



## Inverse Kinematics Modeling and Control based on Closed-Loop Vector Method of a 3DOF Translational Manipulator

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### Abstract

This work is focused on the inverse kinematics (IK) modeling, control and implementation of a 3DOF translational manipulator which is a modified version of the popular Delta robot. In order to distinguish this Delta robot with other existing ones, in this paper, it is tentatively named as MacDelta. MacDelta has a fixed base at top and a moving platform at bottom, connected with 3 identical arms. Each arm is configured with a revolute joint (namely the rotational motor, attached to the fixed base), another revolute joint (attached to center of the top side of the parallelogram four-bar mechanism), a parallelogram four-bar mechanism, and a third revolute joint which connects the center of the bottom side of the four-bar mechanism and the moving platform. The analytic solution is derived in details based on the spatial closed-loop vector method. The IK model is programmable and applied in MacDelta.

**Keywords:** *Inverse Kinematics Control, Closed-Loop Vector, Translational Manipulator, MacDelta, Four-Bar Mechanism*

### 1. Introduction

There are basically two main types of industrial robots in terms of mechanism or robotic kinematic structure, namely serial manipulator and parallel manipulator. Serial manipulators, or robotic arms, have been widely recognized and accepted for several decades. Serial manipulator is usually configured with a single arm between base and the end-effector. The counterpart of serial manipulator is called as parallel manipulator. Stewart platform and Delta robot are two most representative parallel manipulators. Different with serial manipulator, parallel manipulator have multiple arms connecting the base and the end-effector, which makes it possess new features and advantages as a kind of industrial robot [1-9]. Generally speaking, parallel manipulator has larger load capacity, higher stiffness, higher precision, better dexterity, lower inertia, and relatively smaller workspace compared to a serial manipulator with similar physical size. Nowadays, the applications of parallel manipulator are gradually expanded and are not limited to picking-and-placing, packaging, motion simulation, micro/nano positioning, and 3D printing [10-19]. For kinematics modeling of parallel manipulator, it includes the IK issue and forward kinematics (FK) issue. IK means when the pose (position and orientation) of the end-effector is given, to calculate the motor angles as inputs. FK means when the motor angles are known, to derive the pose of the end-effector. Generally, there are multiple solutions of FK and it is

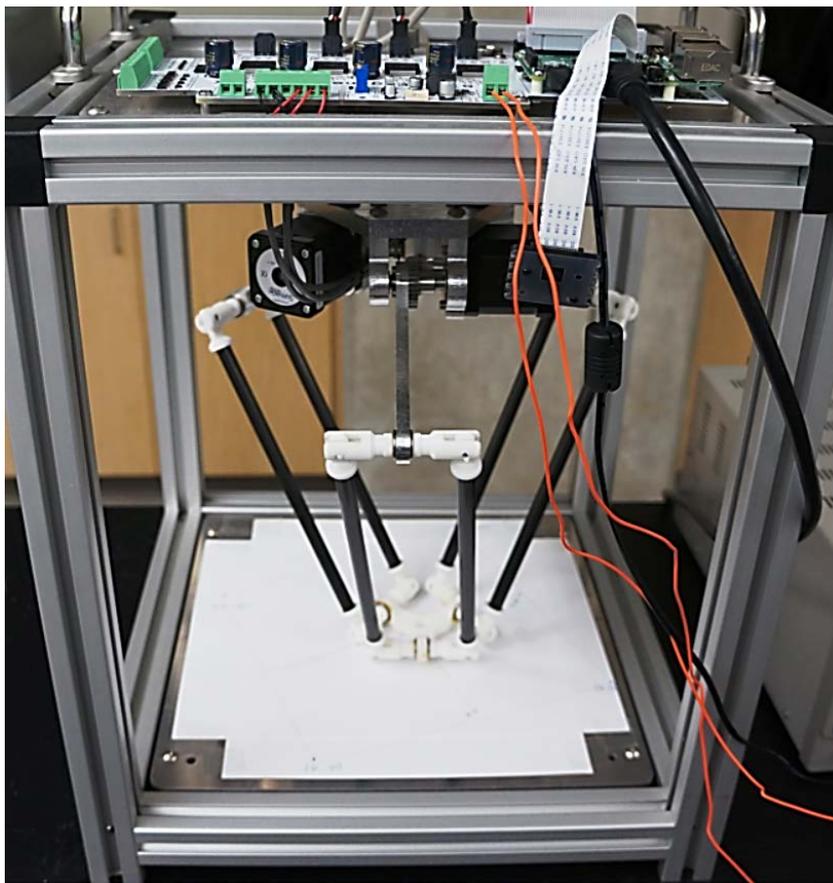
difficult to calculate it. For the kinematic control of parallel manipulator, usually IK is required. That is the reason why the scholars care more about the IK issue for parallel manipulator.

