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Analytical Disturbance Modeling of a Flywheel Due to Statically and Dynamically Unbalances

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Abstract

Unbalances in rotational machines can't delete completely somehow for precise mechanism it is necessary to control vibration due to such disturbances. In this research two common disturbance resources (dynamically and statically unbalances) for a flywheel on a rigid shaft modeled and energy methods used to derive equation of motion in five degrees of freedom. Equations linearized due to small vibration and disturbance forces and torques achieved. The model use to define design criteria for accepted level of unbalances in precise machines like real flywheel with known parameters used in a control system of a satellite.

Keywords: reaction wheel; unbalances; disturbances.

1. Introduction

Earth observing is the most common mission described for satellites nowadays. Because of high altitude and taking image requirements, satellite's stability during payload action is the main goal to achieve. Reaction Wheel -spinning disk- acts as a momentum exchange devices that control satellite's attitude by exchanging momentum with the spacecraft to pointing at scientific targets-.Reaction wheel as many mechanical instruments never manufactured perfectly and induced certain disturbances while spinning. Experiments show [2] spinning disk has three main disturbance sources: 1) imbalances of the fly wheel 2) bearing and motor disturbances and motor driver error 3) flexibility of the wheel. The first is the most significant disturbance which occurs at the same frequency as the wheels spin rate. The model introduced, consist of a balanced

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flywheel on flexible supports and imbalances modeled with lumped masses that are positioned strategically on the wheel. Linear spring and dampers are added to model shaft and bearing flexibility. The model is based on the physical behavior of the wheel and derived using Lagrangian energy method.

The system has five degrees of freedom, two generalized rotation, two generalized translation and the rotation of the wheel which is assumed to be constant. The equation of motion of full system solved in series of stages, First the problem of a balanced rotating flywheel on flexible supports is solved then the static and dynamic imbalance masses are added to complete the model. Finally by assuming small motion, equation linearized thereby four translational and rotational decupled equation obtained.

2. Describing Model

With respect to rotor dynamic concepts for modeling a flywheel and experimental data for unbalanced flywheel on a rotating shaft, there is a reaction wheel model with five degrees of freedom which have translation in axial direction, translation in two radial directions and rotating about two radial axis. This model comprised of 3 dominant model schematically show in figure.1.

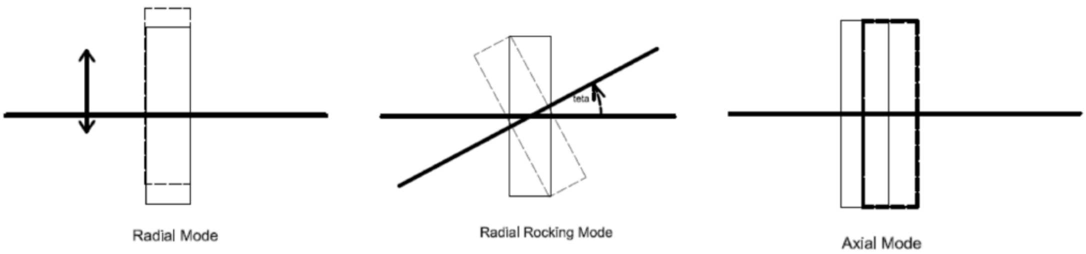

Figure 1: Axial, Radial and Radial Rocking Modes [2]

According to static and dynamic un-balances which discuss in detail in this research, axial translation mode is not induced by such disturbances. A complete model of unbalanced flywheel on rotating shaft introduced in rotor dynamic hand books [9]. This model has five degrees of freedom, 3 rotational and 2 radial translations. The Flywheel of

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mass M and radius R placed on a shaft of length 2D. Flexibility in the shaft and bearings are modeled with 4 linear spring of stiffness k/2 located at a same distance from flywheel center. Linear dashpots with damping coefficient of c/2 placed parallel with the springs at a distance D. figure.2 shows complete model. As show Dynamic and static unbalances modeled as lumped masses placed at radius R on flywheel. This model captures the radial modes, constant rotation, 7, about axis and gyroscopic effect which couple two radial modes.

