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Analytical Study on Motion of a Spherical Solid Particle in Plane Couette Fluid Flow using DTM-Padé

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Abstract

In this paper, motion of a spherical particle in plane Couette fluid flow is investigated employing Differential Transformation Method (DTM) with Padé approximation. Equations of particle's motion in Couette flow in horizontal and vertical directions have been considered. The horizontal and vertical velocity profiles for motion of the particle were obtained using DTM-Padé. The precious contribution of the work is introducing a novel efficient solution of analytical method for motion of a spherical particle in plane Couette fluid flow in comparison with the numerical results. It is shown that the Differential Transformation Method (DTM) with Padé approximate give approximations of a high degree of accuracy for studying particle motion in Couette fluid flow.

Keywords: *Spherical particle; Couette flow; Differential Transformation Method; Padé approximation.*

1. Introduction

Motion of spherical and non-spherical particles in fluid flow is one of the most important problems in engineering which has application in blood flow for medicine and control of particles motion during spraying or injecting processes in industry. The cases that particles come into contact with a solid or fluid boundary and obtaining the drag coefficient in order to predict the free falling velocity of particles have been immense interest of many researchers. The main objective of these problems is to estimate the time and distance required to reach terminal velocity for a given sphere-fluid combination prior to making the reliable determination of the sphere settling velocity. Jalal et al. solved a spherical particle's motion in Couette flow using Homotopy perturbation method (HPM) and got comparable results to numerical ones [1]. Ghasemi et al.[2] discussed about the convergency and accuracy of VIM (Variational Iteration Method) and ADM (Adomian Decomposition Method) for solving the motion of a spherical particle in couette fluid flow. A combined experimental-numerical study was performed for a drag coefficient-Reynolds correlation for a single spherical particle rolling down a smooth plane for three classified regimes of Re from low up to 10^5, in an incompressible Newtonian media by Jan and Chen [3]. Explicit equations of correlation between Reynolds and drag coefficients proposed by Yow et al. for different shapes of particles including spheres, cube octahedrons, octahedrons, cubes, tetrahedrons, discs, cylinders and rectangular parallelepipeds [4]. The unsteady

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rolling motion of a spherical particle restricted to a tube was studied analytically by Jalal and Ganji. An exact solution of particle velocity and acceleration motion under some practical conditions was obtained through applying HPM [5]. Jalal and Ganji proposed an analytically solution for acceleration motion of a spherical particle rolling down an inclined boundary with drag coefficient which is correlated linearly to Re in a specific range using HPM. Various inclination angles were studied and observed that settling velocity, acceleration duration and displacement are proportional to amount of inclination angle while for a constant inclination angle; settling velocity and acceleration duration are decreased by increasing the fluid viscosity [6]. Torabi and Yaghoobi, investigated the better performance of combination of He's polynomials and the diagonal Padé approximants rather the HPM has been shown for calculating approximate solution of the acceleration motion of a single spherical particle moving in a continuous fluid phase [7]. In later study they obtained the acceleration trajectory of a non-spherical particle moving in a continuous fluid phase using VIM- Padé approximants method with acceptable accuracy compared to numerical results [8]. Evolution of angular velocity and radial position of spherical particle in a circular Poiseuille flow for different Reynolds number up to 2200 was numerically studied by Shao et al. [9].

