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Linear simultaneous control of nonlinear systems by state feedback based on TS fuzzy model

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***Abstract*— In this paper, the problem of simultaneous stabilization of nonlinear systems is proposed by using the Takagi-Sugeno (TS) fuzzy modeling and synthesis approach. The key idea of TS fuzzy models is to utilize a group of fuzzy rules combined with a family of local linear models to describe a complex nonlinear system, which are smoothly blended through fuzzy membership functions. Using this feature, analysis and synthesis of such systems based on Lyapunov stability method lead to some linear matrix inequalities (LMIs), which eases synthesis of nonlinear systems. This problem refers to stabilization of some nonlinear systems by utilizing a single linear state feedback controller. a state feedback controller is designed by solving some linear matrix inequalities (LMIs). a numerical example is presented to show that the proposed controller successively stabilizes the three considered nonlinear systems simultaneously.**

Keywords*— *TS fuzzy system, Simultaneous stabilization, Linear state feedback controller

1. INTRODUCTION

Since the work of [1], simultaneous stabilization has received considerable attention. In [2], complexity of simultaneous stabilization has been discussed and it has been proved that the simultaneous stabilizability of three linear systems is rationally undecidable. As a result, the burden of finding linear simultaneous stabilizer is mostly on sufficient conditions and finding methods to reduce conservatism is of high importance. Good literature review on simultaneous stabilization and increasing the feasibility region for linear systems can be found on [3], [4]and references there in.

For nonlinear systems, the simultaneous stabilization problem is more complicated. In [5], it is proved that for any countable family of stabilizable nonlinear systems, a continuous state feedback law, which simultaneously stabilizes the family (non-asymptotically), always exists. Additionally, a sufficient condition for the existence of simultaneously asymptotically stabilizing controllers for a collection of nonlinear systems was provided.

It is known that the fuzzy logic theory provides a powerful approach to tackle the analysis and design problems for complex nonlinear systems. Particularly, the dynamic Takagi-Sugeno (TS) fuzzy model has attracted lots of attentions for most model-based fuzzy analysis approaches, since it is conceptually simple and rigorously effective for synthesizing many highly complex nonlinear systems [6], [7], [8], [9].

Over the past 22 years, TS fuzzy models have been successfully utilized to approximate and control of nonlinear systems [10]. Stability of closed-loop systems with a fuzzy controller has been studied for several years. TS fuzzy models are composed of rules with a conclusion part using a state space representation. They allow consequently the use of the potential of linear theory. Stability results are usually Lyapunov based [11].A more interesting approach, called Parallel Distributed Compensation (PDC). At this stage, the control law has the same structure as the TS fuzzy model. Using the premises and rules of the model, the conclusion part is composed of linear state feedback gains. The PDC synthesis is often performed using a Lyapunov approach and the obtained equations are Linear Matrix Inequalities (LMI), for which powerful resolution tools are available.

This paper proposes a methodology to study the simultaneous stabilization of nonlinear systems by TS fuzzy models. Using this feature, analysis and synthesis of such systems based on Lyapunov stability method lead to some linear matrix inequalities (LMIs), which eases synthesis of nonlinear systems. This problem refers to stabilization of some nonlinear systems by utilizing a single linear state feedback controller. a state feedback controller is designed by solving some linear matrix inequalities (LMIs). Finally, the proposed design conditions are validated on a numerical example that the proposed controller successively stabilize the three considered nonlinear systems simultaneously.

This paper is organized as follows. Section 2 introduces background materials about TS models and the major previously obtained stability conditions will be reviewed. Simultaneous stabilization by using TS fuzzy modeling is formulated in Sections 3. Moreover, LMI

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conditions have been proposed to construct simultaneous stabilizer by state feedback. In section 4 two numerical examples will be given to illustrate the effectiveness of the proposed methods. Finally, the paper is concluded in Section 5.

2. TS fuzzy models

Takagi-Sugeno [10]proposed an effective way to model a complex dynamical system, the dynamic of a system model is built as a convex sum of the dynamics of a fixed number of linear subsystems. Consequently, r linear systems S_i are considered here.

In a multi-model approach, each subsystem may be a linearization of the system around M_i, $i=1,...,r$, point of the state space and the subsystem S_i is then all the more representative of the global system that the state vector $x(t)\in R^n$ is close to $M_i, i=1,...,r$. In this case, the Takagi-Sugeno model is used to deal with uncertainty through the use of an approximation of the system. The weighting function of S_i is called $w_i(z(t))$, with $z(t)=\left[z_1(t),...,z_p(t)\right]\in R^p$, the premise variable vector which depends either linearly or nonlinearly on the state vector.

