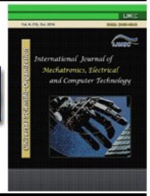
Vol. 7(24), Apr. 2017, PP. 3325-3334

Contents list available at IJMEC

International Journal of Mechatronics, Electrical and Computer Technology (IJMEC)

Journal Homepage: www.aeuso.org

PISSN: 2411-6173 - EISSN: 2305-0543



Model Development and Matrix Representation of Unpowered Laser-Guided Missile

Abdulkadir Iyyaka Audu[1*] and Jibril Danladi Jiya[2]

[1]*Computer Engineering Department, Faculty, of Engineering, University of Maiduguri, Nigeria*

[2]*Mechatronic Engineering Department, Abubakar Tafawa Balewa University, Bauchi,Nigeria*

Phone Number: *+234-0709252528*

****Corresponding Author's E-mail:*** *lm324fairchild@gmail.com*

Abstract

This paper provides for a natural implicit knowledge representation which is centered on model development and matrix representation of unpowered laser-guided missile. Unpowered laser-guided Missile (ULGM) has not only gained a predictably receptive attention but also has successfully adapted to fast changing warfare market. The inaccuracy, which often result in the need to fire many rockets to hit a single target, has led to the search for a means to guide the rocket towards its target. This significant feature in conjunction with the high degree of complexity associated with the generation of state variable representation from the nonlinear control model of ULGM have made linearization and matrix representation of the model very attractive. Of course, it facilitates the understanding of system dynamics of unpowered laser-guided missile.

Keywords: *System dynamics, nonlinear control model, state variable representation,fin control, commandsignal .*

1. Introduction

The Both the radar beam and its reflected illumination have had an extraordinary standing in their use for missile guidance. In such a system, the target is illuminated with a radar beam and the missile is made to track the path of the reflected illumination from the target [1]. The issues of weight, complexity, adaptability, and operational difficulty are known to be associated with beam illuminators [2]. The prominence of ULGM began to emerge in the early 1960s because of the significant feature of a guidance system that steers them towards a pre-selected target and its embedded accuracy and reliability which often reduced the need to fire many rockets to hit a single target. This has significantly contributed to the growth and sophistication of military capability all over the world and consequently, the course of recent wars. The healthy consideration of merging existing rocket technology with enabling technology which provides the potential for very intensive utilization of radar and radio detection devices is uniquely responsible for dawning of the era of high-technology warfare, an era that quickly demonstrate its problems as well as its promise. The launch of projectiles, rockets and missiles has been a laboratory curiosity ever since. What has changed is the generous helping of control theory to provide an exact and balanced knowledge-based understanding of their nonlinear nature, aerodynamic instability caused by the location of centre of gravity behind the centre of drag, and consequently, successful flight. The application of control theory and dynamics of an inverted

Article History:
IJMEC DOI: 649123/10.225123
Received Date: Oct. 08, 2016
Accepted Date: Feb. 23, 2017
Available Online: Mar. 01, 2017

pendulum to the flight of missiles have gained prominence of significant proportion because of associated stability and perturbation issues. Self- tuning strategy concerning the stabilized control system of the rotational inverted pendulum has been emphasized. In the work: "Inverted pendulum systems: rotary and arm-driven - a mechatronic system design case study", the synergistic combination of mechanical engineering, electronics, control systems, and computers has uniquely positioned IP in control design of missile systems [3]. The Rotary Inverted Pendulum (RIP) system is a complex, multivariable, non minimum phase and unstable, electromechanical system with severe nonlinearity and obtaining its stabilizing control is a representative problem in the application of the control theory [4], [5], [7], [8], [9], [10], [11]. The control process can reflect many problems, such as stabilization problems, nonlinear problems and robust problems.

It is more demanding of crew and aircraft, requiring a high standard of basic, unguided bombing accuracy and more attention to the bomb's flight to release their weapons on an unguided, ballistic flight path, activating the designator only to refine the bomb's final impact point. In view of this unique realization, there is a genuine need to develop a control model which with which to foster a good and exceptionally streamlined understanding of dynamic relationship the command signal and other performance parameters of ULGM.