The concept of differential transformation method (DTM) was first introduced by Zhou [10] in 1986 and it was used to solve both linear and nonlinear initial value problems in electric circuit analysis. This method can be applied directly for linear and nonlinear differential equation without requiring linearization, discretization, or perturbation and this is the main benefit of this method. Ghafoori et al. [11] used the DTM for solving the nonlinear oscillation equation. Abdel-Halim Hassan [12] has applied the DTM for different systems of differential equations and he has discussed the convergency of this method in several examples of linear and non-linear systems of differential equations. R. Abazari and M. Abazari [13] have applied the DTM and RDTM (reduced differential transformation method) for solving the generalized Hirota–Satsuma coupled KdV equation. They compared the results with exact solution and they found that RDTM is more accurate than the classical DTM. Rashidi et al. [14] solved the problem of mixed convection about an inclined flat plate embedded in a porous Medium by DTM; they applied the Pade approximant to increase the convergence of the solution. Abbasov et al. [15] employed DTM to obtain approximate solutions of the linear and non-linear equations related to engineering problems and they showed that the numerical results are in good agreement with the analytical solutions. Balkaya et al. [16] applied the DTM to analyze the vibration of an elastic beam supported on elastic soil. Borhanifar et al. [17] employed DTM on some PDEs and their coupled versions.

Moradi and Ahmadikia [18] applied the DTM to solve the energy equation for a temperature-dependent thermal conductivity fin with three different profiles. Moradi [19] applied DTM for thermal characteristics of straight rectangular fin for all type of heat transfer (convection and radiation) and compared it results by ADM and numerical method with fourth order Rang- Kutta method using shooting method. Ghasemi et al.[19] applied the Differential Transformation Method to analyze the temperature distribution in a fin with temperature-dependent heat generation and thermal conductivity. Also, recently some other analytical methods have been applied in many engineering problems by Ghasemi et al [20-23].

In the present letter, analytical solution of motion of a spherical particle in plane Couette fluid flow has been studied by Differential Transformation Method with Padé approximation. For this purpose, after a brief introduction for DTM and description of the problem, we applied DTM to find the approximate solution. Obtaining the analytical solution of the model and comparing with numerical results reveal the capability, effectiveness, simplicity and high accuracy of this method.

2. Description of the Problem

Vander Werff proposed a comprehensive equation of motion of sphere particle in Couette flow [24]. The Vander Werff model for particle motion in Couette flow is adopted in this study while the positive direction rotation of particle is clockwise and combined effects of inertia, gravity and buoyancy are assumed negligible [24]. So, the inertia force in left hand of force balance equation is the product of mass of sphere particle by its acceleration $(\ddot{x}, \ddot{y}, 0)$:

$$T = \frac{4\pi a^3}{3}\rho_s \dot{V} = \frac{4\pi a^3}{3}\rho_s(\ddot{x}, \ddot{y}, 0) \tag{1}$$

Where a, ρ and V are the radius, density and velocity of sphere particle respectively. $\dot{V}$ is first derivative of particle's velocity and $\ddot{x}$ and $\ddot{y}$ are second derivatives of particles' motion in horizontal and vertical directions respect to time.

In order to calculate the drag force the velocities of the sphere particle are considered small adequately so that the Stokes law can be governed:

$$T_{Dx} = 6\pi\mu a V_{rx} = 6\pi\mu a(\dot{x} - \alpha y) \tag{2a}$$

$$T_{Dy} = 6\pi\mu a V_{ry} = 6\pi\mu a\dot{y} \tag{2b}$$

while μ signify the viscosity of fluid.

The rotation and shear portion of the particle's lift force is obtained as:

$$T_{Rx} = \frac{1}{2}\pi a^3 \rho\alpha\dot{y} \tag{3a}$$

$$T_{Ry} = \frac{1}{2}\pi a^3 \rho\alpha(\alpha y - \dot{x}) \tag{3b}$$

$$T_{Sx} = 0 \tag{4a}$$

$$T_{Sy} = 6.46 a^2 \rho \alpha^{1/2} \nu^{1/2}(\alpha y - \dot{x}) \tag{4b}$$

where α is defined as positive proportionality constant.

