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Hamilton-Jacobi Representations of Trapezoidal Uninterruptible Power Supply System

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Abstract

Hamilton-Jacobi Representations of Trapezoidal Uninterruptible Power Supply System is proposed. Fairly recently, Hybid-Fiber/Coax networks which allow for effective segmentation of services in telecommunication industries have been powered using trapezoidal-shaped ac. voltage waveform. The control strategy of Trapezoidal Uninterruptible Power Supply (TUPS) demands the system dynamics be understood. The development of frequency spectrum and subsequent analysis of state variable representation of TUPS using Hamilton-Jacobi approach yields a closed-loop solution in the form of an optimal-control law. This work considers Hamilton-Jacobi approach as an option to seek the solution for a whole family of initial conditions by state feedback instead of only one specific set of initial conditions. A linear quadratic regulator which could provide an optimal control law for the TUPS with a quadratic performance index is considered. The application of Hamilton-Jacobi approach to the control dynamics of TUPS will result in a matrix form of solution in the time domain known as the matrix Ricatti equations. These equations if solved would lead to a control law that is a function of the state variables, indicating that it yields closed-loop control system.

Keywords: *Trapezoidal waveform, optimal control, linear quadratic regulator, matrix Ricatti equations, performance index, frequency spectrum.*

1. Introduction

The first uninterruptible power equipments (then known as no-break power supplies) were of rotary design, and appeared during 1950s, [1]. The market at that time was related to defense equipment such as communication and radar. Today, UPS has become an economic proposition for organizations whose business and associated information technology, and telecommunication equipments demand a vital need to protect against the following: unreliability of electricity supply, electricity supply regulations, the need to protect against power supply breaks and disturbances, and electrical contamination and interference e.g. spikes, surges and dips, and harmonics [1]. In recent years, hybrid-fiber/coax (HFC) distribution networks have emerged as one of the preferred approaches for distributing multimedia services to the customer [2], particularly by the cable TV industry. This hybrid dramatically network connects optical fiber to the neighborhood with coax

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cable to the residence. Compared with all-fiber or all-coaxial networks, this network allows for segmentation of services and high reliability, distribution efficiency, and low cost. Of special fascination is the fact that these networks require single- phase ac power supplies that deliver trapezoidal-shaped voltage waveforms [3]. Additional requirements are high-input power factor, high overall efficiency, and increased reliability. Since the continuity of power supply distribution is not guaranteed, UPS systems that satisfy the above constraints are required [4].

TUPS, an acronym for trapezoidal uninterruptible power supply has the ability to serve as an interface, provide clean, appropriate, and reliable power for Hybrid fiber/coaxial systems. In the particular case of power outage, UPS can supply power for up to 20min. to allow a back-up supply from diesel generating sets to be brought on-line or, if the generators fail to start, for a, controlled shut down of the computers, telecommunication equipments, etc., to be completed.

The voltage waveform, level, and frequency of the network power distribution are still evolving. However, in order to transfer a larger power to the network devices at a given peak voltage and to keep low electromagnetic interference (EMI), the PN (power node) delivers a trapezoidal-shaped voltage waveform rather than a sinusoidal waveform. Based on the information obtained from Bellcore and cable TV operators, 90V/60 Hz is chosen since it provides a good compromise between safety, loading capability, corrosion, and distribution losses.

In view of this unique importance of TUPS, there is a genuine need to develop its small-signal model which would enable us to foster a good and exceptionally streamlined understanding of circuit analysis and design. Therefore, this paper presents the small-signal model of a trapezoidal uninterruptible power supply system based on fluctuating input voltage method.

2. Overview of Proposed TPUS

Trapezoidal Uninterruptible power supply (TUPS) systems have gained critical importance in telecommunication industries where hybrid-fiber/coax (HFC) networks are used. A full-bridge inverter is practically always used for interfacing (HFC) to the mains, [5]. The control of the energy flowing from the DC source, to HFC must be done in order to transfer maximum power and to maintain a sinusoidal mains current with low harmonic distortion and a high power factor, [6] and [7]. In order to lower the generated high frequency current ripple, due to the operation of the inverter, a passive filter consisting of a capacitor and an inductor can be inserted between the inverter operating as a stiff voltage source and the HFC, [8].