2. Overview of unpowered laser-guided missile

Laser guidance is a technique of guiding a missile to a target by means of a laser beam. Laser-guided missiles are generally unpowered, using planar control fins to glide towards their targets. With the help of a command signal which actuates a servo, the fins rotate back and forth to generate forces in the horizontal and/or vertical planes that cause the missile to yaw right and left, pitch up and down, or roll as the missile maneuvers towards a target. The tail controlled all-moving planar control fin topology is the most popular. its mass is above its pivot point.

In the target designator type of laser guidance, on- board sensors are used to pick up laser light reflected from the target. The aircraft/helicopter pilot selects a target, hits the target with a laser beam shot from a target designator, and then launches the missile. The missile's sensor measures the error between its flight path and the path of the reflected light. Correction messages are then passed on to the missile's control surfaces via the electronics suite, steering the missile onto its target.

The stability of an ULGM in flight is only possible when the pivot point (aircraft launcher) moves horizontally as part of a feedback system. Ordinarily, after the launch, the ULGM will accelerate away from the vertical unstable equilibrium in the direction initially displaced, but the control fins try to maintain the missile in this unstable state.

An ULGM system is a control process with input and output. Like most control systems, the desired performance depends on how appropriately the input is chosen [6]. The significant difference between the physical ULGM and the mathematically predicted behavior due to environmental disturbances and parameter variations requires that a control model of the ULGM which would successfully adapt to fast changing warfare market be developed. All derivations, analysis and discussion is centre on the idea depicted in figure 1

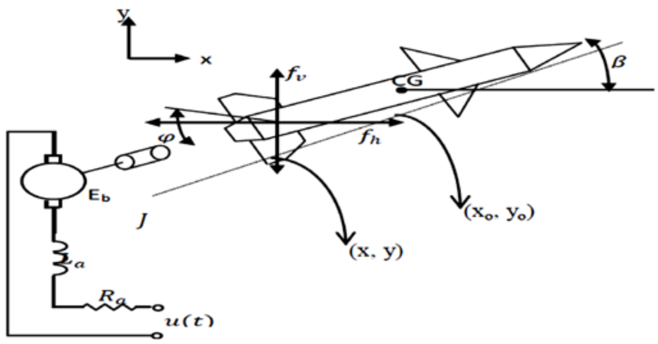


Figure 1: Unpowered laser-guided missile.

3. Control model development of ULGM

Assuming the centre of mass of ULGM has a coordinate (x_o, y_o) in x-y plane, the point at which the ULGM is supported on a system moving in x-direction in x-y plane is (x, y).

Consider the Newtonian acceleration along positive x-direction in the x-y plane of a rectangular coordinate system, we can write

$$M_p \frac{d^2}{dt}(x + L\sin\varphi) = f_h \tag{1}$$

Where, mass of ULGM concentrated at its centre of gravity is M_p, L is it's half length, φ is the fin angle, and f_h is the force component in x-direction. Simplifying (1), we have

$$M_p \frac{d}{dt}\left[\frac{dx}{dt} + L\cos\varphi \frac{d\varphi}{dt}\right] = f_h \tag{2a}$$

$$M_p \frac{d^2x}{dt^2} - LM_p \left(\frac{d\varphi}{dt}\right)^2 \sin\varphi + LM_p\cos\varphi \frac{d^2\varphi}{dt^2} = f_h \tag{2b}$$

$$f_h = M_p \frac{d^2x}{dt^2} - LM_p \left(\frac{d\varphi}{dt}\right)^2 \sin\varphi + LM_p\cos\varphi \frac{d^2\varphi}{dt^2} \tag{3}$$

The Newtonian acceleration in the y-direction is expressed as

$$M_p \frac{d^2}{dt^2} L\cos\varphi = f_v - M_p g \tag{4}$$

g is acceleration due to gravity, f_v is force component in the vertical direction. Eqn. (4) simplifies to

$$LM_p \frac{d}{dt}\left[-\sin\varphi \frac{d\varphi}{dt}\right] = f_v - M_p g \tag{5a}$$

$$-LM_p \sin\varphi \frac{d^2\varphi}{dt^2} - LM_p \cos\varphi \frac{d\varphi}{dt} = f_v - M_p g \tag{5b}$$

$$f_v = -LM_p \sin\varphi \frac{d^2\varphi}{dt^2} - LM_p \cos\varphi \left(\frac{d\varphi}{dt}\right)^2 + M_p g \tag{6}$$

