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Control of a new Chaotic System via Generalized Backstepping Method and Choice of Optimal Controller Mode Based on Electromagnetic Like Algorithm

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Abstract **- In this article, a new chaotic system was chosen as a chaotic system. One of the best control methods that would be used for the stabilization of these systems, was Backstepping. In this article, this technique is improved to Generalized Backstepping Method (GBM). For this new method, exhibit a new theorem and its proof and for showing its abilities, control new chaotic system equation. The generalized Backstepping approach consists of parameters, which accept positive values. The system replied differently for each value. Electromagnetic-like algorithms can select suitable and optimal values for the parameters. EA by minimizing the fitness function can find the optimal values for the parameters. Fitness function forces the system error to decay to zero rapidly that which causes the system to have a short and optimal setting time. Fitness function also makes an optimal controller and causes overshoot to reach its minimum value. This hybrid makes an optimal backstepping controller.**

Keywords: System, Lyapunov, generalized backstepping method, electromagnetic-like algorithm.

I. PREFACE

Due to the complex dynamics and inherent instability of a chaotic system, it seems impossible to control it (to make it behave in a desired manner) Nevertheless, it has been shown that chaotic systems are controllable, and different control objectives can be perceived for them. Nonlinear control concepts can be applied to these systems and moreover, due to their unique characteristics new and unique self-control methods can be utilized. Some of the perceivable control objectives for a chaotic system are as follows:

a. Elimination of the chaotic behavior and stabilization to the equilibrium point.
b. Stabilization of one of the periodic unstable orbits (and creating stable limit cycle).
c. Synchronization of two chaotic systems.
d. Anti-control of chaos (Chaotification).
e. Bifurcation control [12].

A system with chaotic behavior is not desirable. For instance, it can be an unacceptable chaotic oscillation in the rotor of an engine. It is necessary to eliminate such behavior. Synchronization of two chaotic systems entails coupling the two systems such that they will have the same exact behavior. This method has applicability in fields such as communication. Anti-control of chaos has gained attention in recent years and is used to induce chaotic behavior in a system that is not chaotic on its own (e.g. has reached a stable equilibrium point). Chaos is necessary sometimes such as when mixing different materials or regarding heartbeat. Bifurcation, although strictly not a part of the control of chaos, is nonetheless related to it. Bifurcation is a phenomenon that occurs after the system has become chaotic. In a nutshell, Bifurcation means that a change made to a parameter causes a change in the system's fundamental behavior (equilibrium points' position, their stability, number of equilibrium points, or lack thereof, and existence of limit cycle with the same characteristics and states).

In Bifurcation control, the intended objective is either to delay (or advance) bifurcation or to change its nature (stable, unstable). Bifurcation's relation to chaos is obvious as several bifurcations occur before the system becomes chaotic. Naturally delaying bifurcation will delay the onset of chaos [14]. In recent years much has been done to control and stabilize old and new chaotic systems. One of the systems introduced after 1963 was the Lorenz system whose control was first attempted using feedback method [1] & [2]. Later various control methods and chaotic systems were suggested, the most important of which are as follows:

For systems such as the chaotic Duffy system sliding mode, a controller has been used [4].

And for Geresio- Tesi systems adaptive linear feedback controller has been used [5].

And for the new chaotic system introduced by Zheng, a state feedback controller has been used [3].

A backstepping approach was utilized for chaotic systems Qi and Lorenz-stefo [6], [7] & [8].

Also, the new and effective method of Generalized Backstepping has been used for Lorenz chaotic system [9] & [10].

