Twisting Control Algorithm for the Yaw and Pitch Tracking of a Twin Rotor UAV

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Abstract

In this paper, second order sliding mode controller is used to compensate the pitch and yaw angles of multivariable (MIMO) twin rotor system. Twin rotor has 6 state variables including yaw and pitch angles, yaw and pitch angular velocities, and armature currents. Considering the physical structure of twin rotor, the controller aims to track the mentioned angles and stabilize their angular velocities. Depending on the type of sliding control mode in the design process, the armature currents should be placed in an allowable range based on physical characteristics of twin rotor. This helps the controller to do the tracking and stabilizing in a proper way. Second order sliding control design is done based on twisting algorithm and in this regard, two sliding surfaces are defined as the sum of tracking error of yaw and pitch angles and their integrals. The controller performance is studied in the light of tracking accuracy and steady-state error, transient behavior and response fluctuations level and presence or absence of ripple in response. In all cases, the simulation is presented for first and second order sliding mode controllers and their advantages and disadvantages will be investigated.

Keywords: Twin Rotor, First and Second Sliding Mode Control, Twisting Algorithm.

I. Introduction

The most important problem affecting control of twin-engine helicopters movement and control of lateral speed and lateral acceleration applied to the body is the changes in the body lateral speed due to the parametric uncertainty. The reasons for this speed change are: 1) constant and immediate changes in the number of passengers which will affect the helicopter weight; 2) applied force exerted on the helicopter body by strong winds; 3) changes in the mass of fuel tank or hydraulic pumps which in turn affect helicopter weight. Various robust control designs such as sliding control and also adaptive control are investigated in this system in order to obtain a suitable robustness in dealing with uncertainty of model parameters and load disturbances. Sliding mode control is an appropriate approach for controlling systems in the presence of various uncertainties. Sliding mode control is one of the nonlinear control methods for variable structure systems with uncertainties and internal and external disturbances. This method combines high accuracy with robustness. One of the most important problems of sliding controller is called chattering phenomenon. If an optimal sliding surface is not defined, sliding variables will approach the sliding surface in a pendulous and continuous manner, and they will move away from the surface quickly. This may cause a severe damage to durability and robustness of the system. Recently, a new approach called high order sliding mode (HOSM) has been suggested in order to solve the sliding mode problems. The properties of HOSM include using the sliding mode control advantages (being resistant against parametric uncertainty, being insensitive to limited disturbance, and having fast dynamic response) and overcoming the vibration effects at the same time. One of the disadvantages of using HOSM is the inaccessibility of sliding variable high order derivatives. The bad effect of measurement noise on these controllers is
another flaw of this type of control, and sliding surface repeated time derivatives are required. The following methods are suggested for fixing the above-mentioned flaws: 1) second order sliding mode with super twisting algorithm; 2) high order sliding mode differentiation in high order sliding mode controller. In the recent years, many researches are performed to control of twin rotors. A robust control algorithm was presented in [1] for solving the above-mentioned problem. This algorithm was based on a super twisting 2- sliding mode control (2-SMC) and has presented a solution for overcoming the torques caused by disturbances in the center of gravity and it guarantees a smooth flight.

In [2], stabilization of the 2- multivariable 2-rotor helicopter system reference point has been addressed. Based on dynamic model, a nonlinear PID controller at first and a fuzzy PID controller were designed next to stabilize the reference point for immediate implementation of control system on micro-controller. The MIMO system of helicopter in [3] was modeled as a Takagi-Sugeno fuzzy model. LQR fuzzy controller was designed to control pitch and yaw angle positions in TRMS by parallel distribution which is obtained based on Takagi-Sugeno fuzzy model. Using PID fuzzy sliding mode control (PIDFSMC) for controlling a two-rotor system (TRMS) was suggested in [4]. TRMS model was divided into two parts, i.e., the main rotor system and tail system. The sliding mode controller was designed first and PID control and fuzzy logic were developed thereafter.

