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Definite Integral for Real Life Problem Solving in Science and Engineering: An Overview

Musa Adamu[1]*, Tunde Olabiyisi[2] and Abdulkadir Iyyaka Audu[3]
[1]*Department of Computer Science, College of Education and Legal Studies, Nguru, Nigeria*
[2]*Department of Computer Science and Engineering, Ladoke Akintola University of Technology, Ogomoso, Nigeria*
[3]*Department of Computer Engineering, University of Maiduguri, Nigeria*
Phone Number: +234(0)8097252528
**Corresponding Author's E-mail: lm324fairchild@yahoo.com*

***Abstract* - Definite integral is used to calculate length of curves, areas and volumes. It is also used to calculate mass of a body of varying density, displacement, velocity, acceleration, projectile motion and vertical motion of a ball or object. Work done by a variable force, potential energy, total gravitational force, and total kinetic energy of a rotating rod can be obtained by methods of definite integral. Applications of definite integrals are also found in probability computation using probability density function, fluid pressure, finding the mean age and median age of a group of cells, rate of growth, rate of decay and consumption over time, and calculating the minimum costs and maximum profits in economics. They are used to also solve many interesting problems in finance. They however have wide range of applications in engineering, sciences and management, and hence the need to review, assess and evaluate research work put forward to improve means of solving it, in order to point out gaps, that will trigger more contributions to the definite integral research trend. Researchers in the field of mathematics, computer science and engineering are able to provide definite integral solvers in the form of websites, software like MATLAB and MAPLE, Algorithm such as firefly and Neural Network, Calculators with incorporated functionality to solve definite integrals and some device an enhanced manual methods to provide easier means of solving definite integrals when compared with the conventional or traditional methods. The paper assessed definite Integral solvers on websites, software packages, algorithms, devices available to solve definite integrals and research trends aimed at solving definite integral, and identified more contributions that are needed in each category of the solvers. Conclusions are drawn and appropriate recommendations were made to further improve definite integral solvers, to meet up with constant change in technologically driven global village and for overall success in numerous application areas.**

***Keywords*—Definite Integrals, Integral Solvers, Algorithms and Gaps.**

I. INTRODUCTION

In the quest to solve different challenges and problems in human society, scientists and engineers make a lot of decisions based on the results obtained from precise calculations made in designs and models. The decisions made have a profound effect on the quality of technological advancement of a society in all areas of human endeavor that include but are not limited to building environments, power, telecommunications, medicine, and transportation. At the heart of calculations made in designs and models are computer systems and circuits which must be very accurate, fast, and have capacity to handle a wide range of numbers [1]. These computers are relied upon for very precise results which are used as the basis of design of infrastructure in any society.

In literature, it is known that one key area of computation in the design of infrastructure is calculus which is applied in area analysis and volumetric analysis. Calculus is essentially divided into integral calculus and differential calculus. Integral calculus it typically used in the determination of areas and volumes in designs, while differential calculus typically deals with rate of change of an observed phenomenon in a design [1]. The paper review research trend in definite integral so as to boast the research in that area due the fact that infrastructural development is more wide spread in our society and efforts to standardize the process of its development cannot be overemphasized.

The various efforts towards infrastructural design and development in terms of roads, dams, bridges, airports, electricity generating stations, cables, buildings and provision of cars will require effort of computer and engineering professionals in term of accuracy in design. Definite integral equation solvers are aimed at providing design tool to professionals handling various types of projects. Definite Integral applications are widely found in operations research, computer science, mathematics, physical sciences and engineering [2].

Derivation of Solutions to mathematical problems manually is a useful skill when dealing with less complicated problems. However, providing manual solutions is laborious for a great variety of mathematical problem or even not possible and thus computers must be employed for solving these problems conveniently and with greater accuracy [3]. Studies have been conducted to provide high precision definite integral solvers [4]. Researchers have been devising different means to provide solutions to definite integrals [5]. Some definite integrals are difficult to solve and need special methods [6]. Some researchers employ the five columns rule in solving definite integration by parts through transformation of integral limits to provide easy solutions to definite integrals [7]. Some

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employ methods of brackets to provide solutions to special types of definite integrals [8].

