i.e. $q^k y(t) = y(t-k)$) Now using the ARMA model, we have:

$$A(q)y(t) = B(q)e(t) \tag{36}$$

and hence

$$\begin{aligned} \gamma_k &= E\{[A(q)y(t)][A(q)y^*(t-k)]\} = \sum_{l=0}^n \sum_{p=0}^n a_l a_p^* E\{y(t-l)y^*(t-k-p)\} \\ &= \sum_{l=0}^n \sum_{p=0}^n a_l a_p^* r(k+p-l), (a_0 = 1) \end{aligned} \tag{37}$$

for $k=0, \dots, m$. Now inserting the estimated MYW parameters and estimated sample covariances, we get the following estimator of $\{\gamma_k\}$:

$$\hat{\gamma}_k = \sum_{l=0}^n \sum_{p=0}^n \hat{a}_l \hat{a}_p^* \hat{r}(k+p-l) \quad k = 0, \dots, m. \tag{38}$$

For $k=-1, \dots, -m$, we assume that $\hat{\gamma}_k = \hat{\gamma}_{-k}^*$. Finally, the MYW spectrum is estimated by:

$$\hat{\phi}(\omega) = \frac{\sum_{k=-m}^m \hat{\gamma}_k e^{-i\omega k}}{|\hat{A}(\omega)|^2} \tag{39}$$

With

$$\hat{A}(\omega) = 1 + \hat{a}_1 e^{-i\omega} + \dots + \hat{a}_n e^{-in\omega} \tag{40}$$

9- COMPARISON OF DIFFERENT ESTIMATORS VIA AN EXAMPLE

In this section, we examine the properties of non-parametric spectral estimators for a broadband ARMA(4,4) signal with

$$A(q) = 1 - 1.3817q^{-1} + 1.5632q^{-2} - 0.8843q^{-3} + 0.4096q^{-4}$$

$$B(q) = 1 - 0.3544q^{-1} + 0.3508q^{-2} + 0.1736q^{-3} + 0.2401q^{-4}$$

and $\sigma^2 = 1$. The pole-zero diagram of the system is depicted in Fig. 1 and the corresponding power spectral density is illustrated in Fig. 2. We assume that $N = 512$ realizations of the ARMA signal are observed, which are then used to estimate sample covariance estimates required for different spectral estimators. We will use

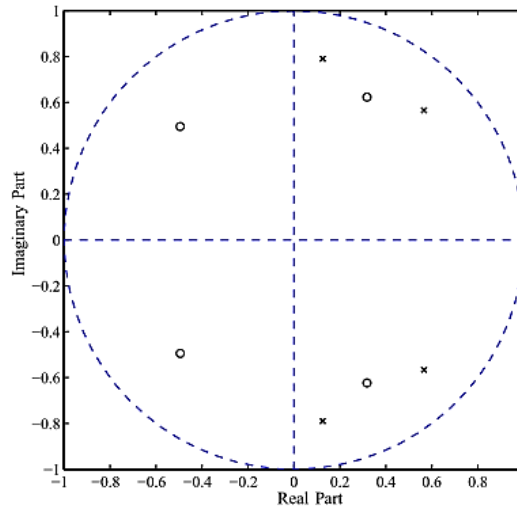


Fig. 1: Pole-zero diagram of the given ARMA signal.

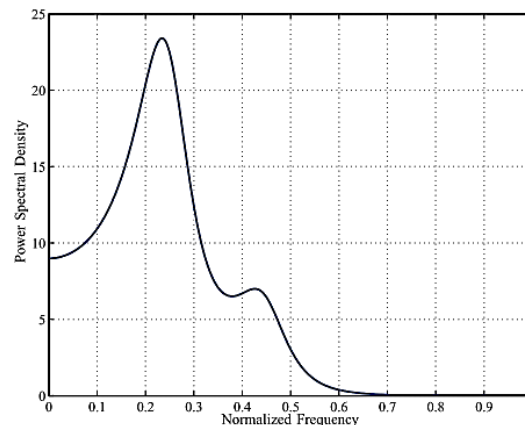


Fig. 2: Power Spectral density of the given ARMA signal.