## 2. System Configuration

The proposed MacDelta is shown in the following figures. The body of the manipulator contains three legs. Each leg, from upper platform to lower platform, is configured with a stepper DC motor which is attached to the upper platform, a rotational joint, another rotational joint, a parallelogram four-bar mechanism, and a third rotational joint which is attached to the lower platform. The motor drive and the controller are installed on top of the frame.

The connection between motor drive and the controller is built based on the GPIO interface. The power supplies of the motor drive and the controller are separated. The joints for the parallelogram four-bar mechanism is replaceable and can be 3D printed.

Note that the coordinate system of the moving platform is not aligned with the lower frame. Thus, the kinematics calibration in the workspace should be conducted when implementing the inverse kinematics control.



**Figure 1:** Prototype

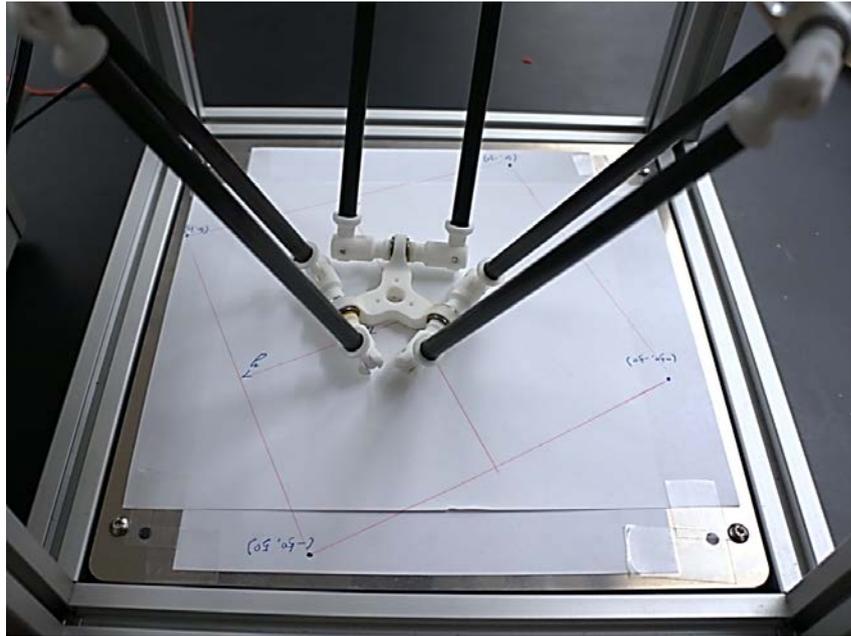


Figure 2: Lower platform

### 3. Inverse Kinematics Modelling and Control

The basic kinematic parameters are measured and given as follows:

$$z = -220 \text{ mm}$$

$$L = 100 \text{ mm}$$

$$l = 230 \text{ mm}$$

$$\text{motor\_distance} = 80 \text{ mm}$$

$$\text{Distance\_from\_center\_of\_lower\_platform\_to\_the\_center\_of\_end\_effector} = 30 \text{ mm}$$

The kinematic structure is drawn in figure 3.

The centers of the upper platform and lower platform are B and P, respectively. The coordinate of B-BxByBz is the global reference frame.

The reference frame of P-PxPyPz is attached to the lower platform, which means when it moves along with the moving platform.

The home position of the lower platform is decided by the initial position of the point P, which is:

$$x = 0$$

$$y = 0$$

$$z = -220 \text{ mm}$$

The length from point Bx ( $x = 1, 2, 3$ ) to Ay ( $y = 1, 2, 3$ ) is L. The length from point Ay to Pz ( $z = 1, 2, 3$ ) is l.

