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# Compare Performance of Fractional-Order Fuzzy PID Controller for tow-link Robotic Manipulator via Evolutionary Algorithms

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## Abstract

Tow-link robotic manipulator is a strongly nonlinear and multi-input multi-output system. Therefore engineers interested in designing a high accurate controller. Currently, the controller method for this type of manipulators has been designed, is a Fractional Fuzzy PID (FOFPID) controller for controlling the robot's position that Genetic Algorithm (GA) and Particles Swarm Optimization (PSO) algorithm are tuned and compared fractional parameters. The results of intended algorithms indicate that PSO Technique has a better performance than GA and with more accurate adjustment of fractional parameters, positioning error of robot deducts.

***Keywords:*** Robotic manipulator, Fractional calculus, FOFPID controller, Fuzzy logic controller, evolutionary algorithm.

## 1. Introduction

In recent years, the use of the robot manipulator in industrial applications has great importance. Various control algorithms such as optimal control, adaptive control, neural networks and fuzzy systems have been proposed to address this problem. Also, in recent years, fractional computing applications has been used increasingly in modeling and controller designing for dynamic systems [1,3]. The reason for considering this increase is fractional calculus method to describe a real system are more accurate than the integer order calculations method [4]. Sharma et al. controlled a two-link robotic manipulator by using a kind of fractional order fuzzy PID controller structures where parameters have been set by the Cuckoo Search Algorithm (CSA) [5]. Delavari et al. published a fractional adaptive PID controller for robotic manipulator in which parameters of PID are updated online whereas other two fractional order parameters are defined offline. The tracking error showed graphically and was claimed to be less. The performance was enhanced with the suggested controller as compared to its integer order design [6]. Nikkhah et al. designed a fractional PI controller for improving the performance of parallel power active filters that has a better performance in comparison with PI controller [7]. Podlubny offered a Fractional Order PID (FOPID) controller that has more effective than integer order PID controller [8]. In Silva et al paper's, performances of the fractional controllers and integer order controllers on a six-legged robot with flexible and friction adhesion legs have been studied [9]. Valerio et al. have evaluated in an article the advantages and disadvantages of different digital implementation for controlling the robotic manipulator via fractional controllers. There are five tunable parameters $\{K_p, K_i, K_d, \lambda, \mu\}$ in the FOPID controller that must be tuned. In



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this paper the fuzzy controller updates the PID parameters adaptively and operator's order has tuned by PSO and GA.

The rest of paper is organized as follows:

In section 2, dynamic equations of the robot manipulator has been described. In Section 3, a description of fractional calculus and definitions in this field has been expressed .The proposed control law for robot manipulator has been brought in section 4. In section 5, the structure of evolutionary algorithms has been discussed. In last section, Simulation and results of Fuzzy PID (FPID) and FOFPID controller has been reported and finally, the conclusion of this paper will be rendered.

## 2. Mathematical Model of Robotic Manipulator

In this section involves the modeling of the two-link robotic manipulator dynamics. The dynamic robotic manipulator equation is as follows [5]:

$$\begin{aligned}\tau_1 = {} & l_2^2 m_2 (\frac{d^2 q_1}{dt^2}) + l_1^2 (m_1+m_2)(\frac{d^2 q_1}{dt^2}) + m_2 g l_2 \cos q_1 \cos q_2 + (m_1+m_2) l_1 g \cos q_1 \\ & - m_2 l_1 l_2 \sin q_2 (\frac{dq_2}{dt})^2 - 2 m_2 l_1 l_2 \times \sin q_2 \, (\frac{dq_1}{dt})(\frac{dq_2}{dt}) + m_2 l_1 l_2 \cos q_2 (2\frac{dq_1^2}{dt^2} + \frac{dq_2^2}{dt^2})\end{aligned} \quad (1)$$

$$\tau_2 = m_2 l_1 l_2 \sin q_2 (\frac{dq_1}{dt})^2 + m_2 l_1 l_2 \cos q_2 (\frac{dq_1^2}{dt^2}) + m_2 l_2^2 (\frac{dq_1^2}{dt^2} + \frac{dq_2^2}{dt^2}) + m_2 l_1 g \cos q_1 \cos q_2 \quad (2)$$

In this paper the numerical values of the tow-link robot manipulator's parameters are adjusted as follow:

$l_1 = 1m$ ، $l_2 = 0.8m$ ، $m_1 = 1kg$ ، $m_2 = 1kg$ and $g = 9.8$ .