When a launching system of mass M_s (aircraft/helicopter) on which the ULGM is supported exerts a force f_a which causes the ULGM to move in x-direction, then the consideration of Newtonian acceleration would yield

$$f_a - f_f - f_h = M_s \frac{d^2x}{dt^2} \tag{7}$$

f_f is the atmospheric drag on M_s. It's assumed M_s does not slip. $f_a - f_f$, is the effective force on the system which supports the ULGM. Therefore, $f_a - f_f$ can be expressed as

$$f_a - f_f = f_h + M_s \frac{d^2x}{dt^2} \tag{8}$$

Using (3) in (8), we get

$$f_a - f_f = M_p \frac{d^2x}{dt^2} - LM_p \left(\frac{d\varphi}{dt}\right)^2 \sin\varphi + \mathrm{L}M_p\cos\varphi \frac{d^2\varphi}{dt^2} + M_s \frac{d^2x}{dt^2} \tag{9}$$

$$(M_p + M_s)\frac{d^2x}{dt^2} = LM_p \left(\frac{d\varphi}{dt}\right)^2 \sin\varphi - \mathrm{L}M_p\cos\varphi \frac{d^2\varphi}{dt^2} + f_a - f_f \tag{10}$$

$$\frac{d^2x}{dt^2} = \left(\frac{1}{M_p+M_s}\right)\left[LM_p\left(\frac{d\varphi}{dt}\right)^2 \sin\varphi - \mathrm{L}M_p \cos\varphi \frac{d^2\varphi}{dt^2} + f_a - f_f\right] \tag{11}$$

$$\text{Let } \left(\frac{1}{M_p+M_s}\right) = C \text{ (constant)} \tag{12}$$

$$\frac{d^2x}{dt^2} = \left[LCM_p\left(\frac{d\varphi}{dt}\right)^2 \sin\varphi - \mathrm{LC}M_p \cos\varphi \frac{d^2\varphi}{dt^2} + Cf_a - Cf_f\right] \tag{13}$$

If the centre of mass ULGM is considered to be a pivot point, then the angular motion about this point is expressed as

$$J\frac{d^2\varphi}{dt^2} = Lf_v \sin\varphi - Lf_h \cos\varphi \tag{14}$$

Where J is defined as the pitch moment of inertia of ULGM about its centre of mass.

$$M_p\frac{L^3}{3}\frac{d^2\varphi}{dt^2} = f_v L \sin\varphi - f_h L \cos\varphi \tag{15}$$

Eqn. (15) is true since

$$J = M_p \int_0^L d^2\, dr = M_p \frac{L^3}{3} \tag{16}$$

Substituting for f_v and f_h in (15) we obtain

$$M_p\frac{L^3}{3}\frac{d^2\varphi}{dt^2} = \left[-LM_p \sin\varphi \frac{d^2\varphi}{dt^2} - LM_p \cos\varphi \left(\frac{d\varphi}{dt}\right)^2 + M_p g\right] L\sin\varphi - \left[M_p\frac{d^2x}{dt^2} - LM_p\left(\frac{d\varphi}{dt}\right)^2 \sin\varphi + \mathrm{L}M_p \cos\varphi \frac{d^2\varphi}{dt^2}\right] L\cos\varphi \tag{17}$$

$$M_p\frac{L^3}{3}\frac{d^2\varphi}{dt^2} = M_p gL\sin\varphi - M_p L^2 \frac{d^2\varphi}{dt^2}[(\sin\varphi)^2 + (\cos\varphi)^2] - M_p L\frac{d^2x}{dt^2}\cos\varphi \tag{18}$$

$$M_p\frac{L^3}{3}\frac{d^2\varphi}{dt^2} = M_p gL\sin\varphi - M_p L\frac{d^2x}{dt^2}\cos\varphi - M_p L^2\frac{d^2\varphi}{dt^2} \tag{19}$$