An effective output low pass filter can be only designed if the model of TUPS system is known. The regulation of the system is performed by the use of a PI control because of the non-linearity between input DC voltage and mains current, [9] and [10].

A TUPS system is a control process with inputs and output. The desired performance depends on how appropriately the inputs are chosen. The significant difference between the physical TUPS and the mathematically predicted behavior due to environmental disturbances and parameter variations [11] requires that a control law for the satellite model be developed. The Hamilton-Jacobi methodology [12] uniquely allows a set of equations to be generated which when solved consequently yields a Ricatti matrix. The control law of the TUPS model is a function of the Ricatti matrix. This work intends to generate Ricatti equations from state variable representation of TUPS system.

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3. Frequency Spectrum of TUPS

However, there is the need to keep the fundamental output voltage at the required value and eliminate (or suppress) the recurrence of the reduced lower order harmonics while the inverter input voltage is fluctuating. Therefore, the following modified Pulse-Width Modulation (PWM) is considered.

Consider a modulating signal defined as

$$v_m(t) = \int_{(\sigma_k - \pi/B)}^{(\sigma_k + \pi/B)} G \sin \omega_o t d\, \omega_o t \tag{1}$$

This can be further simplified as follows

$$v_o(t) = -G[\cos \omega_o t]_{(\sigma_k - \pi/B)}^{(\sigma_k + \pi/B)}$$

$$= -G\left[\cos \sigma_k \cos(\pi/B) - \sin \sigma_k \sin(\pi/B) - \cos \sigma_k \cos(\pi/B) - \sin \sigma_k \sin(\pi/B)\right]$$

$$= 2G \sin \sigma_k \sin(\pi/B) \tag{2}$$

Where G, is the desired fundamental output voltage. Also the carrier signal is defined as

$$v_c(t) = \int_{\sigma_k}^{\omega_o t} v_i(\omega_o t) d(\omega_o t), for\ \sigma_k \leq \ \omega_o t \ < \sigma_{k+1} \tag{3}$$

Where σ_k denotes the leading edge of the kth pulse in the output wave and equals

$$\sigma_k = \frac{k\pi}{F} \tag{4}$$

The falling edge ϕ_k of the k^{th} pulse in the output wave is defined by the intersection of the trapezoidal modulating wave and the quasi-saw carrier wave between σ_k and σ_{k+1} . Therefore, the carrier wave at σ_k is define as

$$v_c(\sigma_k) = 2G \sin(\pi/B) \sin \sigma_k \tag{5}$$

Consider an infinitesimally small element of $v_i(t)$ between points $a^/$ and $a^{//}$ with coordinate $[\sigma_k, v_i(\sigma_k)]$ and $[\phi_k, v_i(\phi_k)]$ respectively as shown in fig. 1. If the small element $a^/a^{//}$ is considered to be a straight line then its slope m, can be written as

$$m \approx \frac{v_i(\varphi_k) - v_i(\sigma_k)}{\varphi_k - \sigma_k} \tag{6}$$

If the same element is projected backward such that it intersect the $v_i(t)$ axis at a point (0 , y), then we can write

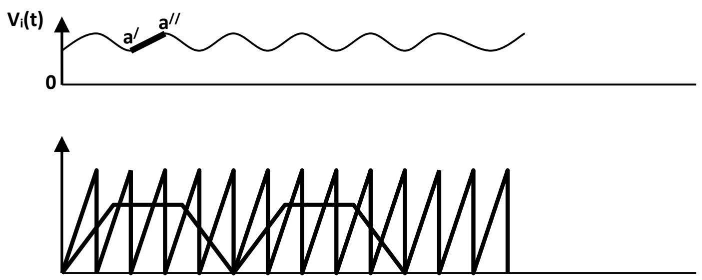

Figure 1: Pulse-width modulation.