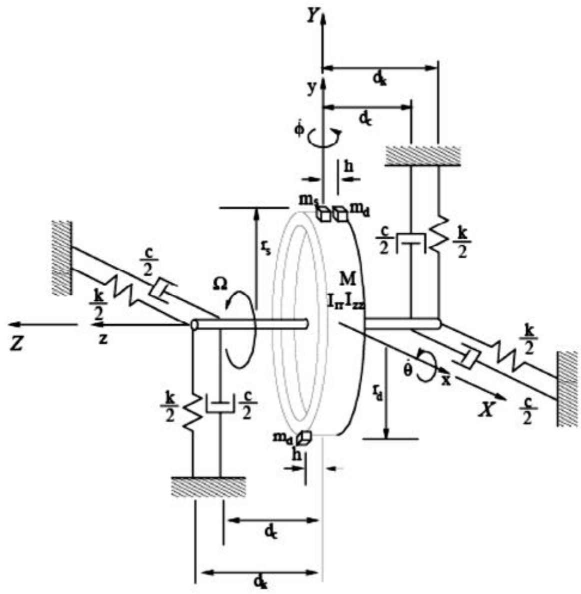

Figure 2: complete model

3. Deriving Equation of Motion

As introduce in the section before, there is a complicated model, Therefore energy methods in series of stages are used to derive equation of motion. At first balanced flywheel on flexible supports is considered.

a. Balanced flywheel

Figure.3 shows a balanced flywheel on flexible supports. The wheel is free to rotate about 3 different axis which are not perpendicular together.

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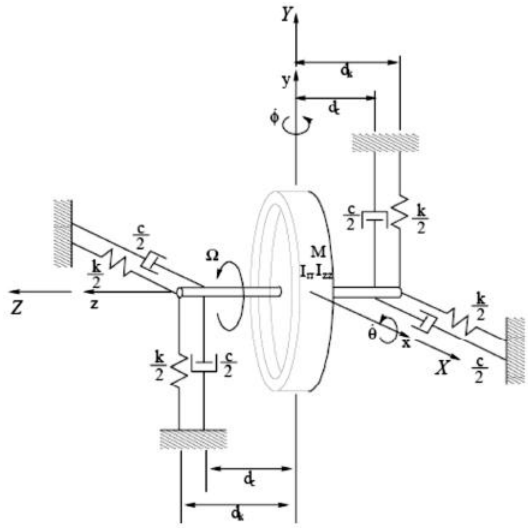

Figure 3: balanced flywheel model

Euler angles used to define rigid body rotations and 4 coordinate frame to simplify the equation. As shown in figure 4, the first rotation, φ, is about Y-axis of the ground fixed inertial frame, "XYZ" which leads to define an intermediate reference frame "abc" that next rotation, θ, is about it's a-axis and rocking frame "x`y`z`" described relatively. This frame is rotating by both $\varphi$ and $\theta$, with respect to ground. Spinning of the wheel, ψ, is about z`-axis and define the final body fixed frame "xyz".

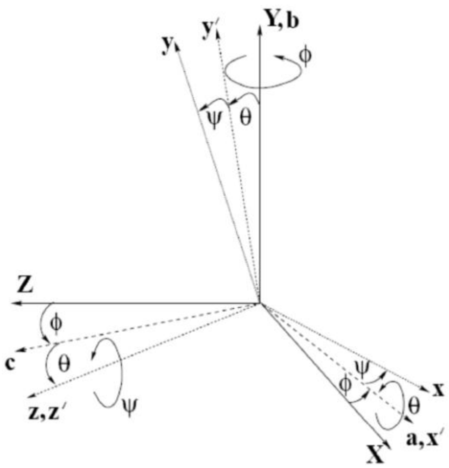

Figure 4: four coordinate frame

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Energy methods require the expressions for the kinetic and potential energies of the system and the external work done on system. The angular velocity of the wheel in terms of the generalized rotations and the constant spin rate is obtained by inspection from the euler angle rotations shown in figure 4.