Finally, application of combined PIDFSMC to main rotor and tail rotor has led to the elimination tracking error and chattering effect in controlling signal behavior. In [5], new fuzzy sliding and fuzzy integral sliding controllers (FSFISC) were designed to put pitch and yaw angles of a two-rotor MIMO system (TRMS) in a proper position. Regarding coupling effects which were considered as uncertainty, the nonlinear TRMS, which has been vastly coupled, was disintegrated into two horizontal and vertical sub-systems (VS).

Estimation of multi-variable two-rotor system (TRS) modes was done in [6]. Accordingly, DC inputs (similar to operational inputs) were created and the outputs were collected from TRS model. Inputs-outputs data were used in a Kalman filter, and the accuracy of obtained estimations about the modes was verified by evaluating state error covariance at first and then, by comparing the evolution of actual and estimated states. In [7], the robust control algorithm was tested for the best possible control of a helicopter rotor motor and great results were obtained. In [8], a combination of PID fuzzy control and genetic algorithm was used for fast and accurate guiding of twin rotor movement angles. In [9], the PID fuzzy controller compensate the twin rotor in an adaptive way based on updating the output scale factor. In addition, a combination of ANFIS fuzzy neural network and reducing fuzzy clustering method was used for improving response time and reducing the complexity.

In some works, modeling and identification of twin rotor system were performed in addition to controlling. In [10], modeling of quasi-linear variable parameter, identification, and control of twin rotor multivariable system were performed. State observer and state feedback controller were used based on pole placement and LMI regions for compensation of system. In [11], a neural network was used for experimental implementation of adaptive-dynamic inversion control rule for a nonlinear model and it was combined with feedback control system in order to compensate the inverse model errors. In [12], a state observer based on Chebyshev neural network was used for estimating unknown nonlinearities of the model in order to reduce the undesirable effects caused by unknown nonlinearities and non-modeled dynamics related to twin rotor system. In [13], twin rotor model was divided into two one input-one output subsystems, and the mutual coupling of these systems was considered as a disturbance for each of them.

A robust deadbeat control program was designed for each sub-system for system compensation. In [14], second order sliding mode controller was designed to stabilize laboratorial twin rotor system under considerable mutual coupling conditions. High order sliding mode control method was first applied to a helicopter movement system in 1998 [15]. In [16], a sliding controller was presented for systems with time switching. This sliding controller was first tested and applied to unmanned aerial aircrafts, and it was designed for helicopters since 2005.

This paper aims to control two-engine helicopter multivariable system by using second order sliding algorithms with twisting algorithms. Also, another important purpose to present is the using desirable features of second order sliding mode control in order to eliminate the chattering
phenomenon. In this regard, two sliding surfaces are defined as the tracking error sum of yaw and pitch angles and their integrals, and to analyze the conditions ruling the twin rotor, second order sliding mode control law is formulated based on twisting algorithm.

The rest of this paper is as follow:
Modeling of twin-rotor is presented in section 2. In section 3, mathematical formulations of first order and second order sliding mode control algorithms will be briefly explained. In section 4, the formulation of problem is presented for designing first and second order sliding mode controls on the twin rotor system. The simulation results and required analyses will be performed in section 5. The conclusion will be included in section 6.

II. TWIN ROTOR SYSTEM

The rotor system can be placed in a helicopter horizontally, just as main rotors which make it possible for helicopters to take off in a vertical direction. Twin rotor is controlled by two inputs, namely $u_1$ and $u_2$. Mutual coupling is one of the main features of twin rotor. According to Fig. 1, twin rotor consists of two thrusts, i.e., primary and secondary, which are run by two independent primary and secondary DC motors. These thrusts are perpendicular to each other and are attached to each other by a bar which can rotate freely on horizontal and vertical planes. Pitch and yaw angles can be justified by changing the voltage of primary and secondary motors, respectively; this is done by controlling the rotation speed of primary and secondary thrusts. Designing an effective controller to track pitch and yaw desirable angles is a difficult task due to the coupling between thrusts.