Another way to use TS model is to write a given nonlinear model under the TS form. All the nonlinearities of the system are rejected in the functions $w_i(z(t))$. Note that the subsystems and the rule systems do not have in general a physical sense.

The TS fuzzy model can be seen as represented by *r* system rules. The ith system rule is:

Model Rule i:

$$\begin{aligned}&IF\ z_1(t)\ is\ M_{i1}\ and\ ...and\ z_p(t)\ is\ M_{ip}\\&\quad THEN\ \dot{x}(t)=A_i x(t)+B_i u(t)\end{aligned}. \qquad (1)$$

where $A_i\in R^{n\times n}$, $B_i\in R^{n\times m}$

Using a standard fuzzy inference, the final state of the fuzzy model is inferred as follows:

$$\dot{x}(t)=\sum_{i=1}^{r}h_i(z)\left(A_i x(t)+B_i u(t)\right). \qquad (2)$$

where $h_i(t)=h_i(z(t))=w_i(z(t))/\sum_{l=1}^{r}w_l(z(t))$ and $w_i(z(t))=\prod_{j=1}^{p}M_{ij}(z_j(t))$.

The function $h_i(z)$ satisfy the convex sum property i.e. , $\sum_{i=1}^{r}h_i(z)=1, 0\le h_i(z)\le 1$.

The continuous open loop fuzzy system, called CFS-OL, is then

$$\dot{x}(t)=\left(\sum_{i=1}^{r}h_i(t)A_i\right)x(t). \qquad (3)$$

The stability analysis of the system described in (3) is performed in the next section.

For the closed-loop system, different control strategies may be investigated. It is supposed in the followings that the state vector is accessible to measurement. Using a simple linear state feedback,

$$u(t)=-Kx(t),\ K\in R^{m\times n}. \qquad (4)$$

The new model is CFS-LIN:

$$\dot{x}(t)=\left(\sum_{i=1}^{r}h_i(z)(A_i-B_iK)\right)x(t). \qquad (5)$$

The interest of such a feedback is that it allows us to do a pole placement. The stability analysis of (5) can be performed in the same way as for (3).

3. Simultaneous Stabilization Based on TS fuzzy models

Theorem 1 [12]**:** Consider the system (3), if there exist a symmetric positive defined matrix $P\in R^{n\times n}$ satisfying

$$A_i^T P+PA_i<0,\quad i=1,...,r. \qquad (6)$$

Then the TS fuzzy model is globally asymptotically stable.

TS model sometimes includes a constant part in the conclusion of its rules, i.e.,

$$\dot{x}(t)=\left(\sum_{i=1}^{r}h_i(z(t))\left(A_i x(t)-D_i\right)\right). \qquad (7)$$

Stability conditions are derived in the same way.

Consider a class of s nonlinear systems,

$$\begin{cases}\dot{x}(t)=f^l(x(t),u(t))\\ y(t)=g^l(x(t),u(t))\end{cases},\qquad l=1,...,s. \qquad (8)$$

Suppose that TS model of these systems can be expressed as the following:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i^l(x)\left(A_i^l x(t) + B_i^l u(t)\right) \\ y(t) = \sum_{k=1}^{r'} h_k'^l(x) C_k^l x(t) \end{cases}, \qquad l = 1,\dots,s \cdot \quad (9)$$

Notation: The term A_i^l means *ith* subsystem of TS fuzzy model for *lth* nonlinear system.

The functions $h_i^l(x)$ and $h_k'^l(x)$ satisfy the convex sum property i.e. , $\sum_{i=1}^{r} h_i^l(x) = 1, \quad 0 \le h_i^l(x) \le 1$, $\sum_{k=1}^{r'} h_k'^l(x) = 1, \quad 0 \le h_k'^l(x) \le 1$

Our main objective is to simultaneously stabilize the class of nonlinear systems of nonlinear systems by state feedback stabilizer.

3.1. State Feedback Simultaneous Stabilization

In this section, the main objective is to design the following state feedback controller,

$$u(t) = -Kx(t),\ K \in R \qquad (10)$$

In order to simultaneously stabilize the class of s nonlinear systems as described in (9).

Substituting (10) in (9), closed loop systems is as follows,

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i^l(x)\left(A_i^l x(t) - B_i^l K x(t)\right), \qquad l = 1,\dots,s \\ &= \sum_{i=1}^{r} h_i^l(x)\left(A_i^l - B_i^l K\right)x(t) \end{aligned} \qquad (11)$$

.