AR(4), AR(8), ARMA(4,4) and ARMA(8,8) methods. For the MYW method, we will use both $M=n$ and $M=2n$. Fig. 3 and 4 illustrate the estimated pole-zero diagram and the estimated power spectral density for the AR estimator with degree 4, for 10 overlaid estimation samples. Fig. 5 and 6 illustrate the same plots as Fig. 3 and 4 but for AR model with degree 8. As the result show by increasing the degree a better performance can be achieved. This is expected since the model with higher degree can predict the spectrum better. Fig. 7 and 8 illustrate the estimated pole-zero diagram and the estimated power spectral density for the ARMA estimator with degrees $n = 4$, $m=4$ and $M=n$, for 10 overlaid estimation samples. Fig. 9 and 10 illustrate the same plots as Fig. 7 and 8 but for the case with $M=2n$. Fig. 11 and 12 illustrate the estimated pole-zero diagram and the estimated power spectral density for the ARMA estimator with degrees $n=8$, $m=8$ and $M= n$, for 10 overlaid estimation samples. Fig. 13 and 14 illustrate the same plots as Fig. 11 and 12 but for the case with $M=2n$. To see the asymptotic behavior of the above schemes, we let the observed sequence to grow arbitrarily large and we compare different

spectral estimation techniques. Fig. 15 and 16 illustrate the estimated pole-zero diagram and power spectral density for 10 overlaid estimation samples using AR(4) signal model. Fig. 17 and 18 illustrate the same but for AR(8) model. Fig. 19 and 20 illustrate the asymptotic behavior of ARMA(4,4) method. Finally, Fig. 21 illustrates the power spectral density estimates using periodogram method. The plot is obtained by averaging over 30 estimation samples.

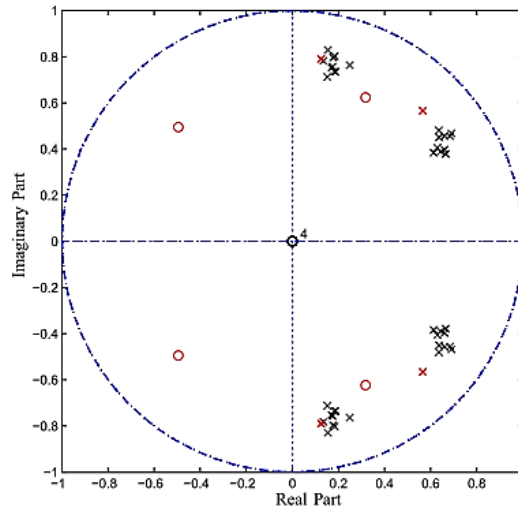


Fig. 3: Pole-zero diagram for 10 overlaid estimations using AR(4) method.

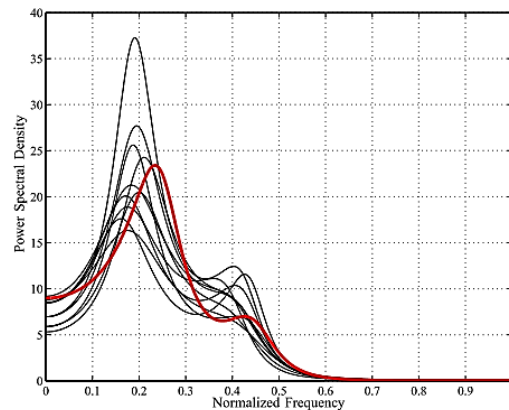


Fig. 4: Power spectral density estimates for 10 overlaid samples using AR(4) method.