$$\frac{v_i(\varphi_k)-v_i(\sigma_k)}{\varphi_k-\sigma_k}=\frac{v_i(\varphi_k)-y}{\varphi_k} \tag{7}$$

Solving for y we get

$$y=\frac{\varphi_k v_i(\varphi_k)-\sigma_k v_i(\sigma_k)}{\varphi_k-\sigma_k} \tag{8}$$

Therefore, when the carrier frequency is chosen such that it is much greater than the maximum input fluctuating frequency (ω_i), then, the equation of the inverter fluctuating input voltage $\upsilon_i(t)$ can be written as

$$v_i(\omega_o t)\approx\frac{v_i(\varphi_k)-v_i(\sigma_k)}{\varphi_k-\sigma_k}\omega_o t+\frac{\varphi_k v_i(\varphi_k)-\sigma_k v_i(\sigma_k)}{\varphi_k-\sigma_k}, for\ \sigma_k\ \le\ \omega_o t\le\varphi_k \tag{10}$$

Therefore, for $\omega_o t=\varphi_K$, (3) becomes, i.e

$$v_c(\varphi_k)\approx\int_{\sigma_k}^{\varphi_k}v_i(\omega_o t)d(\omega_o t)=\frac{v_i(\varphi_k)-v_i(\sigma_k)}{2(\varphi_k-\sigma_k)}[(\omega_o t)^2]_{\sigma_k}^{\varphi_k}+\frac{\varphi_k v_i(\sigma_k)-\sigma_k v_i(\varphi_k)}{\varphi_k-\sigma_k}[\omega_o t]_{\sigma_k}^{\varphi_k}$$

$$v_c(\varphi_k)\approx\frac{1}{2}(\varphi_k-\sigma_k)[v_i(\varphi_k)+v_i(\sigma_k)] \tag{11}$$

Equating (5) and (11) we obtain

$$2G\sin(\pi/_B)\sin\sigma_k=\frac{1}{2}(\varphi_k-\sigma_k)[v_i(\varphi_k)+v_i(\sigma_k)]$$

Therefore, we have

$$\frac{\varphi_k-\sigma_k}{2}=\frac{2G\sin(\pi/_B)\sin\sigma_k}{v_i(\varphi_k)+v_i(\sigma_k)} \tag{12}$$

Since the half-width of the kth output pulse is defined as $W_k=(\phi_k-\sigma_k)/2$, (12) becomes

$$W_k\approx\frac{2G\sin(\pi/_B)\sin\sigma_k}{v_i(\varphi_k)+v_i(\sigma_k)} \tag{13}$$

The output voltage (unfiltered) of the inverter generated with this modified modulation method can be expressed using Fourier series, i.e

$$v_o(t)=\sum_{n=-\infty}^{\infty}\left(d_n e^{jn\omega_o t}\right) \tag{14}$$

Where

$$d_n=\frac{1}{jn}\sum_{k=1}^{F-1}\int_{\sigma_k}^{\varphi_k}v_i(\omega_o t)\sin(n\omega_o t)\,d(\omega_o t) \tag{15}$$

Simplifying the integral term in (15) further we have

$$\int_{\sigma_k}^{\varphi_k}v_i(\omega_o t)\sin(n\omega_o t)\,d(\omega_o t)=\frac{1}{\varphi_k-\sigma_k}\Big[\big(v_i(\varphi_k)-v_i(\sigma_k)\big)\int_{\sigma_k}^{\varphi_k}\omega_o t\sin(n\omega_o t)d(\omega_o t)+\big(\varphi_k v_i(\sigma_k)-\sigma_k v_i(\varphi_k)\big)\int_{\sigma_k}^{\varphi_k}\sin(n\omega_o t)d(\omega_o t)\Big] \tag{16}$$

