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# Computation Adomian Method for Solving Non-linear Differential Equation in the Fluid Dynamic

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## *Abstract*

In this article we investigate to solve non–linear differential equation such that it is produced the in fluid dynamic. Of course the above problem discussed by some authors in three coupled differential equations, but we use some mathematical techniques to transform the coupled equations into a nonlinear differential equation then by Adomian decomposition method we solve the problem. The obtained results are shown the high accuracy in the comparison with similar works.

***Keywords:*** *Differential equation, Adomian decomposition, Nonlinear, Coupled equations.*

## 1. Introduction

Some important problems in sciences and engineering is nonlinear differential equations, so solve of these equations are interesting for mathematicians. Adomian method is a validity technique for solving kinds of problems, such as integral equations and differential equations (see[1,4,5,6,7] ). We consider governing equations with boundary conditions the following form[2,3, 8],

$$\xi^2 \frac{d^2\phi}{d^2y} = -\beta\,(x_+(y) - x_-(y)\ ), \tag{1}$$

$$\frac{d}{dy}\left(\frac{dx_+(y)}{dy} + x_+(y)\frac{d\phi}{dy}\right) = 0, \tag{2}$$

$$\frac{d}{dy}\left(\frac{dx_-(y)}{dy} - x_-(y)\frac{d\phi}{dy}\right) = 0, \tag{3}$$

$$\xi = 0.4, \beta = 188.679, \tag{4}$$

$$\phi(0) = \phi(1) = 0, \tag{5}$$

$$x_+(0) = x_+(1) = 0.00276, \tag{6}$$

$$x_-(0) = x_-(1) = 0.00254. \tag{7}$$

In [8], the coupled system $(1-3)$ was solved by converting to discrete form of the three algebraic systems, but we are going to convert the above coupled problem to a non-linear differential equation. Thus we will have a simple problem, such that it can be solved by Adomian decomposition method. In order we assume $u(y) = x_+(y)$ and $v(y) = x_-(y)$ so, from $(2-3)$ we have



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$$u'(y)+u(y)\ \phi'(y)=a, \tag{8}$$
$$v'(y)-v(y)\ \phi'(y)=b, \tag{9}$$

where $a,b$ are constants. By integrating from Eq. $(8-9)$ in interval $[0,1],$ we can write

$$\phi(1)-\phi(0)=a\int_0^1 \frac{dy}{u(y)},$$

$$\phi(1)-\phi(0)=\ln v(y)-b\int_0^1 \frac{dy}{v(y)},$$

from $\phi(1)=\phi(0),\ v(1)=v(0)=0.00254,\ u(1)=u(0)=0$ and the above relations, we obtain, so, $a=b=0$ in this way by attention to (8-9) andand $b\int_0^1 \frac{dy}{v(y)}=0\ a\int_0^1 \frac{dy}{u(y)}=0$ integrating from them, we have

$$\phi(y)=\ln u(y)+c,$$

Because $\phi(1)=\phi(0),$ then $c=\ln(0.00254).$ In a similar way we have

$$\phi(y)=\ln v(y)-\ln(0.00254). \tag{11}$$

In other words, we can write the last two equations the following form,

$$\phi(y)=\ln(\frac{0.00276}{u(y)})-\ln(\frac{v(y)}{0.00254}),$$

also, between $u(y)$ and $v(y)$ functions is held a relation as follows;

$$u(y)v(y)=7.0104\times10^{-6}. \tag{12}$$

[

According to Eq. $(12)$ it suffices that u (y) or v (y) is computed, since from Eq. $(11)$, $\phi(y)$ is calculable too. Now, we construct a differential equations in terms of $v(y)$, in order to by considering to (11) we obtain,

$$\phi^{/}(y)=\frac{v^{/}(y)}{v(y)},\ \phi^{''}(y)=\frac{v^{''}(y)-v^{/^2}(y)}{(v(y)\ )^2}, \tag{13}$$

by substituting Eq. $(13)$ in Eq. $(1)$ and simplicity we have,

$$\begin{cases} v(y)v^{''}(y)-v^{/^2}(y)+\alpha\, v(y)-\gamma\, v^3(y)=0, \\ v(1)=v(0)=0.00254. \end{cases} \tag{14}$$