A study on the "closed forms of some definite integrals" studies two special types of definite integrals and uses Maple for verification [5]. Research on "evaluating some types of definite integrals" studies three types of definite integrals and compares results obtained with that generated by Maple [6]. The study "Chaotic Firefly Algorithm for Solving Definite Integral" solves definite integrals using chaotic firefly algorithm [2]. A lot of research is put forward to providing easy and quick solutions to definite integrals due to the role they play in real life solutions to problems. Studies were also conducted to access how students of engineering and sciences visualize solutions of definite integrals in terms of problem solving [9]. A lot of work has also been done to improve students' ability to understand and relate solutions of definite integrals to real life problem solving [10]. A research on "Students' Understanding of the Definite Integral Concept" finds out that students' most dominant knowledge was the procedural one, and that they had limited understanding of the application of definite integral [11]. So much work has been done by various researchers in the field of engineering, computer science, information technology and mathematics to provide manual and easier ways of solving definite integrals, software programs to solve definite integrals, algorithms to solve definite integrals and integration of definite integral solvers into certain devices to provide solutions to definite integrals.

II. Riemann Sums and Definite Integrals

Riemann sums is a natural way to calculate mathematical or physical quantities that appear to be irregular when viewed as a whole, but which can be fragmented into regular pieces. Values for the regular pieces are calculated using known formulae and then sum them up to find a value for the irregular whole [12]. This approach to problem solving was around for thousands of years before calculus came along, but it was tedious work and the more accurate you wanted it to be, the more tedious it became. With calculus it became possible to get exact answers for these problems with almost no effort, because in the limit these sums became definite integrals and definite integrals could be evaluated with antiderivatives. With calculus, the challenge became one of fitting an integrable function to the situation at hand (the "modeling" step) and then finding an antiderivative for it [12].

A computer with flexible computer language can allow much more powerful methods of integrations [13]. Definite integration is one of the most important and basic concept in mathematics and it has numerous applications in fields such as physics and engineering [14]. Divergent integrals and integral operators with divergent kernels are widely used in mathematics, applied science and engineering [15]. Definite integral plays a great role in engineering designs.

III. Applications of Definite Integrals to Real Life Problem Solving

Numerous applications of definite integral is found in engineering and science disciplines for the calculation of length of a curve (arc length, length of cable hanging between two poles, and length of curve section of roads, bridges, dams, railways and run ways in airports) [16]. According to [13], calculations of areas (areas between curves, areas under simple curves, area of the region bounded by a curve and a line, surface area, surface area of solid of revolution, area of plane figure, and areas of rectangle, square, and triangle) can be done using definite integrals. Definite integrals are also used to calculate volumes (volume of sphere, cylinder, circular disk, solids of revolution, paraboloid, and cone) obtain by methods of slicing, disk, washers and cylinders [12].

Definite integrals find application in the calculation of mass of a body of varying density, displacement, velocity, acceleration, projectile's motion, vertical motion of a ball or object and initial velocity [12]. Work done by a variable force, potential energy, total gravitational force, and total kinetic energy of a rotating rod are also obtained by methods of definite integral [16]. Applications of definite integrals are also found in probability computation using probability density function, fluid pressure, finding the mean age and median age of a group of cells, rate of growth, rate of decay and consumption over time, and calculating the minimum costs and maximum profits in economics [16]. Definite integrals are used to also solve many interesting problems in finance [16].

IV. Types of Definite Integral Solvers

Definite Integrals are used in calculating values of different physical quantities. These calculated values are used as design parameters for critical real life projects and generation of fast and accurate results is necessary so as to ensure safety, security, robustness, durability and appreciable project life span. An assessment of the recent definite integral equation solvers revealed that websites such as WalframAlpha [17] and Symbolab [18] are online solvers which require internet connectivity, computer or smartphone and takes some time to access and initialize. Matlab is a software package that can calculate integrals but requires system with high computing power to install, also takes time to initialize, and require learning the environment and the language to compute integrals [19]. Maple is also a software package that requires installation, learning the language, and takes time to initialize too [20].

Moreover, calculators such as Texas Instrument (TI-83,84,85 and 89) are used to calculate definite integrals but have a lot of functions for other calculations and hence the process is slow and tedious [20]; these calculators are also 16-bit calculators which means the range of data representations and calculations they perform are limited. In designs involving large-scale computations, these calculators will not perform as required. Different researchers designed various types of Algorithms to solve

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definite integrals such as high precision Chaotic Firefly Algorithm for Solve Definite Integrals [2].

The literature above revealed that definite integral equation solvers are either software to be installed on devices, algorithms to be run on installed software, platforms to be accessed on websites or calculators with incorporated functionality to solve definite integrals.