Differentiating (11) and setting $\frac{d}{dt}\big(v_i(\omega t)\big)=0$, we get

$$v_i(\varphi_k)-v_i(\sigma_k)=0 \tag{17}$$

The first term on the right hand side of (16) cancels out and the entire eqn. reduces to

$$\int_{\sigma_k}^{\varphi_k} v_i(\omega_o t)\sin(n\omega_o t)\,d(\omega_o t) = \frac{1}{\varphi_k-\sigma_k}\left[\left[\left(\varphi_k v_i(\sigma_k) - \sigma_k v_i(\varphi_k)\right)\int_{\sigma_k}^{\varphi_k}\sin(n\omega_o t)d(\omega_o t)\right]\right] \tag{18}$$

$$\int_{\sigma_k}^{\varphi_k} v_i(\omega_o t)\sin(n\omega_o t)\,d(\omega_o t) = \frac{1}{n(\varphi_k-\sigma_k)}\left[\left[\left(\varphi_k v_i(\sigma_k) - \sigma_k v_i(\varphi_k)\right)(\cos n\varphi_k - \cos n\sigma_k)\right]\right] \tag{19}$$

Substituting $\varphi_K = 2W_K + \sigma_K$ in (19) we have

$$\int_{\sigma_k}^{\varphi_k} v_i(\omega_o t)\sin(n\omega_o t)\,d(\omega_o t) = \frac{1}{2nW_k}\left[\left[\left(\sigma_k v_i(\varphi_k) - (2W_k+\sigma_k)v_i(\sigma_k)\right)(\cos n(2W_k+\sigma_k) - \cos n\sigma_k)\right]\right]$$

$$= \frac{1}{n(\varphi_k-\sigma_k)}\left(-2W_k v_i(\sigma_k)\right)\left((-2\sin nW_k)\right)(\sin n\sigma_k \cos nW_k + \cos n\sigma_k \sin nW_k)$$

Replacing $v_i(\sigma_K)$ with $[v_i(\varphi_K) + v_i(\sigma_K)]/2$ we have

$$\int_{\sigma_k}^{\varphi_k} v_i(\omega_o t)\sin(n\omega_o t)\,d(\omega_o t) = \frac{(v_i(\varphi_k)+v_i(\sigma_k))\sin n(\sigma_k+W_k)\sin nW_k}{n} \tag{20}$$

Therefore, (15) becomes

$$d_n = \frac{1}{j\pi}\sum_{k=1}^{F-1}\frac{(v_i(\varphi_k)+v_i(\sigma_k))\sin n(\sigma_k+W_k)\sin nW_k}{n} \tag{21}$$

Further simplification based on the consideration that G is far less than $Bv_i(\omega_O t)$, i.e

$$\sin n(\sigma_k + W_k) \approx \sin\left[n\sigma + \frac{2n\pi G\sin\sigma_k}{B(v_i(\varphi_k)+v_i(\sigma_k))}\right] \approx \sin n\sigma_k \tag{22}$$

Therefore, (21) becomes

$$d_n \approx \frac{G}{jn\pi}\sin\frac{n\pi}{B}\sum_{k=1}^{F-1} 2\sin\sigma_k \sin n\sigma_k \tag{23}$$

$$d_n \approx \frac{G}{jn\pi}\sin\frac{n\pi}{B}\left[\sum_{k-1}^{F-1}\cos\left(\frac{(n-1)k\pi}{F}\right) - \sum_{k-1}^{F-1}\cos\left(\frac{(n+1)k\pi}{F}\right)\right] \tag{24}$$

$$d_n = \begin{cases} 0, for\ n \neq mB\ where\ m = 0,1,2,\ldots. \\ \frac{-FG}{j(mB-1)\pi}\sin\frac{(mB-1)\pi}{B}, for\ n = mB-1 \\ \frac{FG}{j(mB-1)\pi}\sin\frac{(mB+1)\pi}{B}, for\ n = mB+1 \end{cases} \tag{25}$$