Where $\alpha=0.82669704$ and $\gamma=117924.375.$

## 2. Adomian Decomposition Method

Consider a general functional equation that it may be apply in the extension filed of sciences,

$$Lu + Ru + Nu = f(x), \tag{15}$$

where $f(x)$ is a known function, $u(x)$ is the unknown function, $N$ is nonlinear operator, $L$ is a linear and invertible operator and also $R$ is the remainder of the nonlinear operator, so from Eq. $(15)$ we have,

$$u = L^{-1}(f(x)) - L^{-1}(Ru) - L^{-1}(Nu), \tag{16}$$

is the Adomian polynomials $[1-3]$. $A_n$ where In the Adomian method

Also, $u(x) = \sum_{n=0}^{\infty} u_n(x)\lambda^n$ such that $A_n$ is computed by the following formulate,

$$A_n = \frac{1}{n!}(\frac{d}{d\lambda^n}[N(u)])_{\lambda=0}. \tag{17}$$

For solving of Eq. $(14)$ by using Adomian method we consider Eq. $(14)$ as follows;

$$v(y) = \frac{1}{\alpha}(\gamma v^3(y) + v'^2(y) - v(y)v''(y)). \tag{18}$$

where, we choose operators and approximate of solution as,

$$N(v) = \frac{1}{\alpha}(\gamma v^3 + v'^2 - v v''), \tag{19}$$

$$L(v) = v, \tag{20}$$

$$v(x) = \sum_{n=0}^{\infty} v_n(x)\lambda^n. \tag{21}$$

With substituting Eqs. $(19-21)$ in Eq. $(18)$ we have,

$$\sum_{n=0}^{\infty} v_n\lambda^n = \frac{1}{\alpha}\{\gamma(\sum_{n=0}^{\infty} v_n\lambda^n)^3 + (\sum_{n=0}^{\infty} v_n'\lambda^n)^2 - (\sum_{n=0}^{\infty} v_n\lambda^n)(\sum_{n=0}^{\infty} v_n''\lambda^n)\} \tag{22}$$

$$= N(v) = N(\sum_{n=0}^{\infty} v_n\lambda^n) = \sum_{n=0}^{\infty} A_n\lambda^n$$

where $A_n$ is nth Adomian polynomial, by comparing the both sides of Eq. $(22)$, we obtain,

$v_0 = A_0, v_1 = A_1, \ldots$

Also fromEq. $(21)$, we can have,

$$v_0 = A_0 = (N(v))_{\lambda=0} = \frac{1}{\alpha}(\gamma v_0^{\ 3} + v_0'^2 - v_0 v_0''),$$

by choosing $v_0$ as a constant, then from the last above formulate drives, $v_0 = \frac{1}{\alpha}(\gamma v_0^{\ 3})$,

If $v_0 = 0$, then we conclude an obvious solution $v(y) = 0$. So, $v_0 = \sqrt{\frac{\alpha}{\gamma}}$ and similar way we have,

$v_1 = A_1 = (\frac{dN(v)}{d\lambda})_{\lambda=0} = \frac{1}{\alpha}(3\gamma v_1 v_0^{\ 2} - v_0 v_1'')$, in other words

$$(\alpha - 3\gamma v_0{}^2)v_1 + v_0 v_1'' = 0. \tag{23}$$

Because $v_0$ is constant, so we can conclude from Eq. $(23)$ an exponential solution as,

$$v_1(y) = Ae^{r_1 y} + Be^{r_2 y};\ r_1 = -0.000107716,\ r_2 = -1.51219\times 10^{-15}. \tag{24}$$