II. INTRODUCTION

Chaos is one of the most important phenomena observed in systems. Pehlivan introduced a three-degree-of-freedom

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system that showed chaotic behavior [18]. It is difficult to control chaotic systems. In the past decades, control and synchronization of chaotic systems have been extensively studied. The backstepping method cannot be effective for nonlinear feedback systems and MIMO also performs poorly for certain nonlinear systems [13]. The new method known as the Generalized Backstepping Method (GBM) is similar to Backstepping Method (BM), only with broader applicability. Backstepping method is only used in strict feedback systems, however, GBM expands on this class [12]. GBM incurs fewer costs compared to BM. The main contribution of this article is the optimization of the GMB controller with an Electromagnetism-like algorithm, producing an optimized controller by obtaining optimal and proper values for control parameters [16]. Electromagnetism-like algorithm reduces the current value in order to find the minimum Fitness Function. On the other hand, Fitness Function obtains the minimum value by minimizing the square error of the system. This article is organized as follows:

Section 3 describes the GBM algorithm.
Section 4 describes the Electromagnetism-like optimization algorithm.
Section 5 introduces the new chaotic system and its control.
Section 6 is a general discussion on the results of simulation for different systems.

III. THE GENERALIZED BACKSTEPPING METHOD

Generalized Backstepping method will be applied to a certain class of autonomous nonlinear systems which are expressed as follow.

$$\begin{cases} \dot{X} = F(X)+G(X)\eta \\ \dot{\eta} = f_0(X,\eta)+g_0(X,\eta)u \end{cases} \qquad (1)$$

In which $X=[x_1,x_2,\ldots,x_{n-1}]\in\Re^{n-1}$ and $\eta\in\Re$ و [۹] [۱۰]. In order to obtain an approach to control these systems, we may need to prove a new theorem as follow.

Theorem: Suppose equation (1) is available and assume that the relation (1) of the nonlinear control system model is hypothetical. Now consider the scalar function v (X) as follows.

$$V(x_1,x_2,\ldots,x_{n-1})=\frac{1}{2}\sum_{i=1}^{n-1}x_i^2 \qquad (2)$$

Suppose the scalar function $\eta=\Phi_i(x_1,x_2,\ldots,x_{n-1})$ for the i = 1, 2, …, n-1 state could be determined in a manner which by inserting the i term for, the function v (x) would be a positive definite equation 3 with negative definite derivative.

Therefore, the control signal and also the general control Lyapunov function of this system can be obtained by equations 3,4.

$$u=\frac{1}{g_o(x.,\eta)}\left\{\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\frac{\partial\Phi_i}{\partial\Phi_j}[f_i(X)+g_j(X)\eta] - \sum_{i=1}^{n-1}x_i g_i(X) - \sum_{i=1}^{n-1}k_i[\eta-\Phi_i(X)]-f_o(X,\eta)\right\}$$

For: $k_i>o$; i=1,2,...,n-1 (3)

$$V_i(X,\eta)=\frac{1}{2}\sum_{i=1}^{n-1}x_i^2+\frac{1}{2}\sum_{i=1}^{n-1}[\eta-\Phi_i(X)]^2 \quad (4)$$

Proof: Equation 1 can be represented as the extended form of equation 5.

$$\begin{cases} \dot{X} = F(X)+G(X)\eta \\ \dot{\eta} = F_a(X,\eta)+G_a(X,\eta)u \end{cases} \qquad (5)$$

$v(x)$ is always positive definite and therefore the negative definite of its derivative should be examined; it means $w(x)$ in Equation 6 should always be positive definite, so that $\dot{v}(X)$ would be negative definite.

$$\dot{v}(X)=\sum_{i=1}^{n}x_i\dot{x}_i=\sum_{i=1}^{n}x_i[f_i(X)+g_i(X)\Phi_i(X)] =< -w(X) \quad (6)$$

for:
$u_o=f_o(X,\eta)+g_o(X,\eta)u$

by adding and subtracting $g_i(X)\Phi_i(X)$ to the i_{th} of equations 5 and 7 would be obtained.

$$\begin{aligned} \dot{x}_i &= [f_i(X)+g_i(X)\Phi_i(X)]+g_i[\eta-\Phi_i(x)] \\ \dot{\eta} &= u_o \end{aligned} \qquad (7)$$

Now we use the following change of variable.