To study the stability of the fuzzy models, the following Lyapunov functions are defined:

$$V^l\left(x(t)\right) = x^T(t) P^l x(t), \qquad l = 1,\dots,s\,. \qquad (12)$$

with $P^l, l = \{1,\dots,s\}$ symmetric positive definite matrices of $R^{n\times n}$ to be computed later using a LMI approach.

We can calculate the derivative of V^l along the trajectories of the system:

$$\dot{V}^l\left(x(t)\right) = \dot{x}^T P^l x + x^T P^l \dot{x}, \qquad l = 1,\dots,s\,. \qquad (13)$$

Substituting (11) , we have

$$\begin{aligned} \dot{V}^l\left(x(t)\right) &= \left(\sum_{i=1}^{r} h_i^l(x)\left(A_i^l - B_i^l K\right)x(t)\right)^T P^l x \\ &\quad + x^T P^l \left(\sum_{i=1}^{r} h_i^l(x)\left(A_i^l - B_i^l K\right)x(t)\right), \qquad l = 1,\dots,s \end{aligned} \qquad (14)$$

.

Or

$$\dot{V}^l\left(x(t)\right) = x^T \left[\sum_{i=1}^{r} h_i^l(x)\left(\left(A_i^l - B_i^l K\right)^T P^l + P^l\left(A_i^l - B_i^l K\right)\right)\right] x\,. \quad l = 1,\dots,s \qquad (15)$$

Considering convex sum properties of $h_i^l(x),\ l = \{1,\dots,s\}$, the systems (9) is asymptotically stable with state feedback (10), if

$$\left(A_i^l - B_i^l K\right)^T P^l + P^l\left(A_i^l - B_i^l K\right) < 0, \quad i = 1,\dots,r, l = 1,\dots,s \qquad (16)$$

.

The equation (16) is not convex as it contains multiplication of unknown variables. In order to reach convexity, common Lyapunov matrix

$$P = P^1 = \dots = P^s\,. \qquad (17)$$

shall be considered.

Considering common Lyapunov function following by congruence transformation with $P^{-1} = X > 0$ on (16), we have:

$$XA_i^{lT} + A_i^l X - (XK^T B_i^{lT} + B_i^l K X) < 0, \quad i = 1,\dots,r, l = 1,\dots,s \qquad (18)$$

.

Defining $M = K X$, we have,

$$XA_i^{lT} + A_i^l X - (M^T B_i^{lT} + B_i^l M) < 0, \quad i = 1,\dots,r, l = 1,\dots,s \qquad (19)$$

.

The condition is convex with respect to unknown variables $X > 0$ and M and results in the following Theorem.

Theorem 2 : Consider a class of s nonlinear systems (9). The systems are simultaneously stabilizable with state feedback (10) if there exists a positive definite matrix $X > 0$ and M which satisfy the following LMI conditions (19).

If conditions (19) are satisfied then state feedback can be constructed from the following equations and (10):

$$K = M X^{-1}. \qquad (20)$$

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4. Numerical Results

In this section two numerical examples will be given to illustrate the effectiveness of the proposed methods. The implementations are done in MATLAB 7.10.0.499 (R2010a) running on a PC Desktop Intel® Core i3 and 4 GB RAM. We use YALMIP (R14SP3) [13]with LMI solver SDPT3.

***Example* 1**: Consider the following nonlinear models which must be stabilized via state feedback simultaneously:

$$P_1:\begin{cases}\dot{x}_1=-3x_1x_2{}^2+x_2e^{x_1}+0.7x_2x_1{}^2\\ \dot{x}_2=x_1\sqrt{x_2}+x_1\cos(x_1)-2x_1x_2+x_2{}^2+5u\end{cases} \quad (21)$$

$$P_2:\begin{cases}\dot{x}_1=-x_1\cos^2(x_2)+x_1^2x_2{}^3+4e^{\left(x_1{}^2\right)}x_2\\ \dot{x}_2=-2x_1{}^2x_2+x_1{}^2+x_2\sin\left(x_1\right)+x_1x_2+2u\end{cases} \quad (22)$$

$$P_3:\begin{cases}\dot{x}_1=-2x_1\tan^2(x_1)+4e^{(x_1{}^2)}x_2+x_1{}^4x_2\\ \dot{x}_2=x_1x_2-x_1{}^3-2x_1{}^5x_2{}^2+x_2\sin(x_1{}^2)+3u\end{cases} \quad (23)$$