V. Assessment of the Current Classes of Definite Integral Solvers and Available Gaps

Based on the types of definite integral solvers discussed above, it can be deduced that the current classes of the available definite integral solvers are as follows:

i. Websites accessed definite integral solvers
ii. Software packages that solve definite integrals
iii. Calculator with incorporated functionality to solve definite integrals
iv. Algorithms designed to solve definite integrals
v. New Manual Mathematical Methods by different researchers aimed at speeding up and minimizing the conventional or traditional steps required in solving definite integrals.

Websites accessed definite integral solvers:

This classes of definite integrals solvers provides websites containing platform to solve definite integrals. Anybody that wants to solve definite integral equation can first access these sites and follow the steps provided to solve the equation. Appropriate input parameters required should be provided by the user in order to get the answer to the supplied equation.

This class of solvers has the following disadvantages:

1. Delay in the process of results generation due to the need to have a computer system or smartphone to access the website and also internet connectivity.
2. Cost of data and other means of internet connection.
3. Limitation in the number of bits required for computation.
4. Lack of awareness on the part of users of their availability, and how to access and use them.
5. Security threats: These sites are free websites and may have dubious intent to defraud users.

To overcome these shortcomings, programmable hardware-based devices that could be portable with less initialization and run time, moderate cost, and comparably easier to use will be of great advantage and should provide permanent solutions to the above-outlined problems of the internet based solvers.

Software packages that solve definite integrals:

These are definite integral solvers in form of software packages. They are first installed on computers before usage. One has to learn how to work with their environments and also the appropriate language required in performing definite integration. A lot is required of the user to be conversant with these packages. This class of definite integral equation solvers has the following disadvantages:

1. Sophisticated packages that require systems with high processing power and with larger memory.
2. Require great learning and mastery of the environment and language before usage
3. Takes time to initialize and generate results
4. Not easily affordable to most users.

Similarly, to overcome these shortcomings programmable hardware-based devices that could be portable with less initialization and run time, moderate cost, and comparably easier to use will be of great advantage and should provide permanent solutions to the above-outlined problems of the software packages that solve definite integrals.

Software algorithms that require a less sophisticated user environment can also be used to address the problems of definite integral software packages outlined above.

Calculators with incorporated functionality to solve definite integrals:

These calculators such as Texas Instrument (TI-83,84,85 and 89) are used to calculate definite integrals but have a lot of functions for other calculations and hence the process is slow and tedious and requires much learning before efficient usage. They are also 16-bit calculators which means the range of data representations and calculations they perform are limited. This class of solvers has the following disadvantages:

1. Mostly uses 16-bits data and hence limitation in the scope of calculations that will generate accurate results
2. Has a lot of functionality and hence the process of integration becomes tedious with a lot of confusions.
3. Input process is slow and hence results generation becomes relatively slow.
4. Requires learning the user manual before been able to use it and solve definite integrals.

A device with higher bits greater than 16, and with less functionality and or dedicated to solving definite integral will overcome the above outlined shortcomings. The device should have easy input methods with less complexity to enable ease of use. It should have size, cost, power consumption and speed at the heart of the its design. This will produce a cheap device that is less complex and easily affordable to professionals at all time.

Algorithms designed to solve definite integrals:

These are algorithms that run on platforms like computers or smart phones and that solve definite integrals. This class of definite integral solvers has the following disadvantages:

1. Requires platforms to run and be implemented and hence slow and takes time to initialize and generate results.
2. Require skills and knowledge of how to run and implement the algorithm to put it to use.
3. Lack of awareness on the part of the users of its existence and implementation.

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Programmable hardware-based devices that could be portable with less initialization and run time, moderate cost, and comparably easier to use will be of great advantage and should provide permanent solutions to the above-outlined problems of algorithms designed to solve definite integrals.

New Manual Mathematical Methods:

This is conceived by different researchers aimed at speeding up and minimizing the conventional steps required in solving definite integrals. This class of solvers requires computerization to speed it up and generate early and accurate results.

VI. Importance of Providing Improvements on Definite Integral Equation Solvers

Providing an improved definite integral equation solver will help meet up with huge data computations occasioned by the large volume of data in use globally, as a result of technological advancement and also provide more accurate results to Science and engineering professionals, economics and finance experts, project clients, project users, and researchers.

The science and engineering professionals in the design of roads, bridges, dams, energy systems, ports, railway, airports, buildings, cars, mirrors, glasses, cones, cylinders, spheres, cables, and rods, using definite integrals to provide solutions to problems of length, areas, volumes, density, probability, and motion will benefit from the design of an improved definite integral equation solver.