$$\begin{aligned} z_i &= \eta-\Phi_i(X) \\ \rightarrow \dot{z}_i &= u_o-\dot{\Phi}_i(X) \end{aligned} \qquad (8)$$

$$\dot{\Phi}_i(X)=\sum_{j=1}^{n}\frac{\partial(\Phi_i)}{\partial x_j}[f_j(x)+g_j(X)\eta] \qquad (9)$$

Therefore, equation *7* would be obtained as follows:

$$\begin{cases} \dot{x}_i = [f_i(X)+g_i(X)\Phi_i(X)]+g_i(X)[\eta-\Phi_i(X)] \\ \dot{z}_i = u_o-\dot{\Phi}_i\,,\ \ i=1,2,3,\ldots,n \end{cases} \qquad (10)$$

Regarding that zi has n states, the u_o can be considered with n terms, provided that equation *11* would be established as follows.

$$u_o=\sum_{i=1}^{n}u_i \qquad (11)$$

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Therefore, the last term of Equation *10* would be converted to equation *12*.

$$\dot{z}_i = u_i - \dot{\Phi}_i(X) = \Lambda_i \tag{12}$$

At this stage, the control Lyapunov function would be considered as equation *13*.

$$v_t(X,t) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{1}{2}\sum_{i=1}^{n} z_i^2 \tag{13}$$

This is a positive definite function. Now it is sufficient to examine the negative definitely of its derivative.

$$\dot{v}_t(X,\eta) = \sum_{i=1}^{n} \frac{\partial v(x)}{\partial x_i}[f_i(X) + g_i(X)\Phi_i(X)] + \sum_{i=1}^{n} \frac{\partial v(x)}{\partial x_i} g_i(X) + \sum_{i=1}^{n} z_i \Lambda_i \tag{14}$$

So that the function $\dot{v}_i(x,\eta)$ would be negative definite, it is sufficient that the value of Λ_i would be selected as the equation 15.

$$\Lambda_i = -\frac{\partial v(x)}{\partial x_i} g_i(X) - k_i z_i \qquad k_i > o \tag{15}$$

Therefore, the value of would be obtained from the following equation.

$$\dot{v}_t(X,\eta) = \sum_{i=1}^{n} x_i[f_i(X) + g_i(X)\Phi_i(X)] - \sum_{i=1}^{n} k_i z_i^2 \leq -w(X) - \sum_{i=1}^{n} k_i z_i^2 \tag{16}$$

Which indicates that the negative definitely status of the function $\dot{v}_t(X,\eta)$ consequently, the control signal function, using the equations 7, 9, and 11 would be converted to 17.

$$u_o = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial \Phi_i}{\partial x_j}[f_i(X) + g_i(X)] - \sum_{i=1}^{n} x_i g_i(X) - \sum_{i=1}^{n} k_i[\eta - \Phi_i(X)] \tag{17}$$

Therefore, using the variations of the variables which we carried out, equations 3, 4 can be obtained. Now, considering the unlimited region of positive definitely of $\dot{v}_t(X,\eta)$ and negative definitely of $\dot{v}_t(X,\eta)$ and the radially unbounded space of its states, global stability gives the proof [9, 10].

IV. ELECTROMAGNETISM-IKEALGORITHM

In EM algorithm just like swarm intelligence optimization algorithms, first, a cluster of particles (which are in fact algorithms' solving elements) are randomly scattered in the problem's domain. Then using a law similar to Coulomb's law, the location of particles is manipulated in such a way that their general movement is towards a better position. To achieve this, in EM algorithm a particle is assigned a virtual charge, the amount of which is in proportion to how optimally it is positioned.

Similar to natural laws of physics, in EM algorithm each particles' charge determines its force of attraction or repulsion towards other particles. In other words, particles that result in smaller cost function values have a higher charge and therefore attract other particles with a stronger force, while particles with high-cost function values repel other particles (or attract less) [16].

We use minimum error criteria to determine the Fitness or Target Function. Different methods can be used to find the minimum error. If X_i is taken as the system's current state and given that, the ultimate objective is tranquility and stability of the system therefore the final value must be zero (X_{di}).