Substituting for $\frac{d^2x}{dt^2}$ in (19), we get

$$M_p\frac{L^3}{3}\frac{d^2\varphi}{dt^2} = M_p gL\sin\varphi - M_p L\cos\varphi\left[LCM_p\left(\frac{d\varphi}{dt}\right)^2 \sin\varphi - \mathrm{LC}M_p \cos\varphi\frac{d^2\varphi}{dt^2} + Cf_a - Cf_f\right] - M_p L^2 \frac{d^2\varphi}{dt^2} \tag{20}$$

$$\frac{d^2\varphi}{dt^2}\left[M_p\frac{L^3}{3} - L^2CM_p^2(\cos\varphi)^2 + M_pL^2\right] = M_p gL\sin\varphi - \frac{1}{2}L^2CM_p^2\sin 2\varphi\left(\frac{d\varphi}{dt}\right)^2 - M_pCL\cos\varphi\,(f_a - f_f) \tag{21}$$

$$\frac{d^2\varphi}{dt^2} = \frac{M_p gL\sin\varphi - \frac{1}{2}L^2CM_p^2\sin 2\varphi\left(\frac{d\varphi}{dt}\right)^2 - M_pCL\cos\varphi\,(f_a - f_f)}{M_p\frac{L^3}{3} - L^2CM_p^2(\cos\varphi)^2 + M_pL^2} \tag{22}$$

$$\frac{d^2\varphi}{dt^2} = \frac{M_p^2 LC\left[\frac{1}{M_pC} g\sin\varphi\ - \frac{1}{2}L\sin 2\varphi\left(\frac{d\varphi}{dt}\right)^2 - \frac{1}{M_p}\cos\varphi\left(f_a - f_f\right)\right]}{M_p^2 L^2 C\left[\frac{L}{3M_pC}\ - \ (\cos\varphi)^2\ +\ \frac{1}{M_pC}\right]} \tag{23}$$

Multiplying both numerator and denominator of (23) by $^3/_4$, we obtain

$$\frac{d^2\varphi}{dt^2} = \frac{\frac{3}{4}\left[\frac{1}{M_pC} g\sin\varphi\ - \frac{1}{2}L\sin 2\varphi\left(\frac{d\varphi}{dt}\right)^2 - \frac{1}{M_p}\cos\varphi\left(f_a - f_f\right)\right]}{\frac{3L}{4}\left[\frac{L}{3M_pC}\ - \ (\cos\varphi)^2\ +\ \frac{1}{M_pC}\right]} \tag{24}$$

The addition of $\frac{L}{3M_pC}$ and $\frac{1}{M_pC}$ in the denominator of (24) is as follows

$$\frac{3}{4}\left[\frac{L}{3M_pC} + \frac{1}{M_pC}\right] = \frac{L}{4} + \frac{M_sL}{4M_p} + \frac{3}{4} + \frac{3M_s}{4M_p} \tag{25}$$

$$\frac{3}{4}\left[\frac{L}{3M_pC} + \frac{1}{M_pC}\right] = \left[\frac{L}{4} + \frac{3}{4}\right] + \left[\frac{M_sL}{4M_p} + \frac{3M_s}{4M_p}\right] \tag{26}$$

$$\frac{3}{4}\left[\frac{L}{3M_pC} + \frac{1}{M_pC}\right] = \left[\frac{L+3}{4}\right] + \left[\frac{M_sL + 3M_s}{4M_p}\right]$$

$$= \left[\frac{L\left(1+\frac{3}{L}\right)}{4}\right] + \left[\frac{M_sL\left(1+\frac{3}{L}\right)}{4M_p}\right] \tag{27}$$

The application of Binomial theorem helps to simplify (27) further

$$\frac{3}{4}\left[\frac{L}{3M_pC} + \frac{1}{M_pC}\right] = \left[\frac{L(1+3)^{\frac{1}{L}}}{4}\right] + \left[\frac{M_sL(1+3)^{\frac{1}{L}}}{4M_p}\right]$$

$$\frac{3}{4}\left[\frac{L}{3M_pC} + \frac{1}{M_pC}\right] = \left[\frac{4L\left(\frac{1}{L}\right)}{4}\right] + \left[\frac{4M_sL\left(\frac{1}{L}\right)}{4M_p}\right]$$

$$\frac{3}{4}\left[\frac{L}{3M_pC} + \frac{1}{M_pC}\right] = 1 + \left[\frac{M_s}{M_p}\right] \tag{28}$$