Having input voltage of DC motor for main thrust, i.e., $u_v$, and input voltage for secondary motor, i.e., $u_h$, one can determine the armature currents, i.e., $i_v$ and $i_h$, by solving the following differential equations:

\[
\frac{di_v}{dt} = \frac{1}{T_{mr}} (u_v - i_v) \quad (1)
\]

\[
\frac{di_h}{dt} = \frac{1}{T_{tr}} (u_h - i_h) \quad (2)
\]

Where $T_{mr}$ and $T_{tr}$ are time constants of primary and secondary motors, respectively. The angular velocity of primary thrust, i.e., $w_m$, and the angular velocity of secondary thrust, i.e., $w_t$, are nonlinear functions of armature currents $i_v$ and $i_h$, respectively, and are approximated as follows:

\[
w_m (i_v) = 90.99i_v^6 + 599.73i_v^5 - 129.263i_v^4 \\
- 1283.643i_v^3 + 63.45i_v^2 + 1283.4i_v
\]

Fig. 1: Schematic figure of twin rotor with main and tail propulsion [5]
In addition, thrust forces of $F_v$ and $F_h$ which cause the bar to rotate vertically and horizontally, respectively, are defined by a nonlinear function of angular velocities $(w_t)w_m$:

$$ F_v (w_m) = -3.48 \times 10^{-12} w_m^5 + 1.09 \times 10^{-9} w_m^4 + 4.123 \times 10^6 w_m^3 - 1.632 \times 10^4 w_m^2 + 9.544 \times 10^2 w_m $$(5)

$$ F_h (w_t) = -3 \times 10^{-14} w_t^5 - 1.595 \times 10^{-11} w_t^4 + 2.511 \times 10^{-7} w_t^3 - 1.808 \times 10^{-5} w_t^2 + 8.01 \times 10^{-7} w_t $$

(6)

In order to formulate the twin-rotor dynamics, consider the following variables:

- $S_h$: Angular torque of twin rotor on horizontal plane
- $S_v$: Angular torque of twin rotor on vertical plane
- $\alpha_h$: Rotation about axis z (yaw angle)
- $\alpha_v$: Rotation about axis x (pitch angle)
- $\Omega_h$: Yaw angular velocity on horizontal plane
- $\Omega_v$: Pitch angular velocity on vertical plane

Note that $\Omega_h$ and $\Omega_v$ are different from angular velocities of engines, i.e., $w_t$ and $w_m$. Therefore, twin rotor dynamics can be described as follows:

$$ \frac{ds_h}{dt} = l_s S_h F_v (w_v) \cos \alpha_v - k_h \Omega_h $$

(7)

$$ \frac{ds_v}{dt} = l_s S_v F_v (w_m) - g \left( 0.0099 \cos \alpha_v + 0.0168 \sin \alpha_v \right) - k_v \Omega_v - 0.0252 \Omega_v^2 \sin 2\alpha_v $$

(8)

$$ \frac{d\alpha_h}{dt} = \frac{1}{T_h} (u_h - i_h) $$

(9)

$$ \frac{d\alpha_v}{dt} = \frac{1}{T_v} (u_v - i_v) $$

(10)

$$ \Omega_v = \frac{d\alpha_v}{dt} = 9.1(s_x + J_\omega w_t (i_t)) $$

(11)

Table 1 shows the twin-rotor parameters according to above-mentioned equations. By differentiating $\Omega_h$ and $\Omega_v$, we can write:
\[
\Omega_h = \frac{\dot{\Omega}_h}{J_h} = \frac{\Omega_h \dot{\Omega}_h}{J_h} + J_m \omega_m \cos \alpha_m - J_m \omega_m \Omega_m \sin \alpha_m
\]  
(13)

\[
\dot{\Omega}_m = 9.1(\dot{\Omega}_m + J_m \dot{\Omega}_m)
\]  
(14)

where \( J_h = D \sin^2 \alpha_c + E \cos^2 \alpha_c + G \). By defining the new states as \( x_1 = \alpha_h, x_2 = \alpha_m, x_3 = \Omega_h, x_4 = \Omega_m, x_5 = i_h \) and \( x_6 = i_m \), state space of the twin rotor can be described as