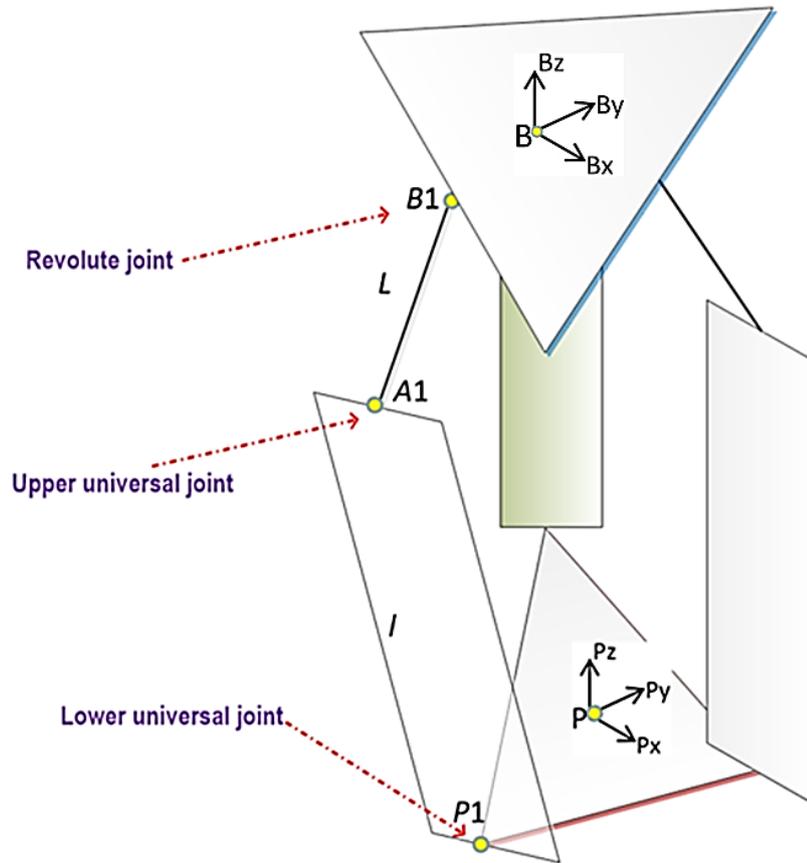


Figure 3: Kinematic structure

In order to solve the inverse kinematics for the proposed manipulator, the method of closed-loop vector is applied in this scenario. As shown in the following figure, there are two ways to describe the vector of  $\overrightarrow{BP_n}$  ( $n = 1, 2, 3$ ). Namely,

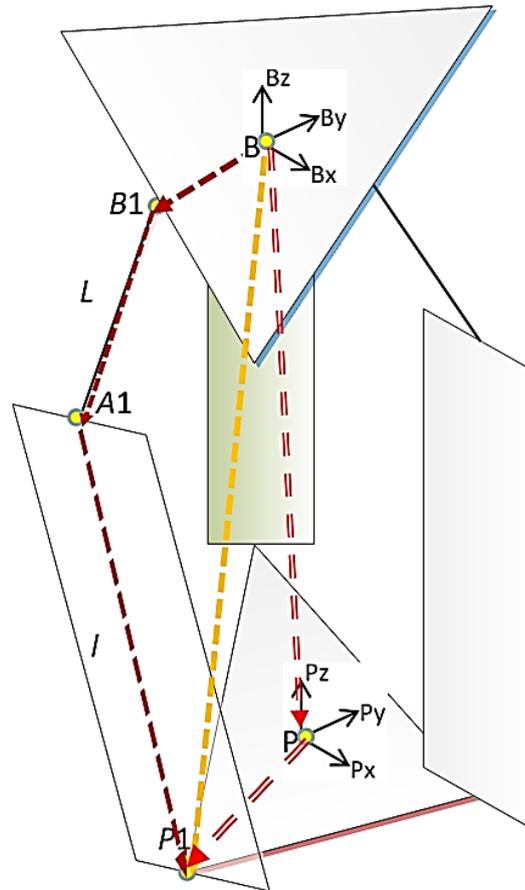
$$\overrightarrow{BB_n} + \overrightarrow{B_nA_n} + \overrightarrow{A_nP_n} = \overrightarrow{BP_n} = \overrightarrow{BP} + \overrightarrow{PP_n} \tag{1}$$