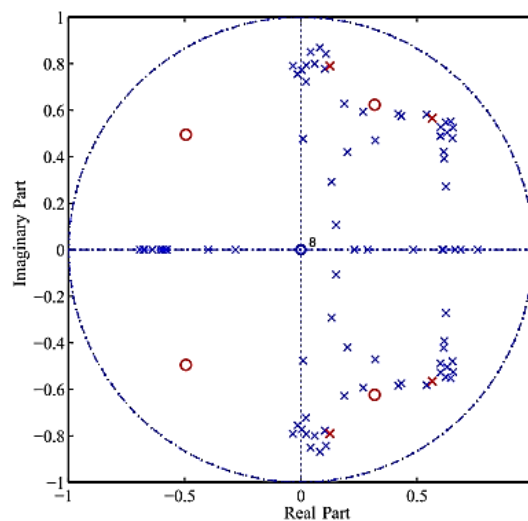


Fig. 5: Pole-zero diagram for 10 overlaid estimations using AR(8) method.

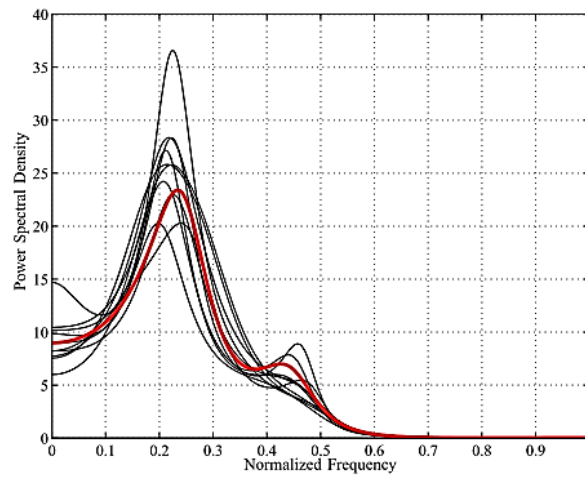


Fig. 6: Power spectral density estimates for 10 overlaid samples using AR(8) method.

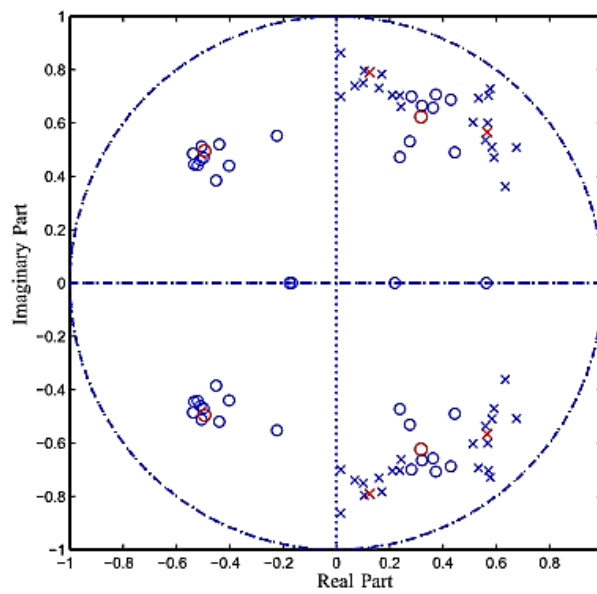


Fig. 7: Pole-zero diagram for 10 overlaid estimations using ARMA(4,4) method with $M = n$.

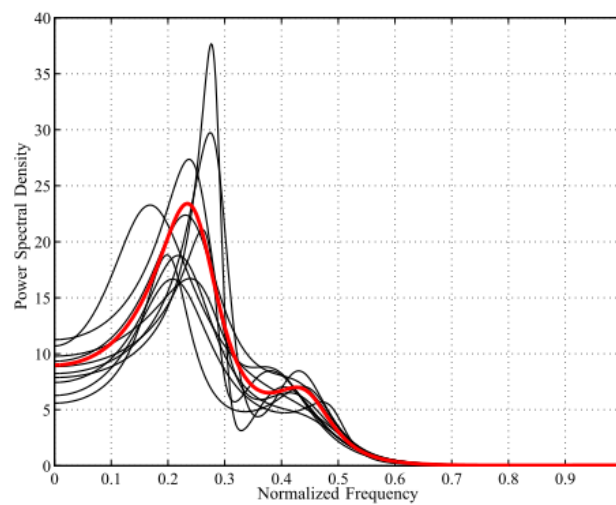


Fig. 8: Power spectral density estimates for 10 overlaid samples using ARMA(4,4) method with $M=n$.