An illustration of the spherical particle in plane Couette fluid flow and exerted forces on particle are shown in Fig.1. The mass of particle is assumed in the center of sphere and the forces caused from the rotation and shear fields and their interactions on drag and lift forces of particle are illustrated in Figs. 1.a and 1.b respectively. By forming the force balance equation of the inertia force to the drag and lift forces; the equations of motion for the particle are driven as:

$$T = T_R + T_S - T_D \tag{5}$$

Eventually by substituting the Eqs. (2) and (4) into Eq. (5) the system of equation of motion of sphere particle in plane Couette flow yields:

$$\frac{4\pi a^3}{3}\rho_s\ddot{x} = \frac{1}{2}\pi a^3 \rho\alpha\dot{y} - 6\pi\mu a(\dot{x} - \alpha y) \tag{6a}$$

$$\frac{4\pi a^3}{3}\rho_s\ddot{y} = (\frac{1}{2}\pi a^3\rho\alpha + 6.46a^2\rho\alpha^{1/2}\nu^{1/2})(\alpha y - \dot{x}) - 6\pi\mu a\dot{y} \quad (6b)$$

$$\begin{cases} \frac{4\pi a^3}{3}\rho_s\ddot{x} = \frac{1}{2}\pi a^3\rho\alpha\dot{y} - 6\pi\mu a(\dot{x} - \alpha y) \\ \frac{4\pi a^3}{3}\rho_s\ddot{y} = (\frac{1}{2}\pi a^3\rho\alpha + 6.46a^2\rho\alpha^{1/2}\nu^{1/2})(\alpha y - \dot{x}) - 6\pi\mu a\dot{y} \end{cases} \quad (7)$$

For simplicity the governing equations have been expressed as:

$$\begin{cases} \ddot{x} = A\dot{y} + B(\dot{x} - \alpha y) \\ \ddot{y} = B\dot{y} - (A + C)(\dot{x} - \alpha y) \end{cases} \quad (8)$$

Where the coefficients A-C are defined as:

$$A = (\frac{3\alpha}{8})(\frac{\rho}{\rho_S})$$
$$B = -(\frac{9\nu}{2r^2})(\frac{\rho}{\rho_S}) \quad (9)$$
$$C = 1.542(\frac{\sqrt{\alpha\nu}}{r})(\frac{\rho}{\rho_S})$$

An appropriate initial condition is required in order to avoid trapping the procedure in nontrivial solution:

$$x(t=0) = 0, \dot{x}(t=0) = u_0 \quad (10)$$
$$y(t=0) = 0, \dot{y}(t=0) = v_0 \quad (11)$$

Since, the procedure of solving the Eq.8 is autonomous of constants A, B and C; so, for generalization and simplyfication of problem for future cases with different physical conditions the constatns which represent physical properties are assumed to be:

$$A = B = C = 1 \quad (12)$$
$$u_0 = v_0 = 1 \quad (13)$$

3. Differential Transformation Method Fundamentals

In this section the fundamental basic of the Differential Transformation Method is introduced. For understanding method's concept, suppose that $x(t)$ is an analytic function in domain *D*, and $t = t_i$ represents any point in the domain. The function $x(t)$ is then represented by one power series whose center is located at t_i. The Taylor series expansion function of $x(t)$ is in form of:

$$x(t) = \sum_{k=0}^{\infty}\frac{(t - t_i)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_i} \qquad \forall t \in D \quad (14)$$

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The Maclaurin series of $x(t)$ can be obtained by taking $t_i = 0$ in Eq. (14) expressed as:

$$x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=0} \qquad \forall t \in D \tag{15}$$

As explained in [12] the differential transformation of the function $x(t)$ is defined as follows:

$$X(k) = \sum_{k=0}^{\infty} \frac{H^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=0} \tag{16}$$