$$\vec{\omega} = \dot{\theta}u_a + \dot{\varphi}u_Y + \Omega u_{z`} \quad (1)$$

Since the balanced flywheel is axisymetric, the kinetic energy can be written in the rocking frame, x`y`z`, which change the angular velocity in form`s below:

$$\omega_{x`y`z`} = \begin{bmatrix} \dot{\theta} \\ \dot{\phi}cos\theta \\ \Omega - \dot{\phi}sin\theta \end{bmatrix} \quad (2)$$

By assuming small rotations in Y-axis and a-axis, the mass moment of inertia tensor can be written in terms of the principle moment of inertia of the wheel

$$I_{x`y`z`} = \begin{pmatrix} I_{rr} & 0 & 0 \\ 0 & I_{rr} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix} \quad (3)$$

Where I_{rr} and I_{zz} are the radial and polar moments of inertia:

$$I_{rr} = \frac{1}{2}MR^2 \ , \ I_{zz} = \frac{1}{4}MR^2 \quad (4)$$

Translational degrees of freedom is:

$$V = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} \quad (5)$$

Kinetic energy is defined as:

$$T = \frac{1}{2} * I\omega^2 + \frac{1}{2} * MV^2 \quad (6)$$

substituting parameters in (6) leads to:

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$$T_w = \frac{1}{2}[(\dot{\theta}^2 + \dot{\varphi}^2 cos^2\theta)I_{rr} + I_{zz}(\Omega - \dot{\varphi}\, sin\,\theta)^2 + M(\dot{x}^2 + \dot{y}^2)]$$

(7)

the potential energy of the flywheel stored in the springs is

$$V = \frac{k}{4}[(x + d_k\, sin\,\varphi)^2 + (x - d_k\, sin\,\varphi)^2 + (y + d_k\, sin\,\theta)^2 + (y - d_k\, sin\,\theta)^2]$$

(8)

Because the wheel is centered axially on the shaft, equation 8 reduced to:

$$V = \frac{k}{2}[{d_k}^2(sin^2\theta + sin^2\varphi) + x^2 + y^2] \quad (9)$$

the external work done on the wheel by dashpots can be written simplicity due to symmetry

$$\delta W = -c[\dot{y}\delta y + \dot{x}\delta x + {d_c}^2(\dot{\theta} cos^2\theta\delta\theta + \dot{\varphi} cos^2\varphi\delta\varphi)] \quad (10)$$

in lagrangian methods, the lagrangian described first which leads to :

$$L_w = \frac{1}{2}\{(\dot{\theta}^2 + \dot{\varphi}^2 cos^2\theta)I_{rr} + (\Omega - \dot{\varphi} sin\theta)^2 * I_{zz} + M(\dot{x}^2 + \dot{y}^2) - k[{d_k}^2(sin^2\theta + sin^2\varphi) + x^2 + y^2]\} \quad (11)$$

The equation of motion for the generalized translations are

$$\begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} + \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \quad (12)$$

And for generalized rotation are:

$$\begin{pmatrix} I_{rr} & 0 \\ 0 & I_{rr} \end{pmatrix}\begin{pmatrix} \ddot{\theta} \\ \ddot{\varphi} \end{pmatrix} + \begin{pmatrix} c{d_c}^2 & \Omega I_{zz} \\ -\Omega I_{zz} & c{d_c}^2 \end{pmatrix}\begin{pmatrix} \dot{\theta} \\ \dot{\varphi} \end{pmatrix} + \begin{pmatrix} k{d_k}^2 & 0 \\ 0 & k{d_k}^2 \end{pmatrix}\begin{pmatrix} \theta \\ \varphi \end{pmatrix} = 0 \quad (13)$$

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These equation have been linearized by assuming small motion about x, y, θ and φ. As equations show translational and rotational degrees of freedom are decoupled due to symmetry in the model. By setting C=0 and solving for the eigenvalues in equation above, two natural frequency obtained, $\omega_{1,2} = \sqrt{\frac{k}{m}}$ for radial translational modes and by assuming $\theta = Ae^{i\omega t}$ and $\varphi = Be^{i\omega t}$ then substituting in to equation 13and setting C=0, two rotational natural frequency obtained.