System P_1 can be written as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-3x_2{}^2 & e^{x_1}+0.7x_1{}^2\\ \sqrt{x_2}+\cos(x_1) & -2x_1+x_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 5\end{bmatrix}u \quad (24)$$

where the nonlinear terms define as:

$$Z_1\equiv-3x_2{}^2 \qquad Z_2\equiv e^{x_1}+0.7x_1{}^2$$
$$Z_3\equiv\sqrt{x_2}+\cos(x_1) \qquad Z_4\equiv-2x_1+x_2 \quad (25)$$

For simplicity, we assume that $x_1,x_2\in[0.5,0.5]$. Of course, we can assume any range for x_1,x_2 to construct a fuzzy model. The minimum and maximum values of nonlinear terms under $x_1,x_2\in[-0.5,0.5]$ are obtained as follows:

$$Max\left(Z_1\right)=0, Max\left(Z_2\right)=1.8237, Max\left(Z_3\right)=1.7071, Max\left(Z_4\right)=1.5$$
$$\min\left(Z_1\right)=-0.75, \min\left(Z_2\right)=0.7794, \min\left(Z_3\right)=0.8776, \min\left(Z_4\right)=-1.5 \quad (26)$$

This nonlinear model can be represented by TS model with the following subsystem matrices:

$$A_1^1=\begin{pmatrix}-0.75 & 0.7794\\ 0.8776 & -1.5\end{pmatrix}, A_2^1=\begin{pmatrix}0 & 0.7794\\ 0.8776 & -1.5\end{pmatrix}$$
$$A_3^1=\begin{pmatrix}-0.75 & 1.8237\\ 0.8776 & -1.5\end{pmatrix}, A_4^1=\begin{pmatrix}0 & 1.8237\\ 0.8776 & -1.5\end{pmatrix}$$
$$A_5^1=\begin{pmatrix}-0.75 & 0.7794\\ 1.7071 & -1.5\end{pmatrix}, A_6^1=\begin{pmatrix}0 & 0.7794\\ 1.7071 & -1.5\end{pmatrix}$$
$$A_7^1=\begin{pmatrix}-0.75 & 1.8237\\ 1.7071 & -1.5\end{pmatrix}, A_8^1=\begin{pmatrix}0 & 1.8237\\ 1.7071 & -1.5\end{pmatrix}$$
$$A_9^1=\begin{pmatrix}-0.75 & 0.7794\\ 0.8776 & 1.5\end{pmatrix}, A_{10}^1=\begin{pmatrix}0 & 0.7794\\ 0.8776 & 1.5\end{pmatrix}$$
$$A_{11}^1=\begin{pmatrix}-0.75 & 1.8237\\ 0.8776 & 1.5\end{pmatrix}, A_{12}^1=\begin{pmatrix}0 & 1.8237\\ 0.8776 & 1.5\end{pmatrix} \quad (27)$$
$$A_{13}^1=\begin{pmatrix}-0.75 & 0.7794\\ 1.7071 & 1.5\end{pmatrix}, A_{14}^1=\begin{pmatrix}0 & 0.7794\\ 1.7071 & 1.5\end{pmatrix}$$
$$A_{15}^1=\begin{pmatrix}-0.75 & 1.8237\\ 1.7071 & 1.5\end{pmatrix}, A_{16}^1=\begin{pmatrix}0 & 1.8237\\ 1.7071 & 1.5\end{pmatrix}$$

The membership functions can be obtained as:

$$m=\frac{Max\left(Z\right)-Z}{Max\left(Z\right)-\min\left(Z\right)}, \quad M=\frac{Z-\min\left(Z\right)}{Max\left(Z\right)-\min\left(Z\right)} \quad (28)$$

Therefore, the membership functions for P_1 are calculated as:

$$m_1=\frac{-Z_1}{0.75},\quad m_2=\frac{1.8237-Z_2}{1.0443},\quad m_3=\frac{1.7071-Z_3}{0.8295},\quad m_4=\frac{1.5-Z_4}{3}$$
$$M_1=\frac{Z_1+0.75}{0.75},\quad M_2=\frac{Z_2-0.7794}{1.0443},\quad M_3=\frac{Z_3-0.8776}{0.8295},\quad M_4=\frac{Z_4+1.5}{3}, \quad (29)$$

Using the proposed LMIs in Theorem 2, a TS state feedback simultaneous controller in the form of (10) with the following gains for systems P_1 , P_2 and P_3 is obtained:

$$K=\begin{bmatrix}38.6212 & 84.9693\end{bmatrix} \quad (30)$$

Using the above mentioned controller, the closed-loop system is simulated. Then, the phase portrait of closed-loop systems P_1 , P_2 and P_3 are presented in Figs. 1 , 2 and 3, respectively.