Where $X(k)$ represents the transformed function and $x(t)$ is the original function. The differential spectrum of $X(k)$ is confined within the interval $t \in [0, H]$, where *H* is a constant value. The differential inverse transform of $X(k)$ is defined as follows:

$$x(t) = \sum_{k=0}^{\infty} (\frac{t}{H})^k X(k) \tag{17}$$

It is clear that the concept of differential transformation is based upon the Taylor series expansion. The values of function $X(k)$ at values of argument *k* are referred to as discrete, i.e. $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete, etc. The more discrete available, the more precise it is possible to restore the unknown function. The function $x(t)$ consists of the *T*-function $X(k)$, and its value is given by the sum of the *T*-function with $(t/H)k$ as its coefficient. In real applications, at the right choice of constant *H*, the larger values of argument k the discrete of spectrum reduce rapidly. The function $x(t)$ is expressed by a finite series and Eq. (17) can be written as:

$$x(t) = \sum_{k=0}^{n} (\frac{t}{H})^k X(k) \tag{18}$$

Some important mathematical operations performed by differential transform method are listed in Table 1.

Table1. Some fundamental operations of the differential transform method

Origin function	Transformed function
$x(t) = \alpha f(x) \pm \beta g(t)$	$X(k) = \alpha F(k) \pm \beta G(k)$
$x(t) = \frac{d^m f(t)}{dt^m}$	$X(k) = \frac{(k+m)! F(k+m)}{k!}$
$x(t) = f(t) g(t)$	$X(k) = \sum_{l=0}^{k} F(l) G(k-l)$
$x(t) = t^m$	$X(k) = \delta(k-m) = \begin{cases} 1, & \text{if k = m,} \\ 0, & \text{if k} \neq \text{m.} \end{cases}$
$x(t) = \exp(t)$	$X(k) = \frac{1}{k!}$

$x(t)=\sin(\omega t+\alpha)$	$X(k)=\frac{\omega^k}{k!}\sin(\frac{k\pi}{2}+\alpha)$
$x(t)=\cos(\omega t+\alpha)$	$X(k)=\frac{\omega^k}{k!}\cos(\frac{k\pi}{2}+\alpha)$

4. Padé Approximation

Padé approximant is the ratio of two polynomials constructed from the coefficients of the Taylor series expansion of a function $y(x)$. The [L/M] Padé approximants to a function $y(x)$ are given by [25-28]:

$$\left[\frac{L}{M}\right]=\frac{A_L(x)}{B_M(x)}, \tag{19}$$

where $A_L(x)$ is polynomial of degree at most *L* and $B_M(x)$ is a polynomial of degree at most *M*. The formal power series

$$y(x)=\sum_{i=1}^{\infty}c_i x^i, \tag{20}$$

$$y(x)-\frac{A_L(x)}{B_M(x)}=O\left(x^{L+M+1}\right), \tag{21}$$

determine the coefficients of $A_L(x)$ and $B_M(x)$ by the equation.

Since we can clearly multiply the numerator and denominator by a constant and leave [L/M] unchanged, we imposed the normalization condition

$$B_M(0)=1.0. \tag{22}$$

finally, we require that $A_L(x)$ and $B_M(x)$ have non-common factors. If we write the coefficient of $A_L(x)$ and $B_M(x)$ as

$$A_L(x)=a_0+a_1x+a_2x^2+\cdots+a_Lx^L, \tag{23a}$$

$$B_M(x)=b_0+b_1x+b_2x^2+\cdots+b_Mx^M. \tag{23b}$$

Then by (22) and (23), we may multiply (21) by $B_M(x)$, which linearized the coefficient equations. We can write out (21) in more details as

$$\left.\begin{aligned} &c_{L+1}+c_Lb_1+\cdots+c_{L-M}b_M=0,\\ &c_{L+2}+c_{L+1}b_1+\cdots+c_{L-M+2}b_M=0,\\ &\vdots\\ &c_{L+M}+c_{L+M+1}b_1+\cdots+c_Lb_M=0, \end{aligned}\right\}, \tag{24a}$$

$$\left.\begin{array}{l} c_0 = a_0, \\ c_0 + c_0 b_1 = a_1, \\ \vdots \\ c_L + c_{L-1} b_1 + \cdots + c_0 b_L = a_L. \end{array}\right\}. \tag{24b}$$