$$\omega_{3,4} = \pm \frac{\Omega I_{zz}}{2I_{rr}} + \sqrt{(\frac{\Omega I_{zz}}{2I_{rr}})^2 + \frac{kd_k{}^2}{I_{rr}}} \tag{14}$$

As shows, $\omega_{3,4}$ are dependent on the spin rate of the wheel **Ω**. The flexibility of the shaft and the gyroscopic precession of the flywheel creates a rocking mode.

b. Static unbalances

Static unbalances is caused by the offset of the center of mass of the wheel from the axis of rotation and can be modeled as a small mass placed at radius R on the wheel.as shown in figure 2. To use lagrangian equation, kinetic energy must be calculated, therefore the position of the mass on the wheel in the XYZ frame is determined first to obtain an expression for velocity of mass.

$$u_{ms} = \begin{Bmatrix} 0 \\ r_s \\ 0 \end{Bmatrix} \tag{15}$$

Transforming a point from the wheel-fixed frame to the inertial ground-fixed frame using direction cosine matrix and considering free translation in the X and Y direction leads to obtain position in new frame:

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$$U_{ms} = \begin{Bmatrix} r_s(\sin\varphi \sin\theta \cos\Omega t - \cos\varphi \sin\Omega t) + x \\ r_s \cos\theta \cos\Omega t + y \\ r_s(\sin\varphi \sin\Omega t + \cos\varphi \sin\theta \cos\Omega t) \end{Bmatrix} \quad (16)$$

By differentiating position vector and adding in kinetic energy, equation 7, equation of motion obtained. Because of decoupling translational and rotational degrees of freedom, the addition of the static imbalance mass does not affect the rotational degrees of freedom.

$$\begin{pmatrix} M + m_s & 0 \\ 0 & M + m_s \end{pmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} + \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = m_s r_s \Omega^2 \begin{pmatrix} -\sin(\Omega t) \\ \cos(\Omega t) \end{pmatrix} \quad (17)$$

The equation of motion for a wheel with static unbalances only change in translational mode-with respect to balanced wheel. Because of small mass (m_s), potential energy does not change by adding static imbalances.

c. Dynamic unbalances

Angular misalignment of the principal axis leads to torque disturbances, it is modeled as two equal masses, m_d, placed 180 apart at a radial distance, r, and an axial distance, h, from the center of the flywheel as shown in figure 2. As static unbalances, to incorporate dynamic unbalances the position of mass in the body fixed frame transform in to the ground-fixed reference frame:

$$U_{md1} = \begin{Bmatrix} r_d(\sin(\varphi)\sin(\theta)\cos(\Omega t) - \cos\varphi \sin(\Omega t)) - h\cos\theta \sin\varphi + x \\ r_d \cos\theta \cos\Omega t + h \sin\theta + y \\ r_d(\sin\varphi \sin\Omega t + \cos\varphi \sin\theta \cos\Omega t) - h \cos\varphi \cos\theta \end{Bmatrix} \quad (18)$$

$$U_{md2} = \begin{Bmatrix} -r_d(\sin(\varphi)\sin(\theta)\cos(\Omega t) - \cos\varphi \sin(\Omega t)) + h\cos\theta \sin\varphi + x \\ -r_d \cos\theta \cos\Omega t - h \sin\theta + y \\ -r_d(\sin\varphi \sin\Omega t + \cos\varphi \sin\theta \cos\Omega t) + h \cos\varphi \cos\theta \end{Bmatrix} \quad (19)$$

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By differentiating these equations the velocity of unbalanced masses obtained and kinetic energy described. Dynamic unbalances does not affect on radial translation mode. To derive equation of motion of full model, figure1, all kinetic energy is combined:

$$T = T_w + T_m + T_{m`}$$

Lagrangian equation is formed as before and a complex expression in terms of the generalized coordinates and there derivatives obtained. By linearizing about small translations and rotations, the equation of motion for the full model in x,y direction formed

$$\begin{pmatrix} M+m_s+m_d & 0 \\ 0 & M+m_s+m_d \end{pmatrix}\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} + \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = m_s r_s \Omega^2 \begin{pmatrix} -sin(\Omega t) \\ cos(\Omega t) \end{pmatrix} \quad (20)$$