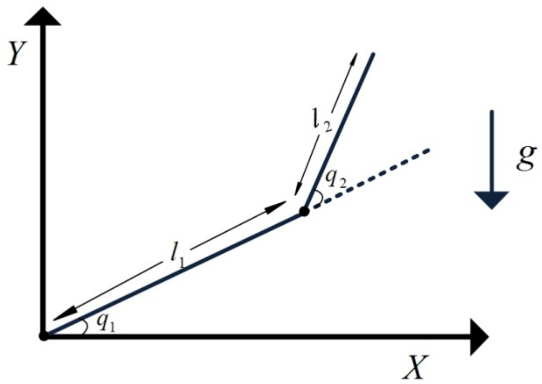


**Figure 1:** A two-links robot manipulator.

## 3. Fractional Calculus

Fractional Calculus is a branch of mathematics that dealing with real number powers of differential or integral operators. It generalizes the common concepts of derivative and integral [7]. ${}_aD_t^{\alpha}$ is a fractional integrator order or derived fractional order which $\alpha$ is operator degree and $a$ and $t$ are lower and upper bounds operator, respectively. $\alpha$ can be positive or negative [11].

Different definitions of fractional derivative and fractional integrator is presented by the authors. Some of the famous methods are Riemann-Liouville (R-L), Grunwald-Letnikov (G-L), Mittag-Leffler (M-L), Caputo definition and oustaloup approximation. Definition of extended Derivative - integral is introduced by equation (3) [12].

$$ {}_{a_0}D_t^{\alpha_0} = \begin{cases} \dfrac{d^{\alpha_0}}{dt^{\alpha 0}} & R(\alpha_0) > 0 \\ 1 & R(\alpha_0) = 0 \\ \int_{a_0}^{t} (d\tau)^{-\alpha_0} & R(\alpha_0) < 0 \end{cases} \tag{3} $$

Where $a_0$ is the initial conditions and $\alpha_0$ is fractional operator.

$$ s^a = k_0 \prod_{k_0=-N}^{N} \frac{s + W_{k_z}}{s + W_{k_p}} \tag{4} $$

$$ W_{k_z} = w_b \left(\frac{w_h}{w_b}\right)^{\frac{k+N+\frac{1}{2}-\frac{a}{2}}{2N+1}} \tag{5} $$

$$ k_0 = w_h^a \tag{6} $$

In this paper, we used the oustaloup approximation to fractional implement which includes the return distribution of zero and poles [13]. Oustaloup fractional approximation controller technique, due to its ability to implement hardware is preferred to other methods [14].

## 4. Control Strategies

### 4.1. Fractional PID controller

Now a days, one of the most wide spread controllers in application is PID controller. Its tangible reality that PID controllers are easy to design and implement. More ever, they are robustness in the uncertainties. In FOPID controllers, I and D operators are usually fractional order. So by controlling the fractional integrator and fractional derivative operators, two more freedom degree, will be added to $K_p$ ،$K_i$ and $K_d$ variables. These two variables, $\lambda$ is for fractional integrator and $\mu$ is for fractional derivative. These two more freedom degrees for variables make FOPID controller which has a better application than integer order PID controller. FOPID transfer function is written as follow [15]:

$$ u(t) = (K_p + K_i D^{-\lambda} + K_d D^{\mu}) e(t) \tag{7} $$

And control law of FOPID controller in the time domain can be written as follow:

$$ C(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{8} $$

In FOPID controller, orders can be non-integer numbers. If $\lambda = 1$ and $\mu = 1$, this hint at classic PID controller. If we choose $\lambda = 1$ and $\mu = 0$, we have a PI controller and finally by the choosing $\lambda = 0$ and $\mu = 1$ we have PD controller. It is obvious that when $\lambda$ and $\mu$ are integer numbers 1 and 0, classic PI, PD and PID controllers. If the numbers be except 0 and 1, it becomes a fractional controller which is more flexible than conventional controllers.