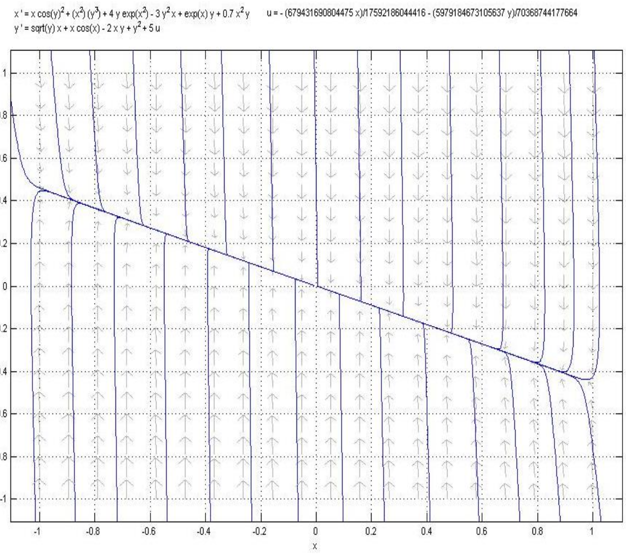

Fig 1. Phase portrait of closed-loop system P_1 in example 1.

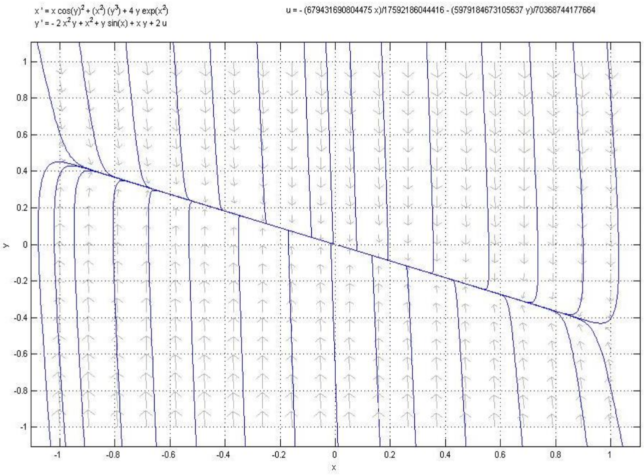

Fig 2. Phase portrait of closed-loop system P_2 in example 1.

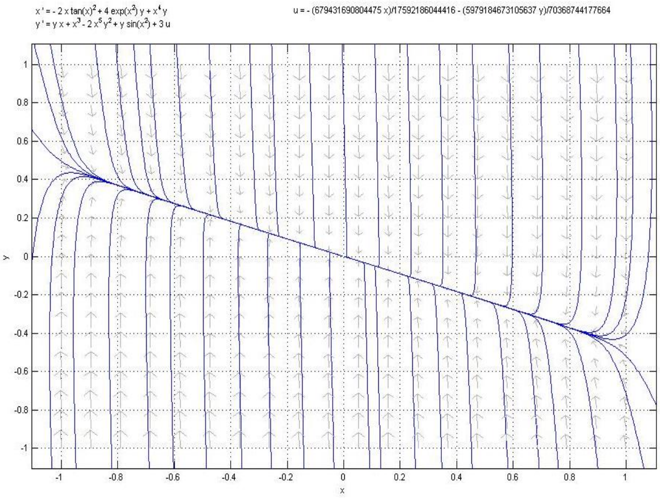

Fig 3. Phase portrait of closed-loop system P_3 in example 1.

Using Figs. 1 , 2 & 3, the proposed TS fuzzy state feedback simultaneous controller successfully stabilizes nonlinear systems P_1 , P_2 and P_3 for various initial conditions.

5. Conclusions

Simultaneous stabilization is the problem of determining a single controller, which simultaneously stabilizes a finite collection of systems. In practice, due to system uncertainty, system variation, failure modes or systems with various modes of operation, the simultaneous stabilization problem is frequently encountered. In this paper a linear state feedback controller structure based on TS fuzzy model is introduced and some conditions on simultaneous stability for the class of nonlinear systems had been reached. Some LMIs had been derived to reach convex problems with respect to variable matrices. Finally, a numerical example was presented to verify the availability of the proposed work.

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