Project clients would benefit indirectly from this design as accurate design parameters lead to perfect end products as required and specified by the owners. Users of completed projects will also benefit from the solver, as sound and solid completed projects ensure robustness and a longer service life span. Students and experts conducting research that require the solution to definite integrals will find the solver helpful. Economics and finance experts solving definite integrals for analysis will also benefit from the solver.

From the assessment of the above types of definite integral equation solvers, to provide an improved solver, the following observations are made:

1. There is a need to design a device purposely dedicated to solving definite integral with less initialization time.
2. The device proposed in one (1) above should be portable and incorporated with high processing power.
3. It should not require internet connectivity to operate.
4. It should not require learning any computer language to operate.
5. Should also be affordable.
6. Capable of being operated to generate quick, efficient, and reliable results.
7. Should be developed using the latest state of the art devices that can be easily upgraded.
8. Capable of requiring just input from the user and generating fast, accurate, and timely results.
9. Adoptable to usage in a wide range of scientific and engineering designs.
10. Cost, size, power consumption, and speed should form the backbone of the design to overcome existence shortcomings identified.

VII. Research Trend Indefinite Integrals

The research on "The Five Columns Rule in Solving Definite Integration by Parts through Transformation of Integral Limits" [7], provides alternative method of solving definite integration by parts. It solves integrals by using algorithm of the tabular integration by parts through transformation of integral limits. Results obtained indicated that the formula of integration by part is changed to a new form after transformation and that the final solutions obtained by this method and standard technique in calculus textbooks are exactly the same [7]. The authors devised the five columns rule to simplify the longer procedure in the conventional methods and the operation of each column is as follows: the first column writes the positive and negative signs alternately, the second column is f(x), and then successive its derivative that lies below it repeatedly such that the derivative equal to zero, the third column is g'(x), and then its integral lies below it, the fourth column writes the lower limit (x = a) and the upper limit (x = b), and the fifth column is the transformation of integral limits, i.e. the lower double limit {u(a)v(a)} and upper double limit {u(b)v(b)} on the integral respectively [7].

The study "Solution of Definite Integrals using Functional Link Artificial Neural Networks" [22], discusses a new method to solve definite integrals using feedforward artificial neural networks. The study built a neural network to be used as a novel alternative to pre-established numerical methods, in which the definite integrals are solved with the help of learning algorithm, through minimizing a well-constructed error function [22]. The results indicated that the algorithm, with respect to existing numerical methods, is effective and precise and well-suited for purposes which require integration of higher order polynomials and all observations made in the research work were recorded and illustrated in tabular and graphical form [22].

The paper "A Study of Definite Integrals Using Parseval's Identity" [23], studies two types of definite integrals in which using Parseval's identity, the infinite series expressions of the two types of definite integrals were determined. Two examples were used for practical calculation and the research method adopted in the study was to find solutions through manual calculations and verify these solutions using Maple [23]. The research method allows the discovery of calculation errors and also helps modify the original directions of thinking and therefore, it gives reason that indicates that Maple provides insights and guidance regarding problem-solving methods [23].

The research "Definite integrals by the method of brackets" [8], presents a new heuristic method for the evaluation of definite integrals and the researchers were of the view that despite the method being heuristic, without a rigorous description, it is a powerful method of integration, quite simple to work with and also generate results of

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definite integrals easily [8]. The evaluation of a definite integral is reduced to solving a linear system of equations, having method of brackets, with origin in methods developed for the evaluation of Feynman diagrams where the operational rules and methods were illustrated using several examples. The method simplifies the evaluation of a large class of definite integrals to the solution of a linear system of equations, with the basic idea behind it, being the assignment of a bracket _a_ to any parameter a [8].

The work "High precision computing of definite integrals with .Net Framework C# and X-MPIR" [4], presents high precision numerical computing of definite integrals in a specific environment, namely .Net Framework in which an application was created using latest state of the art library created for realization of functions and numerical methods of arbitrary precision in a given environment. The main goal of the research work is to provide powerful mutually supporting computational instruments for solving non-trivial problems of numerical integration of definite integrals in the environment [4].The application solves infinite integrals within the interval (a,∞) with the arbitrary real number a or $(-\infty, +\infty)$ and the calculations are done with an arbitrary (given by the user) precision by using a text form function expression that can include algebraic operations and functions (elementary and special) which allows additional parameters and was implemented using C# in .Net Framework, build, targeting the maximal possible portability (32 – or 64-bit windows systems) [4].