### 4.2. Fractional Order Fuzzy PID controller

Fuzzy controller has been used widely in industrial processes. Especially, fuzzy control methods will be effective, when the mathematical model is non-linear system or there is no mathematical model.

The purpose of this paper, is adjusting the controller parameters. It means that $K_p$ ، $K_i$ and $K_d$ parameters are adjusted online by fuzzy rules. Framework of the FOPID controller's structure is shown in figure (2). In this system fuzzy inference is Mamdani min–max inference method. Also for defuzzification, center of gravity (COG) method has been used. Fuzzy Controllers entries are error and fractional order change of error that are considered in position error and velocity error, respectively that is the range of [-3.3]. For all in-puts and out-puts, 7 membership functions with triangular shape has been selected. Range of the membership functions for link (1) are also $k_p=[0,30], k_i=[0,60], k_d=[0,100]$ respectively and for link (2) are $k_p=[0,100], k_i=[0,60], k_d=[0,30]$ respectively. The number of rules that have been written for this system are 49. Each rule follows the IF-THEN. Also, this system has 2 inputs, $\{E, \frac{d^{\mu}}{dt^{\mu}}\Delta E\}$, 3 outputs $\{K_p, K_i, K_d\}$ That $E$ ، $\frac{d^{\mu}}{dt^{\mu}}\Delta E$ ، $\mu$ and $\lambda$ are errors, changes of the fractional error, order of derivative and order of integral, respectively.

Rules of the PID coefficients and FOPID coefficients witch are the same for both links are shown in tables (1-2-3).

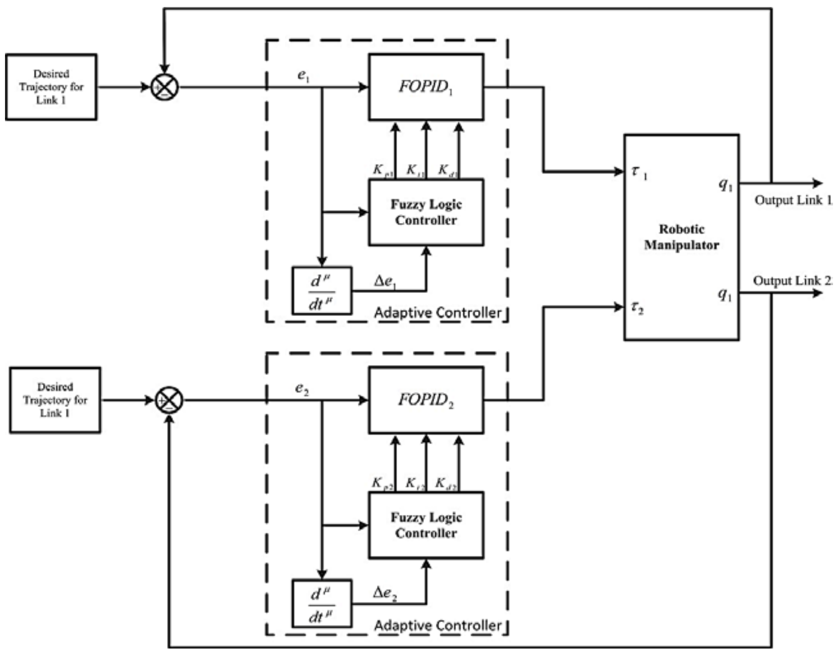


**Figure 2:** structure of the FOFPID controller on a robotic manipulator.

**Table 1:** fuzzy rules for $K_p$

| $K_p$ | | Fractional rate of change of error | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | **NB** | **NM** | **NS** | **ZO** | **PS** | **PM** | **PB** |
| **Error** | **NB** | PB | PB | PM | PM | PS | ZO | ZO |
| | **NB** | PB | PB | PM | PS | PS | ZO | ZO |
| | **NS** | PM | PM | PM | PM | ZO | ZO | NS |
| | **ZO** | PM | PM | PS | ZO | NS | NS | NM |
| | **PS** | PS | PS | ZO | NS | NS | NM | NM |
| | **PM** | PS | ZO | NS | NM | NM | NM | NB |
| | **PB** | ZO | ZO | NM | NM | NM | NB | NB |