\[
\begin{align*}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= \frac{1}{J_h} \left[ J_m \omega_m \cos \alpha_m x_2 - k_v x_4 x_3 + x_4 \left( D - E \right) \sin 2x_2 \\
&- J_m \omega_m \left( i_m - x_6 \right) \frac{d\omega_m \left( i_m \right)}{dx_m} \right] \\
\dot{x}_4 &= 9.1 \left[ k_v x_4 \right] + 9.1 \left[ -0.0252 x_3 \right] + 521.6543 \times 10^{-3} \frac{d\omega_m \left( i_m \right)}{dx_m} \\
\dot{x}_5 &= \frac{1}{T_{tr}} \left( u_h - x_5 \right) \\
\dot{x}_6 &= \frac{1}{T_{tr}} \left( u_m - x_6 \right)
\end{align*}
\]  
(15)

**TABLE I. PARAMETERS OF TWIN Rotor**

<table>
<thead>
<tr>
<th>variable name</th>
<th>definition</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_m )</td>
<td>length of the main part of the beam</td>
<td>0.236 m</td>
</tr>
<tr>
<td>( l_t )</td>
<td>length of the tail part of the beam</td>
<td>0.25 m</td>
</tr>
<tr>
<td>( k_v )</td>
<td>friction coefficient for the vertical axes</td>
<td>0.0095</td>
</tr>
<tr>
<td>( k_h )</td>
<td>friction coefficient for the horizontal axes</td>
<td>0.0054</td>
</tr>
<tr>
<td>( J_m )</td>
<td>moment of inertia in the DC motor for the main propeller</td>
<td>1.6543 \times 10^{-3} \text{kgm}^2</td>
</tr>
<tr>
<td>( J_{tr} )</td>
<td>moment of inertia in the DC motor for the tail propeller</td>
<td>2.65 \times 10^{-4} \text{kgm}^2</td>
</tr>
<tr>
<td>( T_{tr} )</td>
<td>time constant of the main rotor</td>
<td>1.432 sec</td>
</tr>
<tr>
<td>( T_{tr} )</td>
<td>time constant of the tail rotor</td>
<td>0.3842 sec</td>
</tr>
</tbody>
</table>
**III. Second order sliding mode control**

Assume a system with the following nonlinear dynamics is available:

\[
x(t) = F(x(t), t) + G(x(t), t)u(t)
\]  
(16)

where \(x \in X \subseteq \mathbb{R}^n\) and \(u \in U \subseteq \mathbb{R}\) denote state and input vectors, respectively. \(G \in \mathbb{R}^{n \times 1} \to \mathbb{R}^n\) is an unknown, sufficiently smooth vector field. Suppose that the sliding variable is also considered as \(\sigma(t) = \sigma(x(t), t)\).

By having the defined relative degree of the second order, i.e., \(r=2\) and according to control variable of \(u\) and since \(\psi : \mathbb{R}^{n-2} \times \mathbb{R}^2 \to \mathbb{R}^n\) exists, dynamics of internal states of stable \(W(t) \in \mathbb{R}^{n-2}\) is input-output stability. Therefore, system (16) can be decreased in a normal form as follows [7]:

\[
\begin{align*}
\dot{\sigma}(t) &= \varphi(\sigma(t), \dot{\sigma}(t), t) + \gamma(\sigma(t), \sigma(t), \dot{\sigma}(t), t)u(t) \\
\dot{\omega}(t) &= \psi(\sigma(t), \sigma(t), \dot{\sigma}(t), t)
\end{align*}
\]  
(17)

where \(x(t) = \psi(\sigma(t), \sigma(t), \dot{\sigma}(t))\).