Therefore, (24) becomes

International Journal of Mechatronics, Electrical and Computer Technology (IJMEC)
Universal Scientific Organization, www.aeuso.org
PISSN: 2411-6173, EISSN: 2305-0543

$$\frac{d^2\varphi}{dt^2} = \frac{\frac{3}{4}\left[\frac{1}{M_pC} g \sin\varphi \ - \frac{1}{2} L \sin 2\varphi \left(\frac{d\varphi}{dt}\right)^2 - \frac{1}{M_p} \cos\varphi \left(f_a - f_f\right)\right]}{L\left[\ 1 + \frac{M_s}{M_p} \ - \ \frac{3}{4}(\cos\varphi)^2 \ \right]} \tag{29}$$

$$\frac{d^2\varphi}{dt^2} = \frac{\frac{3}{4}\left[\left(1 + \frac{M_s}{M_p}\right) g \sin\varphi \ - \frac{1}{2} L \sin 2\varphi \left(\frac{d\varphi}{dt}\right)^2 - \frac{1}{M_p} \cos\varphi \left(f_a - f_f\right)\right]}{L\left[\ 1 \ + \ \frac{M_s}{M_p} \ - \ \frac{3}{4}(\cos\varphi)^2 \ \right]} \tag{30}$$

Using (29) to substitute for $\frac{d^2\varphi}{dt^2}$ in (13), we have

$$\frac{d^2x}{dt^2} = \left[LCM_p \left(\frac{d\varphi}{dt}\right)^2 \sin\varphi - \mathrm{LC}M_p \cos\varphi \left[\frac{\frac{3}{4}\left[\frac{1}{M_pC} g \sin\varphi - \frac{1}{2} L \sin 2\varphi \left(\frac{d\varphi}{dt}\right)^2 - \frac{1}{M_p}\cos\varphi (f_a - f_f)\right]}{L\left[\ 1 + \frac{M_s}{M_p} - \frac{3}{4}(\cos\varphi)^2 \ \right]}\right] + Cf_a - Cf_f\right] \tag{31}$$

Since, $1 + \frac{M_s}{M_p} = \frac{1}{M_pC}$, (31) becomes

$$\frac{d^2x}{dt^2} = \left[LCM_p \left(\frac{d\varphi}{dt}\right)^2 \sin\varphi - \mathrm{LC}M_p \cos\varphi \left[\frac{\frac{3}{4}\left[\frac{1}{M_pC} g \sin\varphi - \frac{1}{2} L \sin 2\varphi \left(\frac{d\varphi}{dt}\right)^2 - \frac{1}{M_p}\cos\varphi (f_a - f_f)\right]}{L\left[\ \frac{1}{M_pC} - \frac{3}{4}(\cos\varphi)^2 \ \right]}\right] + Cf_a - Cf_f\right] \tag{32}$$

$$\frac{d^2x}{dt^2} = \left[LCM_p \left(\frac{d\varphi}{dt}\right)^2 \sin\varphi - \mathrm{LC}M_p \cos\varphi \left[\frac{\left[\frac{1}{L} g \sin\varphi - \frac{1}{2} M_pC \sin 2\varphi \left(\frac{d\varphi}{dt}\right)^2 - \frac{C}{L}\cos\varphi (f_a - f_f)\right]}{\left[\frac{4}{3} - M_pC(\cos\varphi)^2\ \right]}\right] + Cf_a - Cf_f\right] \tag{33}$$

$$\frac{d^2x}{dt^2} = \left[LCM_p \left(\frac{d\varphi}{dt}\right)^2 \sin\varphi - \left[\frac{\left[M_pCg\cos\varphi\sin\varphi - LM_p{}^2C^2\sin\varphi(\cos\varphi)^2\left(\frac{d\varphi}{dt}\right)^2 - M_pC^2(\cos\varphi)^2(f_a - f_f)\right]}{\left[\frac{4}{3} - M_pC(\cos\varphi)^2\ \right]}\right] + Cf_a - Cf_f\right] \tag{34}$$