**Table 2:** fuzzy rules for $K_i$

| $K_i$ | | Fractional rate of change of error | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | **NB** | **NM** | **NS** | **ZO** | **PS** | **PM** | **PB** |
| **Error** | **NB** | NB | NB | NM | NM | NS | ZO | ZO |
| | **NB** | NB | NB | NM | NS | NS | ZO | ZO |
| | **NS** | NB | NM | NS | NS | ZO | PS | PS |
| | **ZO** | NM | NM | NS | ZO | PS | PM | PM |
| | **PS** | NM | NS | ZO | PS | PS | PM | PB |
| | **PM** | ZO | ZO | PS | PS | PM | PB | PB |
| | **PB** | ZO | ZO | PS | PM | PM | PB | PB |

**Table 3:** fuzzy rules for $K_d$

| $K_d$ | | Fractional rate of change of error | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | **NB** | **NM** | **NS** | **ZO** | **PS** | **PM** | **PB** |
| **Error** | **NB** | PS | NS | NB | NB | NB | NM | PS |
| | **NB** | PS | NS | NB | NB | NB | NM | PS |
| | **NS** | ZO | NS | NM | NM | NS | NS | ZO |
| | **ZO** | ZO | NS | NS | NS | NS | NS | ZO |
| | **PS** | ZO | ZO | ZO | ZO | ZO | ZO | ZO |
| | **PM** | PB | NS | PS | PS | PS | PS | PB |
| | **PB** | PB | PM | PM | PM | PS | PS | PB |

## 5. Evolutionary Algorithms

Classic Optimization methods face some disadvantages such as tapping in local optimized points, increasing the complexity of computing and Inability to apply these methods on a specific set of objective functions. The mentioned disadvantages have resulted to make new innovative optimization methods based on meta-heuristic optimization methods. In recent years, innovative optimized methods have been so developed such as PSO and GA. The following explanation is given about these two algorithms.

### 5.1. Particle Swarm Optimization

This algorithm is inspired by the social behavior of animals such as fish and birds live together in groups large and small, is designed. Some applications of PSO be found in nonlinear dynamical systems, analyze the data, electrical engineering, optimization functions, training artificial neural networks, fuzzy control and many other real-world problems [17,18].

In this algorithm, each bird is a possible solution in search space problem called particle. At first, PSO is initialized by a group of birds that have been produced accidentally and then a quest to best answer begins [19].

In each iteration of the algorithm, particle moves into a better position. Next location to each particle is obtained by according to two values: The first value is the best position that particle has achieved so far (*lbp*[1]). The second one is the best position that ever has been obtained by all particles

[1] - local best position

of the community and is the best $lbp$ in the whole group (*gbp*[2]). This process is repeated until the desired result is achieved (ie, the speed of birds be close to zero), or we achieve maximum number of repetitions which is intended for the PSO algorithm [20].

### 5.2. Genetic Algorithm

In genetic algorithm approach, the search of mechanisms is inspired from natural choice. GA starts with an initial population containing a number of chromosomes where each one represents a solution of the problem which performance is evaluated by a fitness function.

Basically, GA consists of three main stages: Selection, Crossover and Mutation. The application of these three basic operations allows the creation of new individuals which may be better than their parents. This algorithm is repeated for many generations and finally stops when reaching individuals that represent the optimum solution to the problem [21, 22].

## 6. Results and Discussion

In this paper, the simulation is done in MATLAB/Simulink and the implementation of the fractional order is done by FOMCON toolbox [23]. Fitness function includes Integral of Absolute Error (IAE) and Integral of Absolute Change in Controller Output (IACCO) which can be seen in equations (9,10). The fitness function, J, has been minimized by PSO algorithm. The main purpose for the use of this function is minimize the effects of the error between the actual and desired paths and minimizing the effects of changes in the control signal for better tracking path.