Given that the ultimate objective is the tranquility of the system, we consider Fitness Function to be:

$$f(x,y,z) = \frac{1}{n}\sqrt{\sum_{i=1}^{n}(x_i - x_{di})^2} \tag{18}$$

We consider the following as Electromagnetism-like algorithm parameters:

TABLE 1: ELECTROMAGNETISM-LIKE ALGORITHM PARAMETERS

20	size population
75	Max IT
0.03	δ
[1 10]	Search Size (K)

V. CONTROL SYSTEM 3 DEGREE OF FREEDOM

We consider the 3 degrees of freedom below:

$$\dot{x} = a(x-y)$$
$$\dot{y} = -4ay + xz + mx^3$$
$$\dot{z} = -adz + x^3y + bz^2 \tag{19}$$

Where a, b, d, m are constant system parameters. Chaotic behavior occurs when the system parameters have a = 1.8, b = -0.07, d = 1.5, m = 0.12 And consider the following basic conditions: $(x,y,z) = (0.003,00.1,0.002)$.

System equilibrium points are submerged.

$$a(x-y) = 0$$
$$-4ay + xz + mx^3 = 0$$
$$-adz + x^3y + bz^2 = 0 \tag{20}$$

$$J(x,y,z) = \begin{bmatrix} a & -a & 0 \\ 3mx^2 + z & -4a & x \\ 3x^2y & x^3 & 2az - ad \end{bmatrix}$$

$$J(x,y,z) = \begin{bmatrix} 1.8 & -1.8 & 0 \\ 0 & -7.2 & 0 \\ 0 & 0 & -2.7 \end{bmatrix} \tag{21}$$

We have:

$(\Lambda_1 = 1.8, \Lambda_2 = -7.2, \Lambda_3 = -2.7)$

All roots do not have a real negative value, so the system behaves in chaos

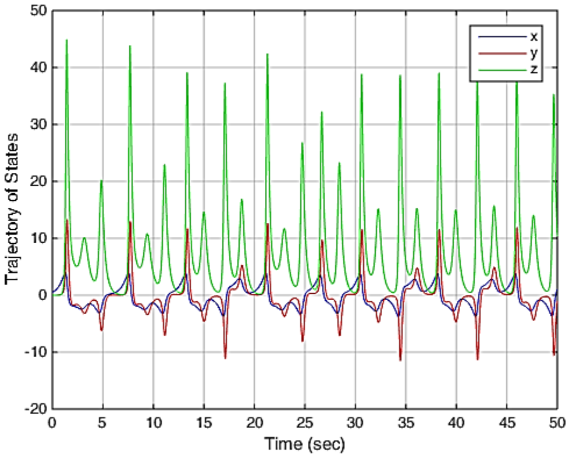

Fig. 1. Modify system states

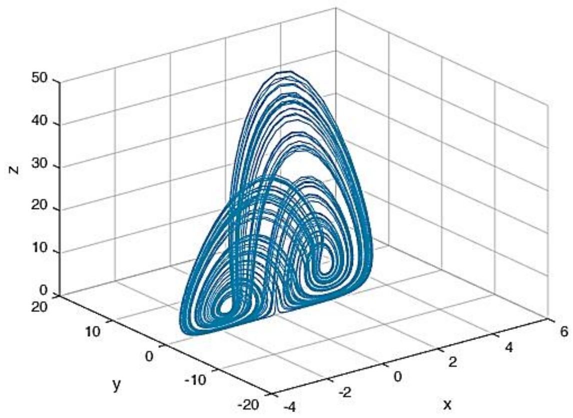

Fig. 2. Xyz three-dimensional diagram of the page-phase system

To stabilize system states with a generalized post transmission method, we add the two control inputs U_1 and U_2 as follows.

$$\dot{x} = a(x-y)$$
$$\dot{y} = -4ay + xz + mx^3 + u_1$$
$$\dot{z} = -adz + x^3y + bz^2 + u_2 \quad (22)$$