To solve these equations, we start with Eq. (24a), which is a set of linear equations for all the unknown *b*`s. Once the *b*`s are known, then Eq. (24b) gives and explicit formula for the unknown *a*`s, which complete the solution. If Eqs. (24a) and (24b) are non-singular, then we can solve them directly and obtain Eq. (25) [25,26], where Eq. (25) holds, and if the lower index on a sum exceeds the upper, the sum is replaced by zero.

$$\left[\frac{L}{M}\right] = \frac{\det\begin{bmatrix} c_{L-M+1} & c_{L-M+2} & \cdots & c_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ c_L & c_{L+1} & \cdots & c_{L+M} \\ \sum_{j=M}^{L} c_{j-M} x^j & \sum_{j=M-1}^{L} c_{j-M+1} x^j & \cdots & \sum_{j=0}^{L} c_j x^j \end{bmatrix}}{\det\begin{bmatrix} c_{L-M+1} & c_{L-M+2} & \cdots & c_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ a_L & a_{L+1} & \cdots & c_{L+M} \\ x^M & x^{M-1} & \cdots & 1 \end{bmatrix}} \tag{25}$$

To obtain diagonal Padé approximants of different order such as [2/2], [4/4], [8/8] or [10/10] we can use the symbolic calculus software such as Maple or Mathematica.

5. Results and Discussions

5.1 Application of DTM-Padé for problem

Consider the following differential equation system:

$$\begin{cases} \ddot{x} = A\dot{y} + B(\dot{x} - \alpha y) \\ \ddot{y} = B\dot{y} - (A + C)(\dot{x} - \alpha y) \end{cases} \tag{25}$$

with the initial conditions

$$x(t=0) = 0, \dot{x}(t=0) = u_0 \tag{25}$$

$$y(t=0) = 0, \dot{y}(t=0) = v_0 \tag{25}$$

Taking the differential transform method from table 1 to Eq. (8) , we obtain

$$(k+1)(k+2)X[k+2]=(k+1)Y[k+1]-(k+1)X[k+1]+Y[k] \qquad (26a)$$
$$(k+1)(k+2)Y[k+2]=-(k+1)Y[k+1]-(k+1)X[k+1]+Y[k] \qquad (26b)$$

rearranging Eq. (12) can be written as:

$$X[k+2]=((k+1)Y[k+1]-(k+1)X[k+1]+Y[k])/(k+1)(k+2) \qquad (27a)$$
$$Y[k+2]=(-(k+1)Y[k+1]-(k+1)X[k+1]+Y[k])/(k+1)(k+2) \qquad (27b)$$

Similarly, the transformed form of boundary conditions can be written as

$$X_0=0, X_1=1 \qquad Y_0=0, Y_1=1 \qquad (28)$$

By solving Eq. (13) and using boundary conditions (Eq. (14)) the DTM terms are obtained as

$$X_2=0$$
$$X_3=-\frac{1}{6}$$
$$X_4=\frac{1}{12}$$
$$X_5=-\frac{1}{40} \qquad (29)$$
$$X_6=\frac{1}{180}$$

.
.
.

$$Y_2=-1$$
$$Y_3=\frac{1}{2}$$
$$Y_4=-\frac{1}{4} \qquad (30)$$
$$Y_5=\frac{1}{24}$$
$$Y_6=-\frac{1}{120}$$

.
.
.