$$f_1 = \int |e_1(t)| dt + \int |e_2(t)| dt \quad (9)$$

$$f_2 = \int |\Delta\tau_1| dt + \int |\Delta\tau_2| dt \quad (10)$$

$$J = w_1 f_1 + w_2 f_2 \quad (11)$$

Where $\Delta\tau_1$ , $\Delta\tau_1, e_1(t)$ and $e_2(t)$ are changes in the output signal of the controller for link 1 and link 2 and errors of the link 1 and link 2 Respectively. Also $w_1$ and $w_2$ are the weights of $f_1$ and $f_2$ . To achieve the conventional FPID controller results, $\mu$ and $\lambda$ are equal to 1 and for FOFPID controller, $\mu$ and $\lambda$ Values are tuned by PSO Algorithm. Iteration number and Population number are equal to 50 and 25, respectively. The results of minimizing the fitness function can be seen in the table 5. Also the controller parameters of the FPID and FOFPID controllers for each of the links are shown in table 4 and they are adjusted by the fuzzy logic, adaptively.

**Table 4:** Tuned values of parameters via PSO and GA

| Controller | Fuzzy FOPID | | | |
|---|---|---|---|---|
| | Link1 | | Link2 | |
| | PSO | GA | PSO | GA |
| $K_p$ | 14.9 | | 50.9 | |
| $K_i$ | 30.1 | | 29.7 | |
| $K_d$ | 33.7 | | 9.8 | |
| $\lambda$ | 0.8190 | 0.7744 | 0.2676 | 0.6496 |
| $\mu$ | 0.8931 | 0.7954 | 0.8384 | 0.2285 |
| $\mu_{CE}$ | 0.1323 | 0.4965 | 1 | 0.9998 |

[2] - global best position

**Table 5:** Comparison performance of controllers via PSO and GA

| | IACOO | | IAE | | Best Cost |
|---|---|---|---|---|---|
| **Controller** | **Link1** | **Link2** | **Link1** | **Link2** | |
| **FOFPID-PSO** | 0.3524 | 0.9737 | 1.0721 | 0.9715 | 1.6849 |
| **FOFPID-GA** | 0.3617 | 0.8251 | 1.2027 | 1.0783 | 1.73404 |
| **Fuzzy PID** | 1.7223 | 4.1300 | 1.2442 | 0.9432 | 4.032109 |