And for the θ and φ which are much more complex obtained:

$$\begin{pmatrix} I_{rr} + 2m_d h^2 + (2m_d {r_d}^2 + m_s {r_s}^2)cos^2\Omega t & \frac{1}{2}(2m_d {r_d}^2 + m_s {r_s}^2)sin2\Omega t \\ \frac{1}{2}(2m_d {r_d}^2 + m_s {r_s}^2)sin2\Omega t & I_{rr} + 2m_d h^2 + (2m_d {r_d}^2 + m_s {r_s}^2)sin^2\Omega t \end{pmatrix}\begin{pmatrix} \ddot{\theta} \\ \ddot{\varphi} \end{pmatrix} +$$

$$\begin{pmatrix} c{d_c}^2 - \Omega(2m_d {r_d}^2 + m_s {r_s}^2)sin2\Omega t & I_{zz}\Omega + 2\Omega(2m_d {r_d}^2 + m_s {r_s}^2)cos^2\Omega t \\ -I_{zz}\Omega - 2\Omega(2m_d {r_d}^2 + m_s {r_s}^2)sin^2\Omega t & c{d_c}^2 + \Omega(2m_d {r_d}^2 + m_s {r_s}^2)sin2\Omega t \end{pmatrix}\begin{pmatrix} \dot{\theta} \\ \dot{\varphi} \end{pmatrix} + \begin{pmatrix} k{d_k}^2 & 0 \\ 0 & k{d_k}^2 \end{pmatrix}\begin{pmatrix} \theta \\ \varphi \end{pmatrix} =$$

$$2m_d r_d h\Omega^2 \begin{pmatrix} cos\Omega t \\ sin\Omega t \end{pmatrix} \quad (21)$$

Results

To have a preliminary view of forces and torques produced by unbalances, equation of motion solved for actual reaction wheel parameters using numerical method solution for differential equation in MAPLE. Table.1 shows the parameters.

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Table 1: Actual reaction Wheel Parameters

parameter	value	unit
M_t	689.4	gr
R	5	cm
r_s	5	cm
r_d	5	cm
h	2.58	cm
k	784.78	$\frac{N}{\mu m}$
m_d	0.31	gr
m_s	0.095	gr
c	6000	$\frac{kg}{s}$
d_c, d_k	0.75	cm
I_{rr}	0.00067	Kg.m^2
I_{zz}	0.0012	Kg.m^2

Disturbing forces and torques calculated from translational and angular displacement through the relations below:

$$F_x = kx(t) \qquad T_x = K_\theta \theta(t)$$
$$F_y = ky(t) \qquad T_\varphi = K_\varphi \varphi(t)$$

Charts show these torques and forces for parameters.

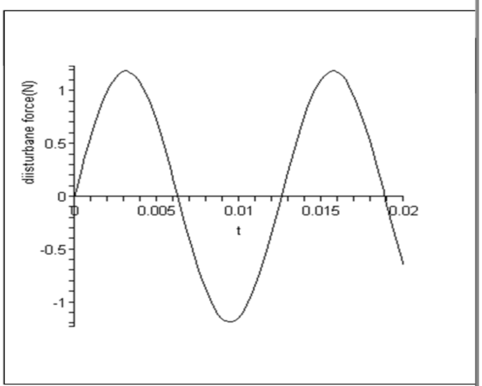

Figure 4: disturbance forces (N) figure

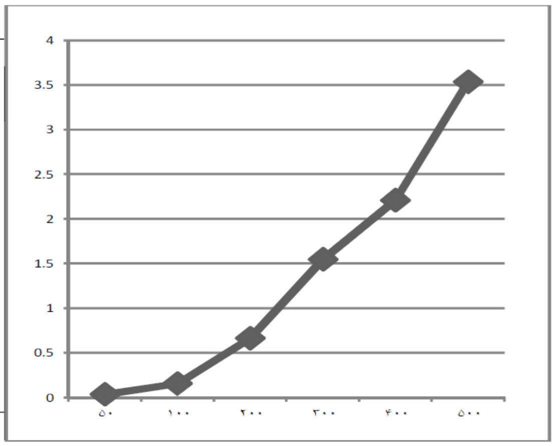

Figure 5: Maximum disturbance torque

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in x and y direction in maximum wheel rate (N.M) in y direction in 6 wheel rate