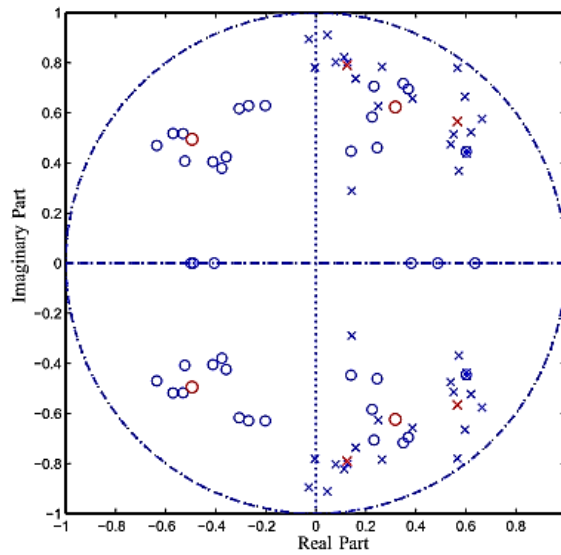


Fig. 9: Pole-zero diagram for 10 overlaid estimations using ARMA(4,4) method with $M = 2n$.

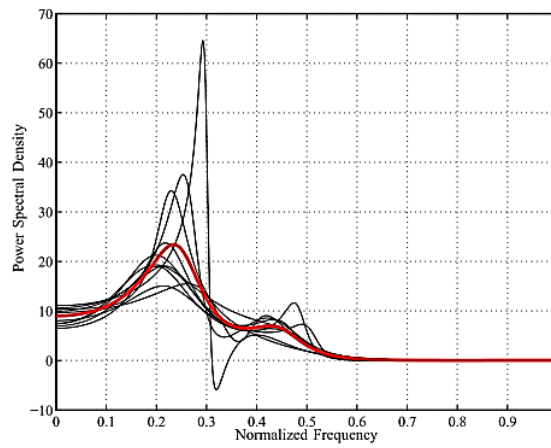


Fig. 10: Power spectral density estimates for 10 overlaid samples using ARMA(4,4) method with $M=2n$.

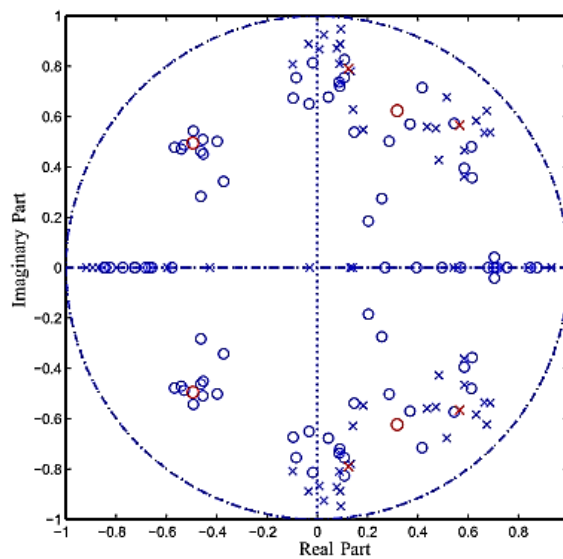


Fig. 11: Pole-zero diagram for 10 overlaid estimations using ARMA(8,8) method with $M = n$.