$$\frac{d^2x}{dt^2} = M_pLC\left(\frac{d\varphi}{dt}\right)^2 \sin\varphi \left[1 + \frac{M_pC(\cos\varphi)^2}{\frac{4}{3} - M_pC(\cos\varphi)^2}\right] - \left[\frac{\frac{\sin 2\varphi}{2} M_pCg}{\frac{4}{3} - M_pC(\cos\varphi)^2}\right] + \left[\frac{M_pC^2(\cos\varphi)^2(f_a - f_f)}{\frac{4}{3} - M_pC(\cos\varphi)^2} + C(f_a - f_f)\right] \tag{35}$$

$$\frac{d^2x}{dt^2} = \frac{M_pLC\left(\frac{d\varphi}{dt}\right)^2 \sin\varphi\left[\left(\frac{4}{3} - \ M_pC(\cos\varphi)^2\right) + M_pC(\cos\varphi)^2\right] - \left(\frac{\sin 2\varphi}{2} M_pCg\right) + \left[\frac{4}{3}(f_a - f_f)\right]}{\frac{4}{3} - \ M_pC(\cos\varphi)^2} \tag{36}$$

$$\frac{d^2x}{dt^2} = \frac{\frac{M_pL}{(M_p+M_s)}\left(\frac{d\varphi}{dt}\right)^2 \sin\varphi\left[\left(\frac{4}{3}- M_pC(\cos\varphi)^2\right)+\left(\frac{M_p}{M_p+M_s}\right)(\cos\varphi)^2\right]-\left[\left(\frac{M_p}{M_p+M_s}g\right)\frac{\sin 2\varphi}{2}\right]+\left[\frac{4}{3}(f_a-f_f)\right]}{\frac{4}{3}-\left(\frac{M_p}{M_p+M_s}\right)(\cos\varphi)^2} \quad (37)$$

4. Model of fin positional control

Missile fins are turned using servo mechanisms that rotate the fin to a new deflection angle. In order to eliminate loading problem, we consider armature-controlled dc motor as the servo mechanism. The torque T as a function of time generated by the servo is expressed

$$\mathrm{T(t) = CI_a(t)I_f(t)} \quad (38)$$

In armature-controlled dc servo, the field current $\mathrm{I_f(t)}$ is kept constant or the field circuit is replaced by a permanent magnetic field. The torque generated when a command signal $\mathrm{u(t)}$ is applied to the armature circuit is given as

$$\mathrm{T(t) = C_cI_a(t)} \quad (39)$$

Where $\mathrm{C_c = CI_f(t)}$ = constant and $\mathrm{I_a(t)}$ is armature current. The back electromotive force $\mathrm{E_b}$ developed when the servo drives the control fins (load) through an angle $\varphi(t)$ is

$$\mathrm{E_b = C_b\frac{d\varphi(t)}{dt}} \quad (40)$$

$\mathrm{C_b}$ is a constant of proportionality. The armature circuit can be described as

$$\mathrm{L_a\frac{dI_a(t)}{dt} + E_b + R_aI_a = u(t)} \quad (41)$$

$$\mathrm{L_a\frac{dI_a(t)}{dt} + C_b\frac{d\varphi(t)}{dt} + R_aI_a = u(t)} \quad (42)$$

The resistance and inductance of the armature circuit is $\mathrm{R_a}$ and $\mathrm{L_a}$ respectively. If $\mathrm{L_a}$ is very small compared to $\mathrm{R_a}$ as often the case in practice, $\mathrm{L_a}$ can be neglected and (42) becomes

$$\mathrm{I_a = \frac{u(t)}{R} - \frac{C_b}{R}\frac{d\varphi(t)}{dt}} \quad (43)$$

Substituting for $\mathrm{I_a}$ in (39), we have

$$\mathrm{T(t) = C_c\left[\frac{u(t)}{R} - \frac{C_b}{R}\frac{d\varphi(t)}{dt}\right]} \quad (44)$$

If J is the total moment of inertia of the shaft, the rotor of the motor, and the load; f, the viscous friction coefficient of the bearing; and $\varphi(t)$, the angular displacement of the control fins, then $\mathrm{T(t)}$ can be expressed as

$$\mathrm{T(t) = j\frac{d^2\varphi(t)}{dt^2} + f\frac{d\varphi(t)}{dt}} \quad (45)$$