Equation 1 can be rewritten as,

$$\overrightarrow{BB_1} + \overrightarrow{B_1A_1} + \overrightarrow{A_1P_1} = \overrightarrow{BP} + \overrightarrow{PP_1} \tag{2}$$

$$\overrightarrow{BB_2} + \overrightarrow{B_2A_2} + \overrightarrow{A_2P_2} = \overrightarrow{BP} + \overrightarrow{PP_2} \tag{3}$$

$$\overrightarrow{BB_3} + \overrightarrow{B_3A_3} + \overrightarrow{A_3P_3} = \overrightarrow{BP} + \overrightarrow{PP_3} \tag{4}$$



**Figure 4:** Kinematic structure with closed loop vector

For the issue of inverse kinematics, based on the above three equations, the input parameters of the three stepper motors can be solved with analytic solution, which is unique.

A human-machine interface (HMI) is built as shown in figure 5. The first four inputs, namely side base, end-effector, bicep length and forearm, are structural dimensions which can be measured when the robotic system is fabricated.

The positions of  $x$ ,  $y$  and  $z$  are the required position of the moving platform. With the aforementioned input information, when the function button 'Calculate Inverse Kinematics' is pressed, the three motors' feed data would be correspondingly calculated.

Generally speaking, forward kinematics has multiple solutions. However, if the robot structure is well designed, due to the existence of mechanical interference, unique solution in general case is also possible. That is why in the real application of parallel robot, forward kinematics is also utilized in some specific scenario.