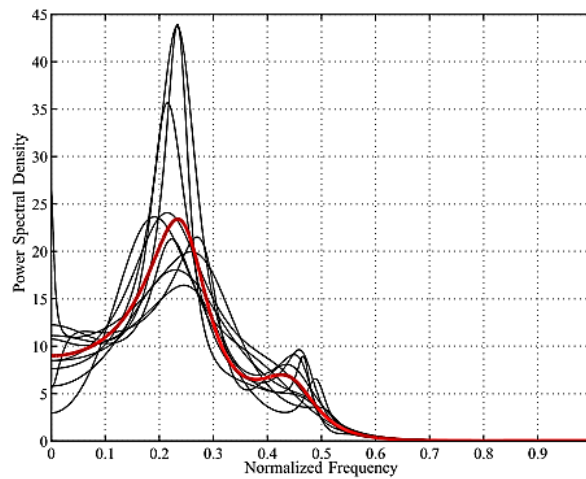


Fig. 12: Power spectral density estimates for 10 overlaid samples using ARMA(8,8) method with $M=n$.

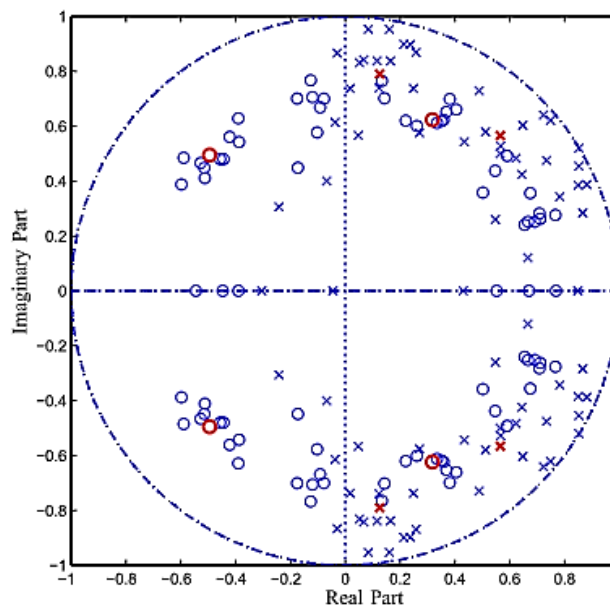


Fig. 13: Pole-zero diagram for 10 overlaid estimations using ARMA(8,8) method with $M = 2n$.

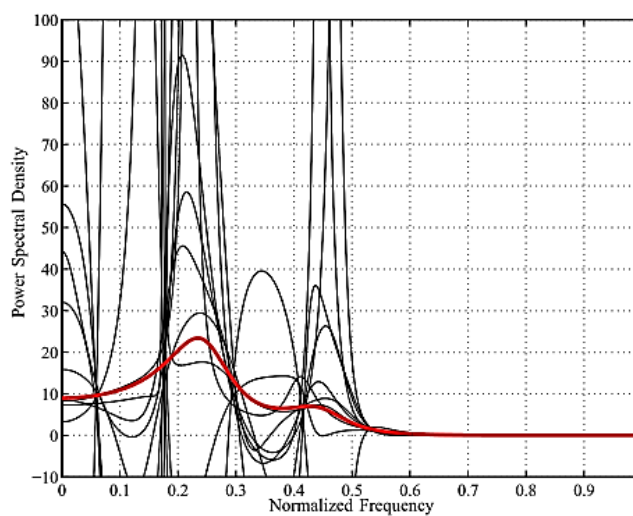


Fig. 14: Power spectral density estimates for 10 overlaid samples using ARMA(8,8) method with $M=2n$.

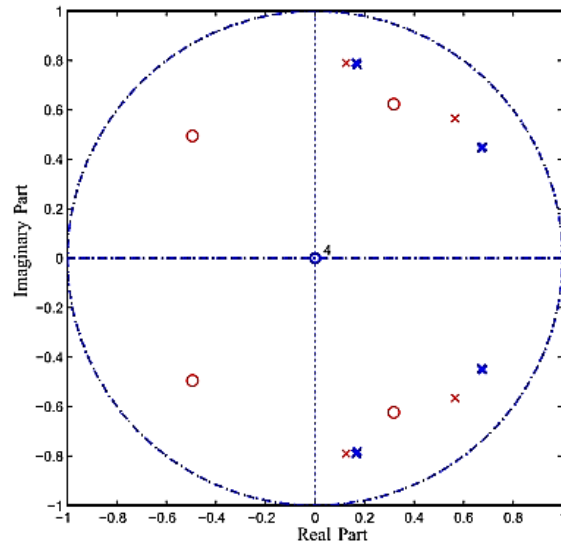


Fig. 15: Pole-zero diagram for 10 overlaid estimations using AR(4) method with many observations.