Suppose that the following second order input-output dynamics is limited generally, and let the controlling gain signal be constant and known:

\[
\dot{\sigma}(t) = \varphi(\sigma(t), \sigma(t), \dot{\sigma}(t), t) + \gamma(\sigma(t), \sigma(t), \dot{\sigma}(t), t)u(t)
\]  
(18)

Then, second order sliding mode control problem for system (16) will cause system (17) limited time to be stable in such a way that it satisfies general boundary conditions.

\[
\begin{align*}
\left|\varphi(\sigma, \sigma, \dot{\sigma}, t)\right| &\leq \phi \\
0 < \Gamma_w &\leq \gamma(\sigma, \sigma, \dot{\sigma}, t) \leq \Gamma_M
\end{align*}
\]  
(19)

Let the sliding variable and total time derivative signal be accessible for feedback. Stability problem is introduced and solved in different ways. When the goal of this system is maintained at border areas, it is assumed possible that the limiting conditions at (19) are maintained locally.

**A. Twisting Algorithm**

Twisting algorithm is one of the first algorithms proposed in the desired class of second order sliding mode. This algorithm is based on knowing \(\sigma\) and \(\dot{\sigma}\) signals.

\[
u = -\alpha_1 \text{sign}(\sigma) + \alpha_2 \text{sign}(\dot{\sigma}), \quad \alpha_1 > \alpha_2
\]  
(20)
By setting $\beta$ parameter to zero in the ideal type, the control law (20) causes the system to have a similar path from twisting algorithm with $u - \alpha$, and $\alpha = \alpha U$. The variable sliding phase paths of $\sigma$ twist at the starting of two-dimensional phase plane, and it indicates that sliding variable cannot have a steady behavior; furthermore, maximum values of $|\varphi|\Omega$ are directly dependent on $u$ value. Convergence conditions are achieved for second order sliding mode by setting $\beta = 0$ in (21).

$$\begin{align*}
U &> \frac{\phi}{\Gamma_m} \\
\alpha^* &\geq \frac{2\phi + \Gamma_m U}{\Gamma_m U}
\end{align*}$$

(21)

**B. Designing second order sliding mode control for twin rotor**

If the rotor system dynamic model is as follows:

$$\dot{y} = f(x) + b(x)u$$

(22)

Therefore, sliding mode control law will be equal to:

$$u = -b^{-1}(x)(f(x) + k\text{sign}(s))$$

(23)

If $b(x)$ is not square, it must be used of pseudo inverse of $b(x)$, that is

$$u = -b^*(x)(f(x) + k\text{sign}(s)),$$

$$b^*(x) = b^T(x)(b^T(x)b(x))^{-1}$$

(24)

According to system model, it is not possible to use all of the state variables for sliding mode control due to singularity of $b'b$ matrix. In this regard, only 4 first state variables should be used in the process of compensation designing, because singularity of above-mentioned matrix will be avoided under these conditions. However, by doing this, $x_5$ and $x_6$ variables are abandoned and their divergent behavior causes the closed-loop system to be unstable. In order to compensate for the above-mentioned system, state variables should be located at a specified range that is determined based on system technical features [1].

This is a solution for instability problem of closed-loop system which was caused by the behavior of state variables 5 and 6. Limiting state variables, especially variables 5 and 6, prevents them from having a divergent behavior, and this makes the sliding mode controller perform the compensation for the first 4 state variables without worrying about the above-mentioned variables. It should be noted that the initial conditions of state variables and their reference values should be in their range for each one of them. The range of state variables is as follows:

$$
\begin{align*}
0 &\leq x_1 \leq 0.4346 \\
0.7 &\leq x_2 \leq 0.7 \\
0 &\leq x_3 \leq 0.1 \\
0 &\leq x_4 \leq 0.35 \\
0 &\leq x_5 \leq 1.112 \\
0 &\leq x_6 \leq 0.556
\end{align*}
$$

In order to design the control signal of second order sliding mode, with the aim of eliminating chattering at system controlled states, the following actions take place:
\[
\begin{align*}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= f_1 + f_s u_1 \\
\dot{x}_4 &= f_3 + f_s u_2
\end{align*}
\] (25)