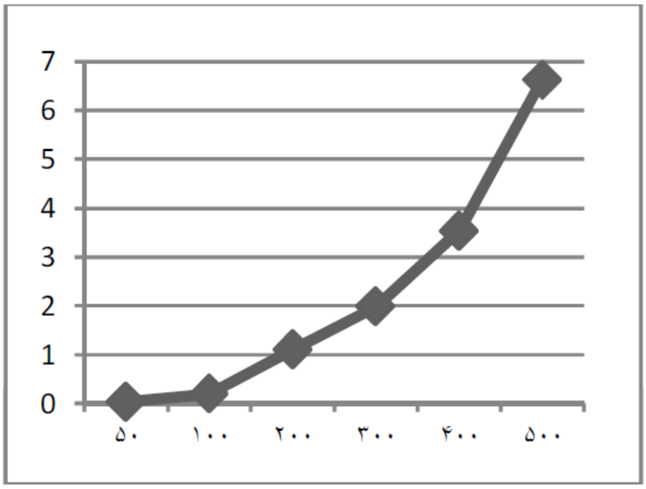

Figure 5: Maximum disturbance torque (N.M) in x direction in 6 wheel rate (50-500)

Conclusion

As graph shows, maximum disturbance forces in x and y direction is less than 1.3N for maximum wheel rate (500 rad/sec). Preliminary data stored from acceleration sensors set up in x and y direction in a reaction wheel assembly shows much less forces for two spin rate of wheel (50 & 100rad/sec)**.** Torque disturbance have a high increase rate against wheel spin. This torque must be decrease and control for avoiding inaccuracy in satellite's operation. In a satellite design process, defining payload requirements leads to define subsystem's requirements. For pointing accuracy of a camera, disturbance forces and torques must be characterized. The model can be used as a preliminary design tool for engineers to limit maximum allowable unbalances. Also by creating a FEM [11], model of a satellite couple with this model, accurate result will get to predict payload action against reaction wheel disturbances.

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Available online at: http://www.aeuso.org

References

[1] M. E. Laila, E. Dekens, "A methodology for modeling the mechanical interaction between a reaction wheel and a flexible structure", Massachusetts institute of technology,

[2] R. Masterson, D. Miller, R. Grogan, "Development of empirical & analytical Reaction Wheel Disturbance Model", AIAA, 1999

[3] J. Brien, W. Gregory, W. Melody, J. Calvet, "six-axis vibration isolation technology to space born interferometers", Jet propulsion laboratory, California institute of technology.1996.

[4] J.R. Wertz," Spacecraft Attitude Determination and Control John Wiley & sons", 1997

[5] MIT, space system product development, "Reaction Wheel Design", spring2003.

[6] J. S. Rao, "VibratoryCondition monitoring", Alpha science, Delhi, 2000

[7] V. Wowk, "machinery vibration Alignment", McGraw Hill,2000.

[8] B. Biakle,"A compilation of reaction wheel induced spacecraft Disturbances", 20th Annual American Society Guidance and control conference, February 1997.

[9] F. Ehrich, Handbook of Rotor dynamics, Mc. Graw Hill inc, 1992

[10] L. James taylor,"vibration analysis Handbook", 2001

[11] L. Olivier, Dweck, D. Miller, J. Mallory, G. Mosier, "Integrated modeling and dynamic simulation for Next Gene, ration Space Telescope(NGST)",NASA Goddard space flight center,1999.

[12] L. Kuo-Chia, M. Peiman, "Reaction Wheel Disturbance Modeling, Jitter Analysis, and Validation Tests for Solar Dynamics Observatory". AIAA Guidance, Navigation and Control Conference, Hawaii, 2008.

[13] C. Dong, J. J. eun, H. suk, "reaction wheel disturbance reduction method using disturbance measurement table", journal of astronomy and space science,28(4), 311-317, 2011.