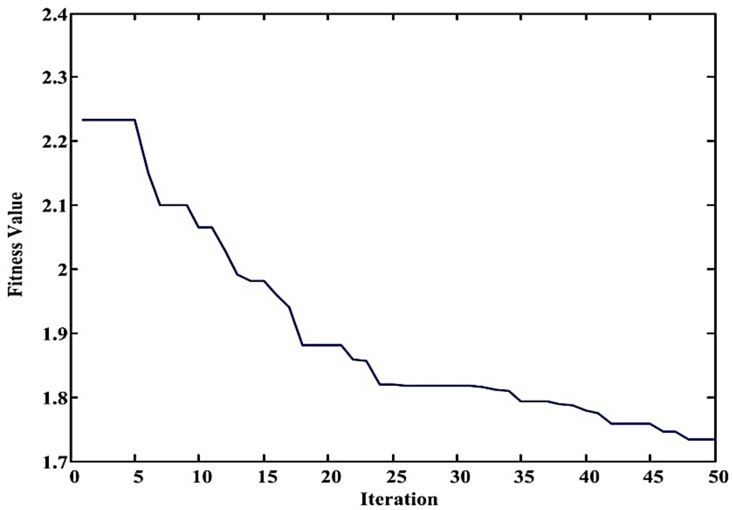


**Figure 3:** A graph of the fitness value in the iteration-GA.

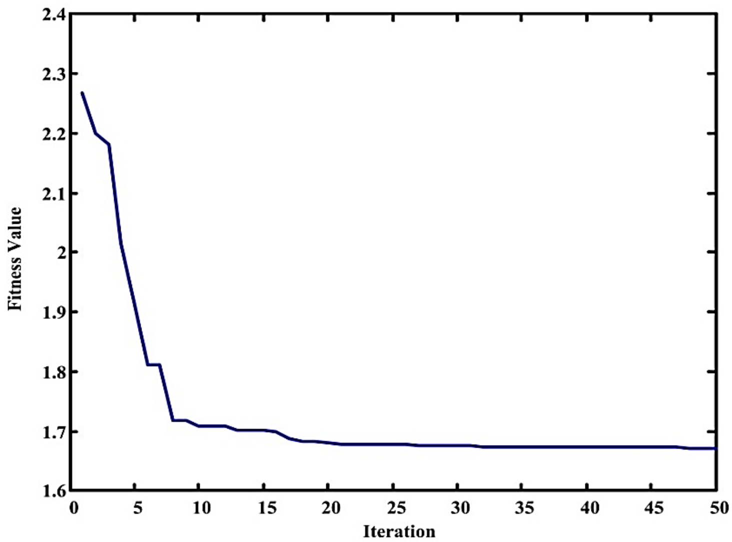


**Figure 4:** A graph of the fitness value in the iteration-PSO.

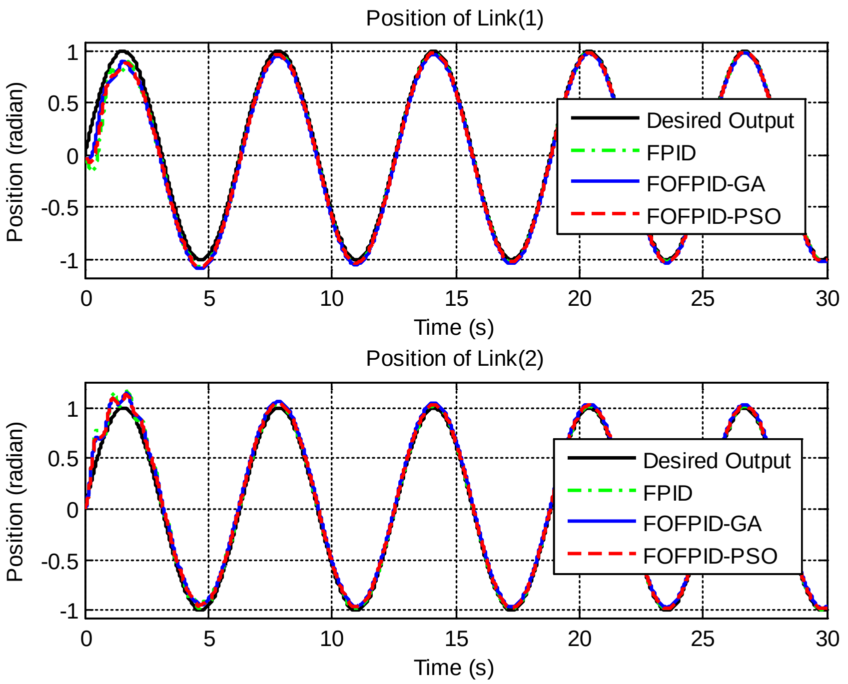


**Figure 5:** Trajectory tracking for FPID, FOFPID-GA, FOFPID-PSO in both links.

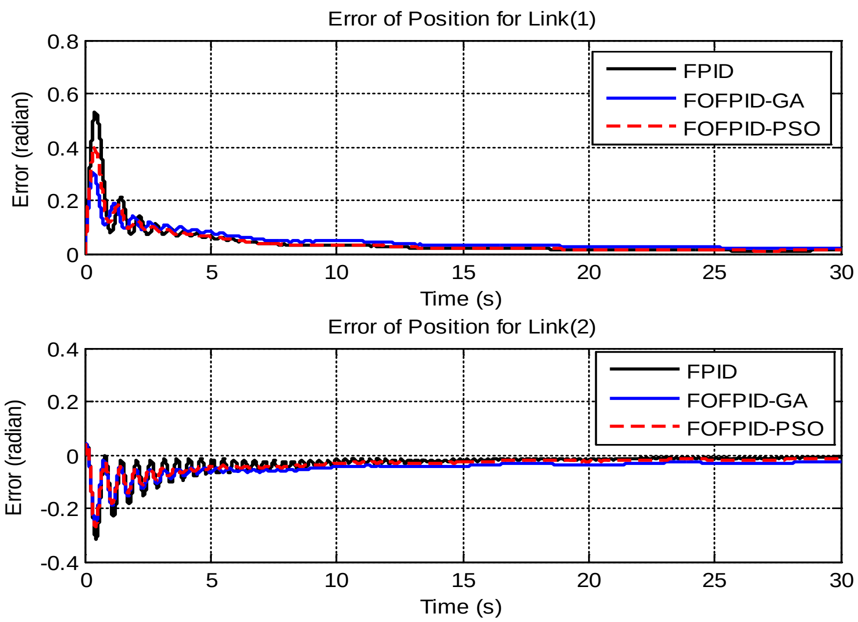


**Figure 6:** Position error for FPID, FOFPID-GA, FOFPID-PSO in both links.

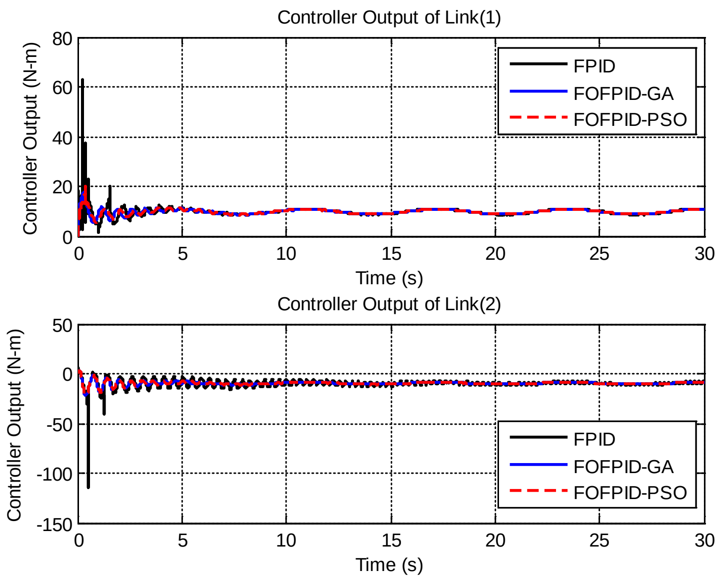


**Figure 7:** Controller output for FPID, FOFPID-GA, FOFPID-PSO in both links.

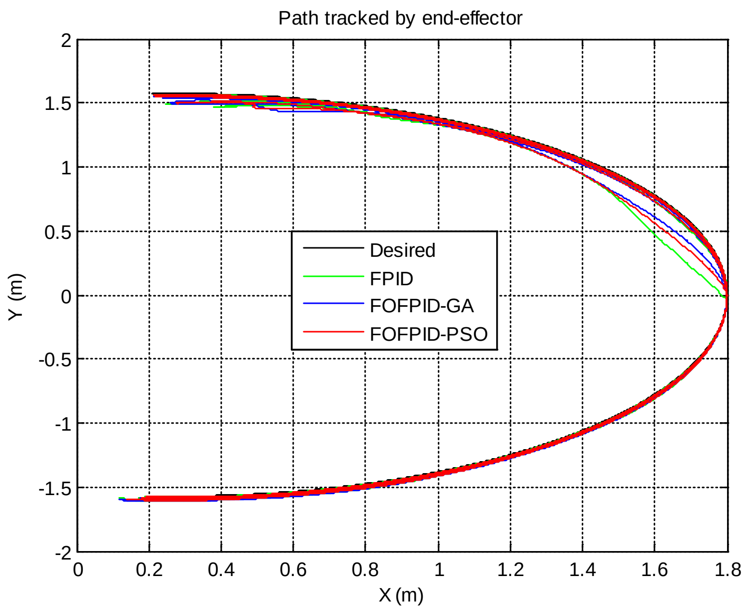


**Figure 8:** Path tracked by end-effector.

## Conclusion

Robotic manipulator is a nonlinear time-invariant system and control the link vibrations and making a flexible system, is extremely difficult. So in this type of system overshoot should be reached in least its value. However, these limitations been resolved by conventional controllers but FOFPID controllers improve the performance and robustness of the system The results obtained from this controller show that proposed controller's overshoot is less than traditional Fuzzy PID controller. Also the oscillation amplitude is less than its normal mode. In addition, controller performance was evaluated with two proposed algorithms, PSO and GA. Fitness function, J, was obtained equal to 4.0321 by Fuzzy PID controller. Whereas by using PSO algorithm and the proposed controller J was obtained equal to 1.684 and with GA algorithm it's value increased to 1.734. Among of these two algorithms, PSO has better performance and shows its superiority compared to genetic algorithms.

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