By differentiating the above equations, we can write:
\[
\begin{align*}
\ddot{x}_1 &= \dot{x}_3 = f_1 + f_s u_1 \\
\ddot{x}_2 &= \dot{x}_4 = f_3 + f_s u_2
\end{align*}
\] (26)

Now, the sliding surface is defined as
\[
s = e + \int_0^t e(t) dt
\] (27)

where, error signal is
\[
e = \begin{bmatrix} x_{d1} \\ x_{d2} \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\]

By differentiating the above equation again, the following expression is obtained:
\[
\dot{s} = \dot{e} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\] (28)

In order to set the second derivative of sliding surface, we have to:
\[
\ddot{s} = \ddot{e} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} - \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_3 \end{bmatrix} - \begin{bmatrix} f_2 & 0 \\ 0 & f_4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
\] (29)

Also, in order to set the second derivative of sliding surface to zero, we have to:
\[
\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} f_2^{-1} & 0 \\ 0 & f_4^{-1} \end{bmatrix} \begin{bmatrix} f_1 \\ f_3 \end{bmatrix} + f_s
\] (30)

The switching part of controller (\( f_s \)) is designed by which \( s \) and \( \dot{s} \) become zero. To this end, there are various type of switching functions which a typical of it is as follows:
\[
f_s(s) = -(k_1 \text{sign}(s) - k_2 \text{sign}({\dot{s}})), \ k_1 \geq k_2
\] (31)

The evidence for proving the fact that this method sets \( s \) and \( \dot{s} \) to zero is available in related literature, and it shows that if the second derivative of the sliding surface is considered as
\[
\ddot{s} = -k_1 \text{sign}(s) - k_2 \text{sign}({\dot{s}}), \ k_1 \geq k_2
\] (32)

It will set \( s \) and \( \dot{s} \) to zero under any initial condition. Under these conditions, the chattering phenomenon is no longer observed on the sliding surface because its derivative has been set to zero.

In addition to controller, a precise differentiator for robust real time has been added in order to take the derivative of \( S \).

Differentiation is as follows:
\[
\begin{align*}
\dot{z}_0 &= -\gamma_1 L_1^2 \left[ z_0 - \frac{1}{\sqrt{2}} \text{sign}(z_0 - s) + z_1 \right] \\
\dot{z}_1 &= -\gamma_1 L \text{sign}(z_1 - \dot{z}_0) 
\end{align*}
\] (33)

Where \(z_0\) and \(z_1\) are real time estimations of \(s\) and \(\dot{s}\). Differentiator parameters of \(\gamma_1\) are selected experimentally. \(\gamma_1 = 1.10\) and \(\gamma_2 = 1.5\) are assumed in [17].

**IV. SIMULATION RESULTS**

The sampling time is equal to \(T_s=0.01\) sec. and the simulation time is 50 sec. The initial conditions of state variables and control signals for both first and second order sliding modes are as follow:

\[
\begin{align*}
\mathbf{x}(0) &= \begin{bmatrix} 0 & 0.05 & 0.05 & 0 & 0.05 & 0.02 \end{bmatrix}^T \\
\mathbf{x}(1) &= \begin{bmatrix} 0 & 0.06 & 0.06 & 0.02 & 0.03 & 0.02 \end{bmatrix}^T \\
\mathbf{u}(0) &= [0.1, 0.2], \mathbf{u}(1) = [0.6, 0.5].
\end{align*}
\]

Reference values for tracking the first two states are equal to 0.2 rad and 0.4 rad, respectively. First order sliding mode control signal parameters are equal to \(k_1 = 0.1, k_2 = 0.1, \lambda = 1\).

The selected sliding surface is as follows:

\[
s = e + \lambda \int_0^t \text{edt}
\] (34)

where, the error signal is \(e = \begin{bmatrix} x_{d1} \\ x_{d2} \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\).