By changing the following variable.

$$\begin{cases} \eta_1 = y \\ \eta_2 = z \\ u_{a1} = -4ay + xz + mx^3 + u_1 \\ u_{a2} = -adz + x^3y + bz^2 + u_2 \end{cases} \quad (23)$$

We rewrite the equations (22) as follows.

$$\dot{x} = a(x_1 - \eta_1)$$
$$\dot{\eta}_1 = u_{a1}$$
$$\dot{\eta}_2 = u_{a2} \quad (24)$$

We select the virtual control vectors as follows.

$$\begin{cases} \varphi_{11} = -k_1 x \\ \varphi_{21} = 0 \end{cases} \quad (25)$$

Now we obtain the control vectors u_1, u_2 as follows.

$$u_1 = k_1\big(a(x-y)\big) + x - k_2(y - \varphi_{11}) - (-4ay + xz + mx^3)$$
$$u_2 = -k_3 z - (-adz + x^3y + bz^2) \quad (26)$$

The Lyapunov function of the entire system (27) is also obtained in the form below.

$$V(x,y,z,w) = \frac{1}{2}x^2 + \frac{3}{2}y^2 + \frac{1}{2}z^2 + \frac{1}{2}(y - \varphi_{11})^2 + \frac{1}{2}(z - \varphi_{21})^2 \quad (27)$$

The system controller is stable.

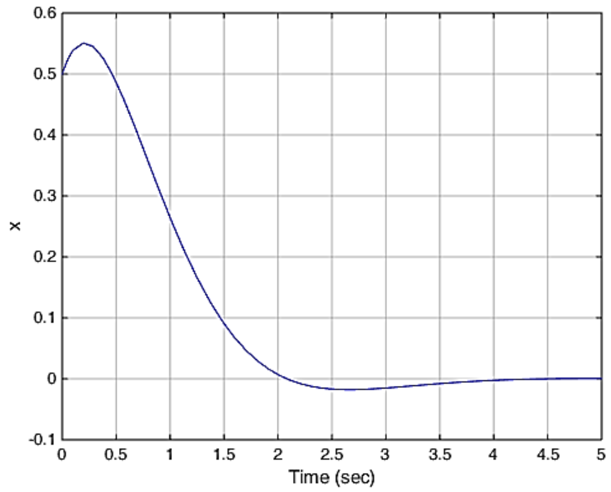

Fig. 3. Change x states

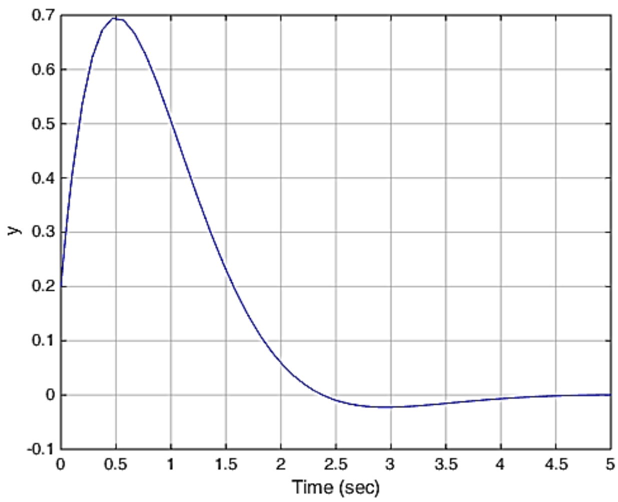

Fig. 4. Change y states

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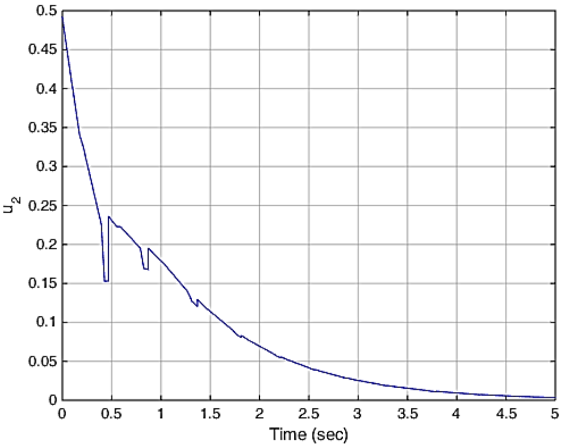