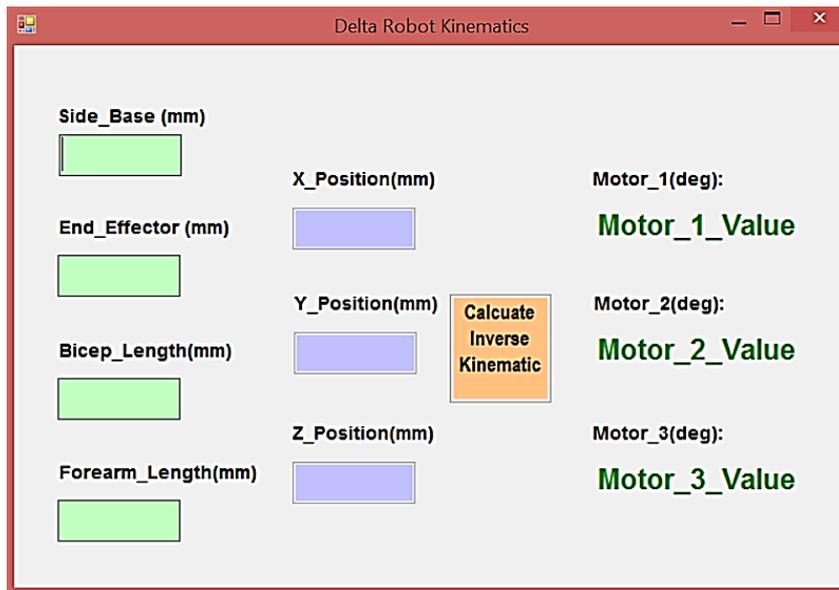


Figure 5: The HMI of inverse kinematics control

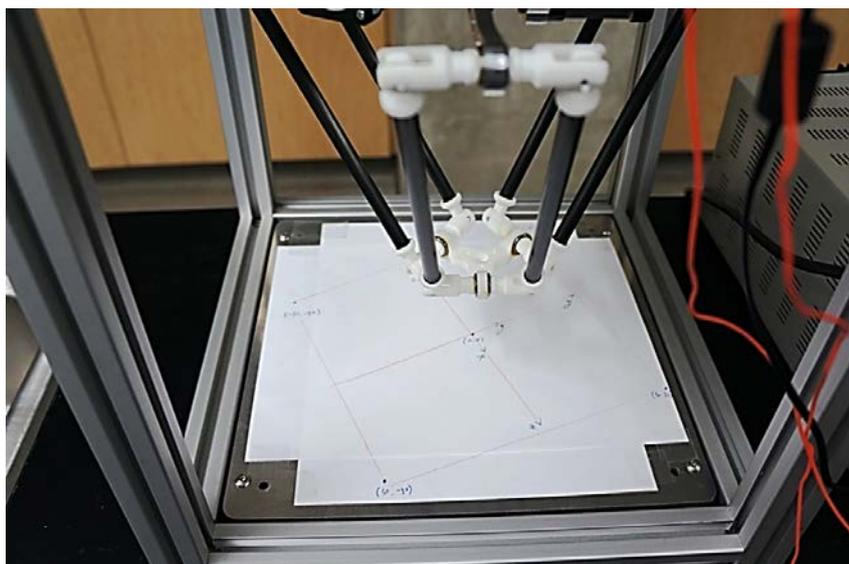
#### 4. Implementation

To implement the inverse kinematic control of the proposed manipulator, the function `dk.inverse(x, y, z)` is programmed to take the position coordinates of the moving platform and derive the angle that each stepper motor must move through to move the end effector to that desired position. In a word, `dk.inverse(x, y, z)` returns the input angles of three motors.

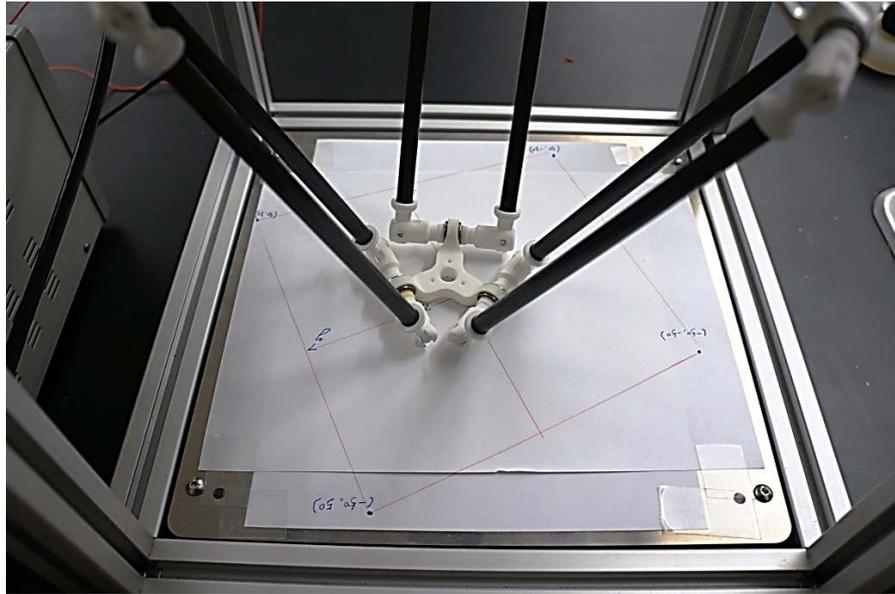
In order to convert the angles into the number of steps, following three commands should be implemented.

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distance_1 = angles[1] * Steps_per_degree
distance_2 = angles[2] * Steps_per_degree
distance_3 = angles[3] * Steps_per_degree
    