Now by applying Eq. (17) In to Eqs. (29) and (30), x(t) and y(t) will be obtained as following :

$$x(t)=t-\frac{1}{6}t^3+\frac{1}{12}t^4-\frac{1}{40}t^5+\frac{1}{180}t^6+\ldots \qquad (31)$$

$$y(t) = t - t^2 + \frac{1}{2}t^3 - \frac{1}{6}t^4 + \frac{1}{24}t^5 - \frac{1}{120}t^6 + \ldots \tag{32}$$

So for vertical velocity and horizontal velocity we have :

$$V(x) = 1 - \frac{1}{2}t^2 + \frac{1}{3}t^3 - \frac{1}{8}t^4 + \frac{1}{30}t^5 + \ldots \tag{33}$$

$$V(y) = 1 - 2t + \frac{3}{2}t^2 - \frac{2}{3}t^3 + \frac{5}{24}t^4 - \frac{1}{20}t^5 + \ldots \tag{34}$$

And by using Padé Approximation we have:

$$V(x) \quad (\text{DTM Padé } [10/10]) = \frac{\begin{array}{l}(439537341354814999296000 + 5323837100748245593113600t + 28202383053827924387846400t^2 - \\ 19535954635790756581017600t^3 + 4730291013414232419753600t^4 - 661160187241044787296000t^5 + \\ 60663709617076648882800t^6 - 3800088132000699464640t^7 + 160329202208626348260t^8 - \\ 4203204357633168360t^9 + 52917372648013801t^{10})\end{array}}{\begin{array}{l}(439537341354814999296000 + 5323837100748245593113600t + 28224359920895665137811200t^2 + \\ 7068579623238644217907200t^3 + 1101841521468181532438400t^4 + 118340902210513217114880t^5 + \\ 9129120065658686293200t^6 + 508895259049998025920t^7 + 19908443574085094820t^8 + \\ 500228133767903640t^9 + 6215116576374743t^{10})\end{array}}$$

$$V(y) \quad (\text{DTM Padé } [10/10]) = \frac{\begin{array}{l}(37777954503666896320768000 + 50812488210721798118638080t - 74828377616122226768031360t^2 + \\ 29234634619224682189301760t^3 - 5959099593330639001646400t^4 + 765480359980559489666400t^5 - \\ 66789299432175064152408t^6 + 4046226004464667410816t^7 - 166691392376220230706t^8 + \\ 4292606714871975170t^9 - 53297471246333025 7t^{10})\end{array}}{\begin{array}{l}(37777954503666896320768000 + 50888047300795136045053440t + 26891047667913041877264000t^2 + \\ 6709845367211584118505472t^3 + 1041513767878931881934400t^4 + 111317413109536166236704t^5 + \\ 8538586204632365025144t^6 + 472785779583107939520t^7 + 18347090795972902062t^8 + \\ 456468814907015350t^9 + 5601828558831480 9t^{10})\end{array}}$$

Figure 1 shows the DTM Padé in comparison Numerical method for horizontal velocity of spherical particle in plane Couette fluid flow. This figure emphasis on accuracy and efficiency of Differential Transformation Method with Padé approximation.

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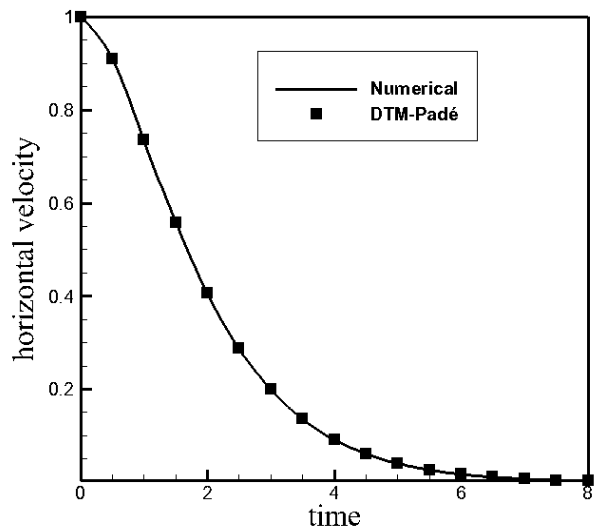

Figure 1. Comparison between the DTM-Padé and numerical solution for horizontal velocity.