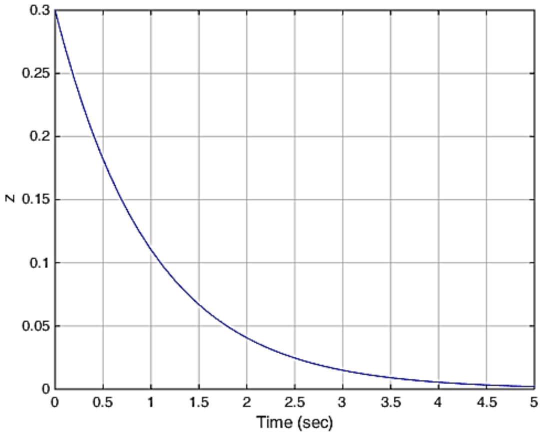

Fig. 5. Change z states

Control signals are as follows.

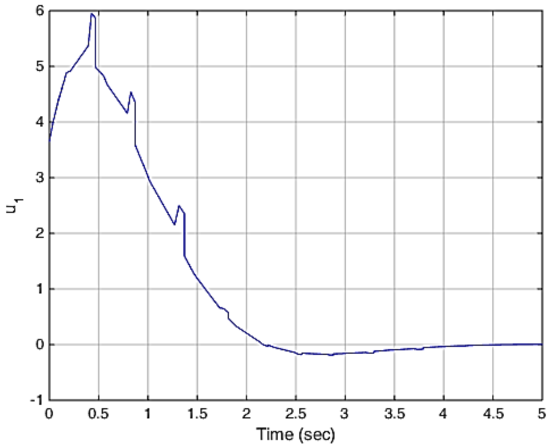

Fig. 6. Control signal u_1

Figure (7). Control signal u_2

After applying the electromagnetic induction algorithm, the controller coefficients are as follows.

$$k_1 = 9.96 \;\; k_2 = 2.59 \;\; k_3 = 9.51$$

With the optimal control of the system, it achieves sustainability, and the speed of the system's state of equilibrium is reduced.