Comparing (44) and (45), we get

$$\mathrm{j\frac{d^2\varphi(t)}{dt^2} + \left[f + \frac{C_bC_c}{R}\right]\frac{d\varphi(t)}{dt} = \frac{C_c}{R}u(t)} \quad (46)$$

International Journal of Mechatronics, Electrical and Computer Technology (IJMEC)
Universal Scientific Organization, www.aeuso.org
PISSN: 2411-6173, EISSN: 2305-0543

Integrating (46), yields

$$j\frac{d\varphi(t)}{dt}+\left[f+\frac{C_bC_c}{R}\right]\varphi(t)=\frac{C_c}{R}u(t)t=\frac{C_c}{R}[(u(t)t+1)-1] \quad (47)$$

The application of Binomial theorem to the right-hand side of (47) results in

$$j\frac{d\varphi(t)}{dt}+\left[f+\frac{C_bC_c}{R}\right]\varphi(t)=u(t) \quad (48)$$

Eqn. (48) models the fin positional control system.

5. Transfer function and state variable representation

Since $90-\beta=\varphi$, where β is the pitch angle, (30) becomes

$$\frac{d^2\beta}{dt^2}=-\frac{\frac{3}{4}\left[\left(1+\frac{M_s}{M_p}\right)g\sin\varphi\ -\frac{1}{2}L\sin 2\varphi\left(\frac{d\varphi}{dt}\right)^2-\frac{1}{M_p}\cos\varphi\,(f_a-f_f)\right]}{L\left[1\ +\frac{M_s}{M_p}\ -\frac{3}{4}(\cos\varphi)^2\right]} \quad (49)$$

Laser-guided missiles are generally unpowered (f_a), using small wings to glide towards their targets. Therefore, if the atmospheric drag is negligible, f_a-f_f would be zero. Also if φ is small, then $\sin\varphi=\varphi$, $\cos\varphi=1$, and $\left(\frac{d\varphi}{dt}\right)^2=0$. Eqn. (49)

$$\frac{d^2\beta}{dt^2}=-\frac{\frac{3}{4}\left(1+\frac{M_s}{M_p}\right)g\varphi}{L\left[\frac{1}{4}\ +\frac{M_s}{M_p}\ \right]} \quad (50)$$

$$\frac{d^3\beta}{dt^2}=-\frac{\frac{3}{4}\left(1+\frac{M_s}{M_p}\right)g}{L\left[\frac{1}{4}\ +\frac{M_s}{M_p}\ \right]}\frac{d\varphi}{dt} \quad (51)$$

From (48), substituting for $\frac{d\varphi}{dt}$ in (51), we obtain

$$\frac{d^3\beta}{dt^2}=-\frac{\frac{3}{4}\left(1+\frac{M_s}{M_p}\right)g}{L\left[\frac{1}{4}\ +\frac{M_s}{M_p}\ \right]}\frac{1}{j}\left(u(t)-\left[f+\frac{C_bC_c}{R}\right]\varphi(t)\right) \quad (52)$$

$$\frac{d^3\beta}{dt^2}+\frac{1}{j}\left[f+\frac{C_bC_c}{R}\right]\frac{d^2\beta}{dt^2}=-\frac{1}{j}\frac{\frac{3}{4}\left(1+\frac{M_s}{M_p}\right)g}{L\left[\frac{1}{4}\ +\frac{M_s}{M_p}\ \right]}u(t) \quad (53)$$

International Journal of Mechatronics, Electrical and Computer Technology (IJMEC)
Universal Scientific Organization, www.aeuso.org
PISSN: 2411-6173, EISSN: 2305-0543

If $u(t)$ is a negative dc voltage command signal, then (53) becomes

$$\frac{d^3\beta}{dt^2}+\frac{1}{j}\left[f+\frac{C_bC_c}{R}\right]\frac{d^2\beta}{dt^2}=\frac{1}{j}\frac{\frac{3}{4}\left(1+\frac{M_s}{M_p}\right)g}{L\left[\frac{1}{4}+\frac{M_s}{M_p}\right]}u(t)$$

(54)