To find unknowns Eq. $(24)$ we use boundary condition Eq. $(14)$ and $\lambda = 1$, so

$$\begin{cases} v(0) = v_0(0) + v_1(0) + v_2(0) + \dots, \\ v(1) = v_0(1) + v_1(1) + v_2(1) + \dots. \end{cases} \tag{25}$$

on the other hand since $v_0 = \sqrt{\dfrac{\alpha}{\gamma}}$, then $v_0(0) = v_0(1) = \sqrt{\dfrac{\alpha}{\gamma}}$ and from Eq. $(25)$ we obtain

$$v_1(0) = v_1(1) = 0.00254 - \sqrt{\frac{\alpha}{\gamma}}, \tag{26}$$

$$v_i(0) = v_i(1) = 0,\ \forall i \geq 2.$$

According to Eqs. $(24)$ and $(26)$ the solution of Eq. $(23)$ is the following form :

$$v_1(y) = -0.000107716\, e^{-24.9892y} - 1.51219\times 10^{-15}\, e^{24.9892y}. \tag{27}$$

Approximation of Adomian method in terms of two iterations, is shown in fig1.

$$v(y) = v_0(y) + v_1(y) = 0.00264772 - 0.000107716\, e^{-24.9892y} - 1.51219\times 10^{-15}\, e^{24.9892y} \tag{28}$$

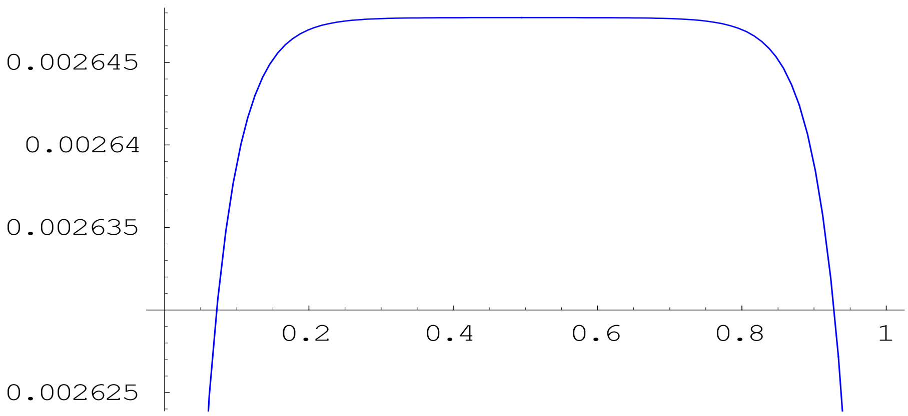


**Figure 1:** Approximation of $v(y)$ by two iterations

Also, for computing $u(y)$ and $\phi(y)$ we use from Eqs. $(11-12)$ and Eq. $(28)$. The compare of $v(y)$ and $u(y)$ is shown in fig2,

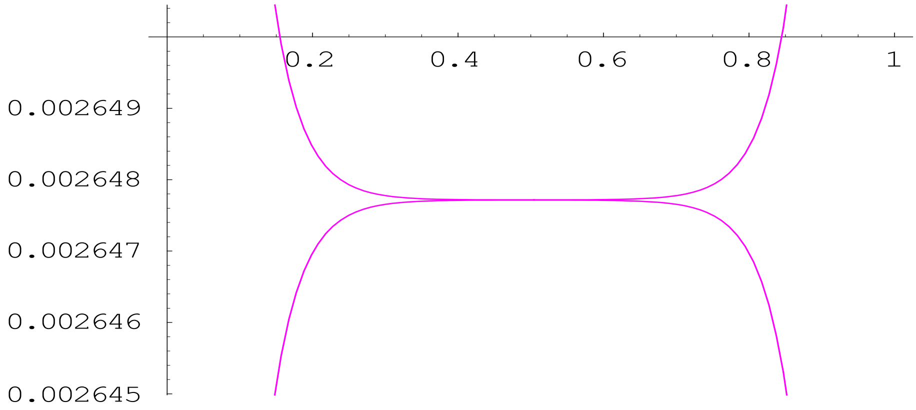


**Figure 2:** The compare of $v(y)$ and $u(y)$.

## Conclusion

In this article we use Adomian decomposition method for solving nonlinear problem that in compare with some of the similar method [8], this method is simple and also by simplicity of the coupled problem, we solved the problem with only an ordinary nonlinear differential equation and two other functions in the problem obtained from solution of the above ordinary nonlinear differential equation to simplicity.

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