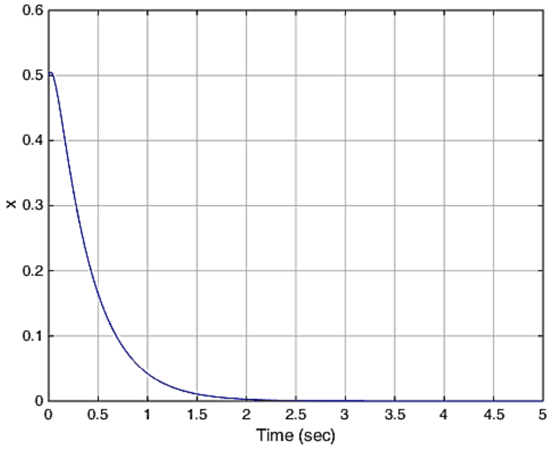

Figure (8). Change x states

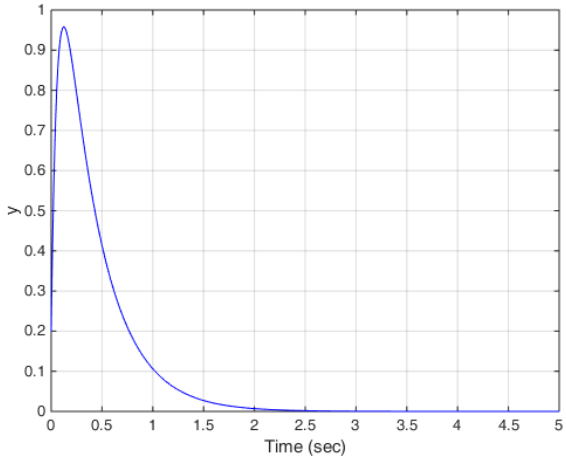

Figure (9). Change y states

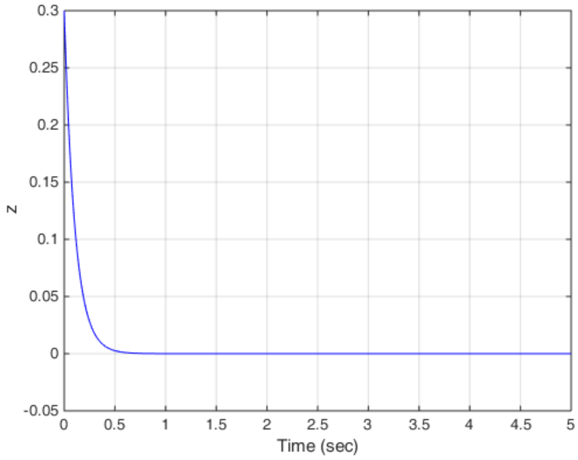

Figure (10). Change z states

It can be seen that an optimal control signal is obtained to stabilize the state of the system.

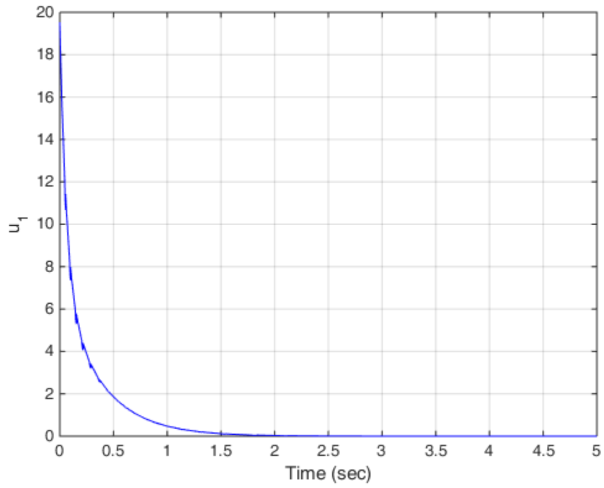

Figure (11). Control signal u_1

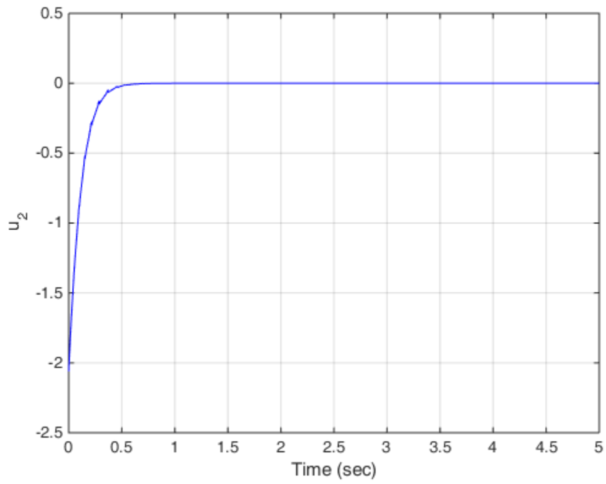

Figure (12). Control signal u_2

VI. CONCLUSION

This article made use of a controller based on GBM for chaotic systems. The design was achieved by controlling every part of the system and generalizing it to the entire system, resulting in the stabilization of system states. The results obtained using GBM show that the chaotic system can be controlled with an acceptable approximation as demonstrated in the given figures. Using the Electromagnetism-like optimization method, which is one of the best methods in this control system, we managed to optimize the controller even more than the original level which was already good and acceptable. This article shows that chaotic systems can be controlled and optimized to ideal levels for different purposes.

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