In Figure 2 the vertical velocity of spherical particle in plane Couette fluid flow wiht DTM Padé and Numerical method is depicted.
This figure indicate that the DTM Padé has a good agreement with Numerical results for vertical velocity.

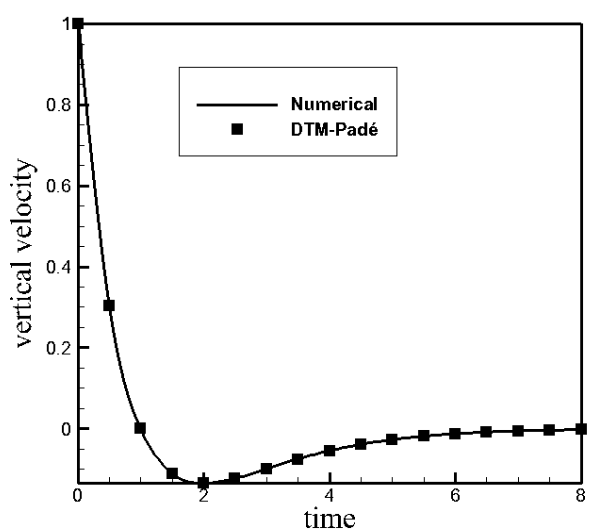

Figure 2. Comparison between the DTM-Padé and numerical solution for vertical velocity.

Table 2 shows the error of DTM-Padé in comparison Numerical method for horizontal velocity and vertical velocity when A=B=C=1. The low maximum error in this table emphasis on accuracy of Differential Transformation Method and confirms the validity of the proposed solution.

Table2. Comparison between DTM-Padé solutions and numerical results when A=B=C=1.

t	V_x			V_y		
	Numerical	DTM-Padé	Re. Error	Numerical	DTM-Padé	Re. Error
0	1	1	0	1	1	0
0.5	0.909796	0.909796	2.57E-08	0.303265	0.303265	1.12E-07
1	0.735759	0.735759	3.77E-08	-4.1E-08	-1.7E-15	1
1.5	0.557825	0.557825	5.79E-08	-0.11157	-0.11157	4.68E-07
2	0.406006	0.406006	6.15E-08	-0.13534	-0.13534	3.04E-07
2.5	0.287298	0.287297	8.5E-08	-0.12313	-0.12313	3.52E-07
3	0.199148	0.199148	1.16E-07	-9.96E-02	-9.96E-02	4.54E-07
3.5	0.135888	0.135888	1.57E-07	-7.55E-02	-7.55E-02	7.09E-07
4	0.091578	0.091578	1.58E-07	-5.49E-02	-5.49E-02	7.88E-07
4.5	0.061099	0.061099	1.32E-07	-3.89E-02	-3.89E-02	9.38E-07
5	0.040428	0.040428	3.63E-08	-2.70E-02	-2.70E-02	1.31E-06
5.5	0.026564	0.026564	5.73E-07	-1.84E-02	-1.84E-02	1.35E-06
6	0.017351	0.017351	1.71E-06	-1.24E-02	-1.24E-02	8.69E-07
6.5	0.011276	0.011276	4.77E-06	-8.27E-03	-8.27E-03	2.89E-06
7	0.007295	0.007295	1.62E-05	-5.47E-03	-5.47E-03	1.63E-05
7.5	0.004701	0.004701	5.08E-05	-3.60E-03	-3.60E-03	6.15E-05
8	0.003019	0.00302	0.000166	-2.35E-03	-2.35E-03	0.000211

Conclusion

In this study, a new analytical method to solve a couple of equations of a spherical particle motion in a plane Couette fluid flow has been presented. Differential Transformation Method (DTM) with Padé approximation have been applied for this problem and the results compared to numerical results. Two dimensional velocity profile of incompressible Newtonian flow in horizontal and vertical motions as the result of solutions. Obtaining the analytical DTM- Padé solution of the problem and comparing with numerical results reveal the facility, effectiveness, and high accuracy of this method.

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