The first order sliding mode control obtained as follows:

\[
u = -b^{-1} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} - 2 \begin{bmatrix} x_{d1} - x_1 \\ x_{d2} - x_2 \end{bmatrix} + \begin{bmatrix} k_1 \text{sign}(s) \\ k_2 \text{sign}(s) \end{bmatrix}
\] (35)

Parameters of second order sliding mode control are as follow

\[k_1 = 0.01, k_2 = 0.01, k_{s1} = 0.009, k_{s2} = 0.009, \lambda = 1\]

The sliding surface is similar to one defined for first order sliding mode. The sliding surface derivative is required for second order sliding mode control and it is determined by:

\[
s = e + \lambda \int_0^t \text{edt} \rightarrow \dot{s} = \dot{e} + \lambda e
\]

\[
\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} x_{d3} - x_3 \\ x_{d4} - x_4 \end{bmatrix} + \lambda \int_0^t \begin{bmatrix} x_{d3} - x_3 \\ x_{d4} - x_4 \end{bmatrix} \text{edt}
\] (36)

Therefore, signal control can be obtained as follows:
The following results are obtained for first and second order sliding mode controls according to presented conditions. Fig. 3 and 4 display the first two state variables.

As it can be observed, the chattering behavior caused by first order sliding mode control signal is eliminated by second order mode. Furthermore, the response state error is set to zero by second order sliding mode, and the derived transient response behavior is better than the first order mode, in such a way that it has a lower maximum overshoot. Figures 5 and 6 display the related responses to angular velocities of yaw and pitch. According to these results, sliding mode controller has successfully performed their convergence, whereas an oscillatory behavior is observed for responses derived from first order sliding mode controls. Figures 7 and 8 display the fifth and sixth variable behavior, i.e., armature currents. According to the obtained results, although the two above-mentioned variables are not controlled by first and second order sliding modes, the proposed algorithm has prevented them to diverge, so that both states have converged to certain values after a time period. According to the presented results in Fig. 9 and 10, the changes in $u_h$ control signal occur more in first order sliding mode than second order sliding mode.

![Fig. 3: tracking of yaw angle using first and second sliding mode control](image)

![Fig. 4: tracking of pitch angle using first and second sliding mode control](image)
Fig. 5: stabilizing of yaw angular velocity using first and second sliding mode control

Fig. 6: stabilizing of pitch angular velocity using first and second sliding mode control

Fig. 7: behavior of first armature current using first and second sliding mode control
Fig. 8: behavior of second armature current using first and second sliding mode control

Fig. 9: control signal $u_h$ obtained by first and second sliding mode control

Fig. 10: control signal $u_v$ obtained by first and second sliding mode control
Conclusion

In this paper, second order sliding mode control method is presented for compensation of yaw and pitch angles and also stabilization of yaw and pitch angular velocities. According to the investigation performed on twin rotor structure, it was revealed that each state of rotor should be located in a specified physical range practically in order to maintain the desired performance of the system. The presence of such a limitation forced the design process to locate armature currents in their allowable ranges as fifth and sixth state variables, so that the user is never allowed to move these states out of their range. In this case, these two variables are out of sliding mode control range and they are abandoned in their allowable range.

Therefore, control laws were determined based on the equations governing the first four state variables. Two sliding surfaces were defined as the sum of tracking error of first and second state variables and their integrals for design. The control law of first order sliding mode is determined by differentiating from above-mentioned surfaces once, and the control law of second order sliding mode was determined by differentiating from sliding surfaces twice. By performing first and second order sliding mode algorithms, the obtained results revealed that the second order sliding mode algorithm has the following desirable characteristics compared to first order sliding mode algorithm:

1. The guidance of yaw and pitch angles in second order sliding mode algorithm has less tracking errors compared to first order sliding mode algorithm. As a result, steady state response behavior derived from second order sliding mode has more accuracy than first order sliding mode.
2. The rate of fluctuations and changes related to the above-mentioned angles is less in second order mode compared to first order mode. Control signals range in second order sliding mode is smaller than first order sliding mode range. As a result, second order sliding mode control requires less control costs.
3. In comparison with first order sliding mode control, the results obtained by second order sliding mode control are chattering free.

References