```



(a)



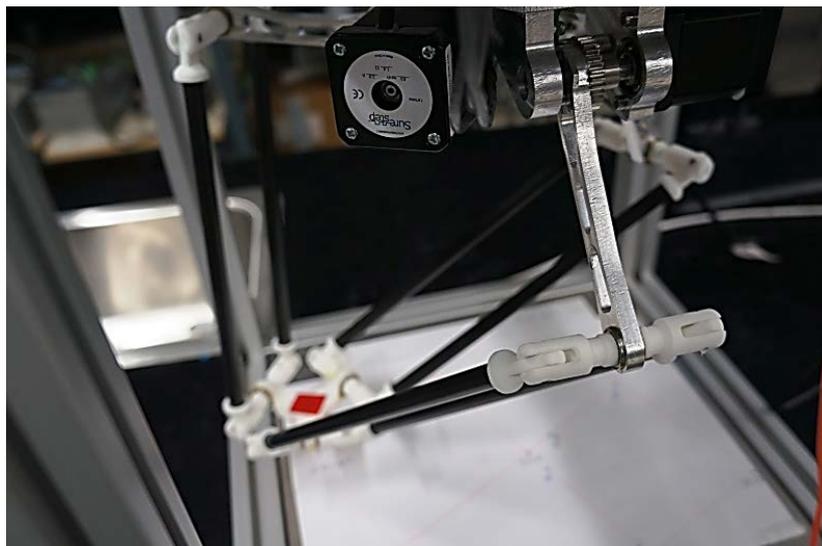
(b)

**Figure 6:** Demo experiments

When this robotic system is ready to run, the end-effector is located in the home position, which is right above the origin of the coordinate system on the lower frame with the distance of 22cm, as shown in figure 6 (a).

The end-effector has the 3 translational movements which can be triggered by keyboard event. The motion in a pure direction is achievable. Obviously, the hybrid motion with the combination of 2 or more than 2 directions is feasible by pressing more than one key.

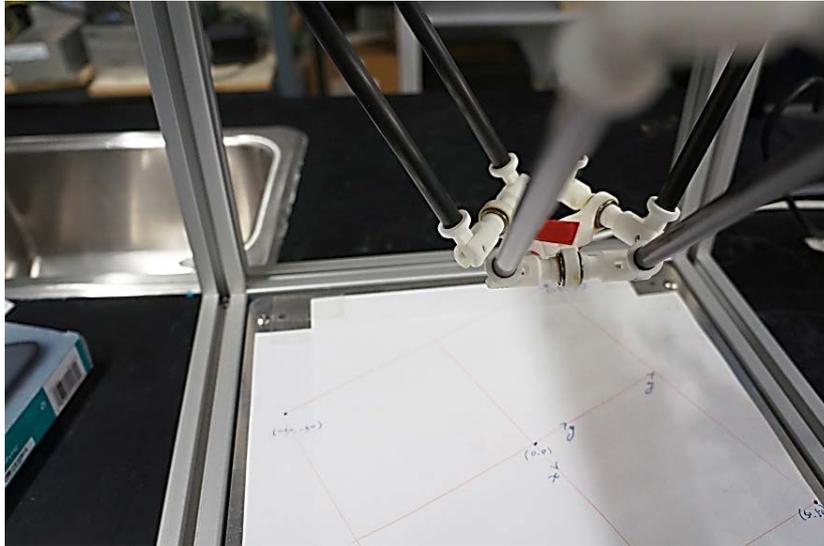
Three representative poses are selected to show the relationship between the position of end-effector and the angles of motors, as depicted in figures 7-9 and tables 1-3.



**Figure 7:** Representative pose 1

**Table 1:** Parameters for representative pose 1

Position of end-effector (mm)	x-axis	y-axis	z-axis
	-50	-49	-245
Angles of motors (deg)	Motor 2	Motor 2	Motor 3
	-60.32	0.05	-46.68



**Figure 8:** Representative pose 2

**Table 2:** Parameters for representative pose 2

Position of end-effector (mm)	x-axis	y-axis	z-axis
	-37	-50	-165
Angles of motors (deg)	Motor 2	Motor 2	Motor 3
	30.48	18.4	-1.08



**Figure 9:** Representative pose 3

**Table 3:** Parameters for representative pose 3

Position of end-effector (mm)	x-axis	y-axis	z-axis
	49	50	-245
Angles of motors (deg)	Motor 2	Motor 2	Motor 3
	-8.97	-66.81	-26.93

## Conclusions

A parallel manipulator called MacDelta is introduced in this work. It put the emphasis on the modeling, control and implementation of the IK issue. The system configuration is illustrated to describe the mechanical structure of the proposed robot. The method of closed-loop vector is applied to derive the analytic solution of the IK model. A HMI is built to generate the result of inverse kinematics automatically by giving the input conditions. This HMI is generic and it can solve the IK issue of different kind of delta robots. For the future work, real-life application will be considered to by using the IK and visual servo.

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