The article "Generalization of Risch's Algorithm to Special Functions" [24], deals with the evaluation of integrals in closed form in which an overview of Risch's algorithm highlighting key areas and recent developments were discussed and it proved suited for both indefinite and definite integration. The research appeared worthwhile in that it can be used to compute linear relations among integrals, to also find identities for special functions given by parameter integrals and was set to achieve two aims: to introduce the reader to some basic ideas of differential algebra in the context of integration and to raise awareness in the physics community of computer algebra algorithms for indefinite and definite integration [24].

The research "Solving the Linear Integral Equations Based on Radial Basis Function Interpolation" [3] introduces the radial basis function (RBF) method, especially the multiquadric (MQ) function, in solving linear integral equations that uses solution procedure of the multiquadric function which expresses the unknown function, firstly in linear combination forms of radial basis functions. The process transforms the integral equation into collocation matrix of RBFs, before finally, solving the matrix equation to obtain an approximate solution and the superior interpolation performance of MQ, enables the method to acquire higher precision with fewer nodes and perform low computations [3]. This gives MQ method more advantages over thin plate splines (TPS) method and the implementation uses two types of integration schemes, the Gauss quadrature formula and regional split technique, where the numerical results indicated that the MQ solution can achieve accuracy of $1E$-5, hence the MQ method is suitable and promising for integral equations [3].

The Journal "An Algorithm to find Definite Integrals using Simpson Rule" [1], presents an algorithmic approach to find definite single and double integrals using Simpson's rule, through an easy practical way of calculation with less computation time (run time) and storage space to engineers and scientists. The work identified a gap to design a Simpson method which is different from the traditional one available in literature to provide numerical solution of ordinary differential equations and after the development of the new Simpson method, numerical experiments were performed to show the validity of the algorithm [1].

The Research "Evaluating Some Types of Definite Integrals" [6], undertook study of three types of definite integrals by obtaining infinite series forms of these integrals using Parseval's theorem and using some examples provided for practical calculations which were verified with Maple. The research indicated that, the applications of the theorem are extensive, can be employed easily in solving many difficult problems and some recommendations were made to conduct further studies on calculus and engineering mathematics problems and verify the results obtained using Maple [6].

The paper Chaotic Firefly Algorithm for Solving Definite Integral [2], presented an Improved Firefly Algorithm with Chaos (IFCH) for solving definite integrals which satisfies the question of conducting parallel calculation of numerical integration in sciences and engineering and provided an adaptive segmentation points in the algorithm. After conducting several numerical simulations, the results indicated that the algorithm offers an efficient way to calculate the numerical value of definite integrals, with a high convergence rate, high accuracy and robustness [2]. The research "The Definite Integral and Computer" [13], presents the results of the learning concept of definite integral and numeric integration with the computer, where the computer does all the tedious work, which leaves the teacher and the pupils with enough time to discuss the problem, try out multiple ideas and approaches to solving, and, finally, compare and analyze them. The computer does all the tedious work, which leaves the teacher and the pupils with enough time to discuss the problem, try out multiple ideas and approaches to solving, and, finally, compare and analyze them [13].

Another research "Engineering students' visual thinking of the concept of definite integral" [9], undertakes a study that is not only to extend understanding of students' difficulties and strengths associated with visualisation, but also to identify the types of visual image they utilise while solving integral problems. Detailed analyses of students' work and verbal protocols indicated that, high proportions of students with high visualisation ability use imagination images along with algebraic representations, and linking these two representations leads them to successful problem solving [9]. Students with low visualisation ability use memory images and the study shows that imagination images produce by students play a significant role in the

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problem-solving process. Therefore, visualization process allows an articulation between representations to produce another representation that could help students to solve given problems [9]. A study "Definitions and Images for the Definite Integral Concept" [25], presents definitions and images, as well as the relation between them and the definite integral concept, where 41 English high school students were examined, through a questionnaire designed to assess the cognitive schemes of the definite integral concept evoked by the students. One question aimed to check whether the students knew how to define the concept of definite integral and five others were designed to categorize how students use the concept of definite integral and how this is related to the definition. The results show that only 7 students out of 41 of the sample knew the definition [25].

The paper "Application of definite integral methods in solving the problem of digitization" [16], revealed that definite integral methods are widely used in solving practical problems in geometry, physics, and economics and it also observed that mastering some certain integral calculation methods helps in solving some real life practical problems. The few examples, used in the research work, indicated that to solve practical problems of definite integral, the most important thing to do is to digitize the problem, and then write out the formula using the mathematical theory, and finally calculating the results using integral principle [16].

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