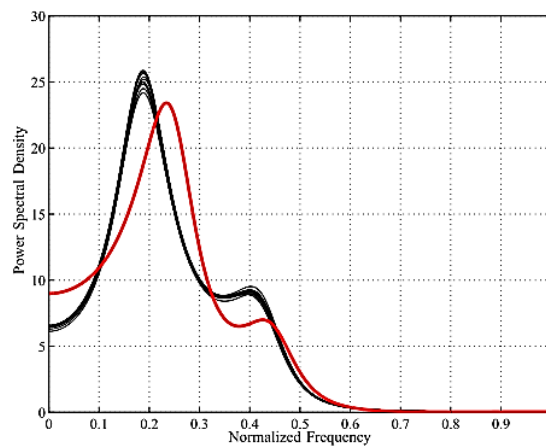


Fig. 16: Power spectral density estimates for 10 overlaid samples using AR(4) method with many observation.

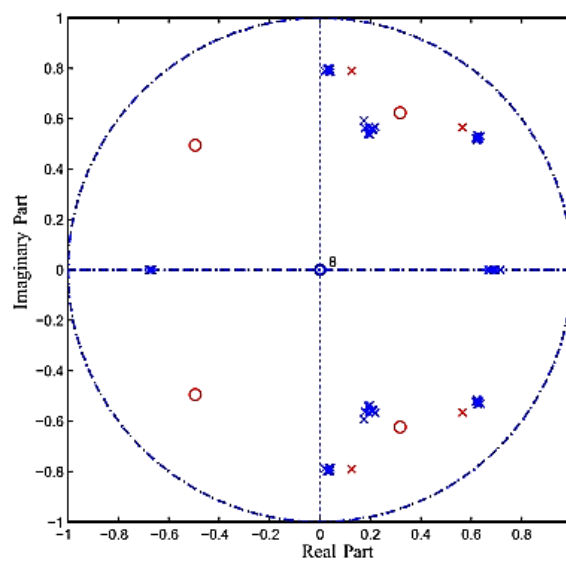


Fig. 17: Pole-zero diagram for 10 overlaid estimations using AR(8) method with many observations.

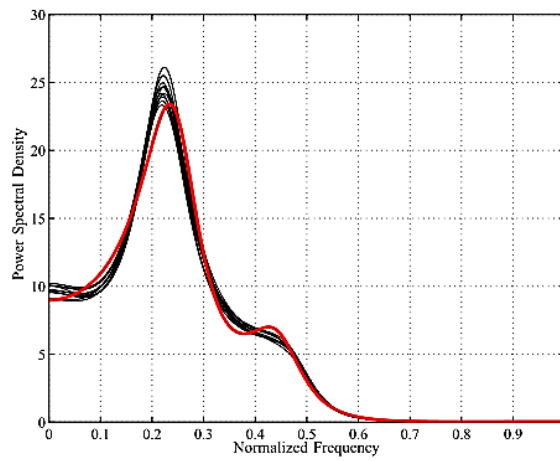


Fig. 18: Power spectral density estimates for 10 overlaid samples using AR(8) method with many observation.