Taking the Laplace transform of (54), yields

$$\frac{\beta(s)}{u(s)}=\frac{\frac{1}{j}\frac{\frac{3}{4}\left(1+\frac{M_s}{M_p}\right)g}{L\left[\frac{1}{4}+\frac{M_s}{M_p}\right]}}{s^3+\frac{1}{j}\left[f+\frac{C_bC_c}{R}\right]s^2}$$

(55)

Eqn. (55) relates the pitch angle to the command signal and hence is the transfer function of the entire ULGM system. Let a1, a2, and a3, be the coefficients of $\frac{d^3\beta}{dt^3}, \frac{d^2\beta}{dt^2}$, and $u(t)$ respectively. Eqn. (54) becomes

$$a_1\frac{d^3\beta}{dt^3}+a_2\frac{d^2\beta}{dt^2}=a_3u(t)$$

(56)

The state variable representation of LGM in terms of phase variables is expressed as follows

$$\dot{x}(t)=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & -a_2\end{bmatrix}x(t)+\begin{bmatrix}0\\0\\1\end{bmatrix}u(t),\quad \text{state equation} \tag{57}$$

$$y=[a_3 \quad 0 \quad 0]x(t),\quad \text{output equation} \tag{58}$$

However, the generalized state variable representation is

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{59}$$

$$y=Cx(t)+Du(t) \tag{60}$$

Where coefficient matrix of ULGM is$A$, B is the driving matrix, C is the output matrix, D is the transmission matrix. y is the output. Comparing (57), (58) and (59), ((60) respectively, we can write that

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & -a_2\end{bmatrix},\quad B=\begin{bmatrix}0\\0\\1\end{bmatrix},\quad C=[a_3 \quad 0 \quad 0] \tag{61}$$

6. Discussion and conclusion

This paper has addressed two issues associated with ULGM: model development and state variable representation which is derived from transfer function. The ULGM model is a second-order differential equation. The model is derived based on the critical assumption of an effective force on the system which supports the ULGM. This method further allows for the computation of state variable representation of the system which when decomposed could yield the system, input and output matrices. When effectively manipulated, the derived model could also be a basis for the study and estimation of the dynamic stability of the system.

The model proposed in this paper can be enhanced by handling the case where atmospheric drag is not negligible and fin angle is large. In the case where the differential of the fin angle is not negligible, the differential of the pitch angle becomes dependent on the differential of the fin angle. In this case, the complexity associated with simplification leading to the derivation of state variable representation increases. This limitation could be an interesting avenue for future work.

12. References

[1] Jr. Jennings, B. Walter, B. M. William, and D. M. Richard, "Missile system using laser illuminator." U.S. Patent 4 143 835, March 13, 1979.

[2] "Laser-guided projectile system." U.S. Patent 3 860 199, January 14, 1975.

[3] S. Awtar, N. King, T. Allen, I. Bang, M. Hagan, D. Skidmore, and K. Craig. "Inverted pendulum systems: rotary and arm-driven-a mechatronic system design case study," Mechatronics, vol. 12, no. 2, 357-370, 2002.

[4] A. Isidori, Nonlinear Control Systems: An Introduction. New York: Springer-Verlag, 1989.

[5] K. Ogata, Modern control engineering, New York: Prentice-Hall, 1997.

[6] A. K. Mandal, Introduction to Control Engineering: Modeling, Analysis and Design, New Delhi: New Age International, 2006.

[7] J. J. Craig, Adaptive Control of Mechanical Manipulators. Reading, MA: Addison-Wesley, 1988.

[8] J. J. E. Slotine and W. Li, “Adaptive manipulator control: A case study,” IEEE Trans. Automat. Contr., vol. 33, pp. 995–1003, 1988.

[9] J. J. E. Slotine and W. Li, “On the adaptive control of robot manipulators,” Int. J. Robot. Res., vol. 6, pp. 49–59, 1987.

[10] H. Nijmeijer and A. J. van der Shaft, Nonlinear Dynamical Control Systems, 3rd ed. New York: Springer-Verlag, 1996.

[11] D. Taylor, P. V. Kokotovic´, R. Marino, and I. Kanellakopoulos, “Adaptive regulation of nonlinear systems with unmodeled dynamics,” IEEE Trans. Automat. Contr., vol. 34, pp. 405–412, 1989.