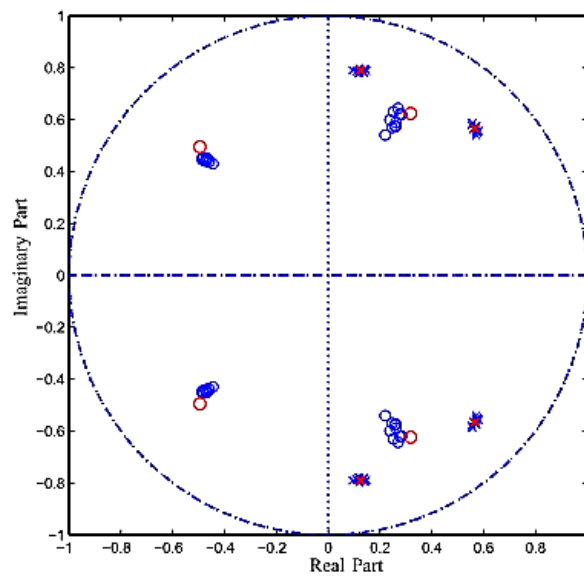


Fig. 19: Pole-zero diagram for 10 overlaid estimations using ARMA(4,4) method with many observations.

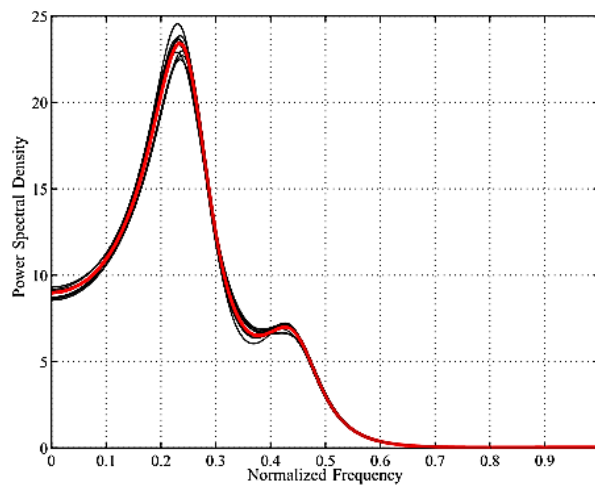


Fig. 20: Power spectral density estimates for 10 overlaid samples using ARMA(4,4) method with many observation.

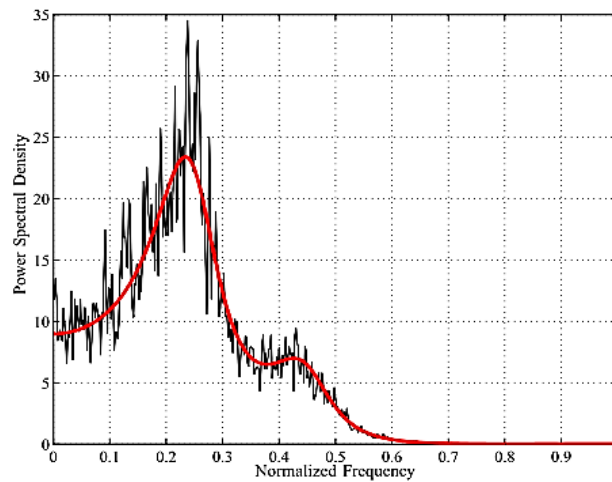


Fig.21: Power spectral density estimates using periodogram method. The results are averaged over 30 estimation instances.

Conclusion

The several methods for the estimation of Power spectral density of noise using Parametric Methods were demonstrated. They led to a reduced variance in the estimate of Power Spectral density of noise. The accuracy of power spectral density estimation can greatly affect the results of the methods that use it. Those can be demonstrated on speech enhancement by the spectral subtraction method with various types of interference and various signal-to-noise ratios. Although the mean value of improvement in the signal-to-noise ratio changes very little depending on the method of Power spectral density estimation used, marked differences can be seen in the variance of improvement in the signal-to-noise ratio. It can be said that methods of power spectral density estimation with a smaller variance of estimation provide a smaller variance of the improvement of signal-to-noise ratio. Hence these methods are more suitable because the improvement in the signal-to-noise ratio is less dependent on the type of interference or on the input signal-to-noise ratio.

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