Therefore, (14) becomes

$$v_o(t) \approx \sum_{m=-\infty}^{\infty}\frac{FG}{j(mB+1)\pi}\sin\frac{(mB+1)\pi}{B}e^{j(mB+1)\omega_o t} - \sum_{m=-\infty}^{\infty}\frac{FG}{j(mB-1)\pi}\sin\frac{(mB-1)\pi}{B}e^{j(mB-1)\omega_o t} \tag{26}$$

$$v_o(t) \approx \sum_{m=-\infty}^{\infty} \frac{FG}{j(mB+1)\pi} \sin\frac{(mB+1)\pi}{B} \left[2j\sin(mB+1)\omega_o t - e^{-j(mB+1)\omega_o t}\right] - \sum_{m=-\infty}^{\infty} \frac{FG}{j(mB-1)\pi} \sin\frac{(mB-1)\pi}{B} \left[2j\sin(mB-1)\omega_o t - e^{-j(mB-1)\omega_o t}\right] \quad (27)$$

Simplifying further we obtain

$$v_o(t) = \frac{BG}{\pi}\sin\left({}^{\pi}/_{B}\right)\sin\omega_o t + \sum_{m=1}^{\infty} \frac{FG}{j(mB+1)\pi} \sin\frac{(mB+1)\pi}{B} \sin(mB+1)\omega_o t - \sum_{m=1}^{\infty} \frac{FG}{j(mB-1)\pi} \sin\frac{(mB-1)\pi}{B} \sin(mB-1)\omega_o t \quad (28)$$

It can be seen that the fundamental value of the output voltage and the frequency spectrum are almost insensitive to the input voltage fluctuation when ω_i is far less than $B\omega_o$ and E_d far less than $B\upsilon_i$.

4. TUPS State Variable Representation

We will now use $v_o(t)$, (the fundamental component) in the derivation of the state variable model of TUPS system. The PWM inverter, output filter, and R load are modeled as

$$m = \frac{1}{T_i}\int_{-\infty}^{t}\left(v_{o,ref} - K_f v_o\right)d\tau \quad (29)$$

$$v_o = L_o\frac{di_{Lo}}{dt} + v_l \quad (30)$$

$$i_{Lo} = C_o\frac{dv_l}{dt} + \frac{v_l}{r_l} \quad (31a)$$

m = modulating signal, $\upsilon_{o,ref}$ = output-voltage reference, υ_o = actual output voltage, i_{Lo}= inductor current, v_l = load voltage, L_o = load filter inductor, C_o = load filter capacitor, T_i = integrator time constant, r_l = resistance, k_f = feedback factor, Λ_Δ=amplitude of sawtooth carrier signal. Since a carrier-based PWM technique with fluctuating input voltage to the inverter is used the output voltage is approximated by

$$v_o = \frac{v_{dc}}{K} \quad (31b)$$

where

$$v_{dc} = \text{inverter dc signal}$$

$$K = \frac{1}{T_i\Lambda_\Delta} = \text{gain of optocoupler}$$

Therefore, the model expressed by (29) to (31) can be written

$$\frac{dm}{dt} = \frac{1}{T_i} v_{o,ref} - K_f \frac{v_{dc}}{K} \quad (32)$$

$$\frac{di_{Lo}}{dt} = \frac{v_a}{LK} - \frac{1}{L} v_l \quad (33)$$

$$\frac{dv_l}{dt} = \frac{1}{C_o} i_{Lo} - \frac{1}{r_l C_o} v_l \quad (34)$$

$v_o = v_l$. Expressing capacitor voltage, inductor current and modulating signal in state variables form i.e. $x = [m i_{Lo} v_l]^T$. We can write these as

$$\left.\begin{aligned} x_1 &= m \\ x_2 &= i_{Lo} \\ x_3 &= v_l \end{aligned}\right\} \quad (35)$$

Also the system perturbation vector is defined as **Error! Bookmark not defined.** $u = [r_l v_{dc}]^T$

Differentiating (35), we get

$$\left.\begin{aligned} \dot{x}_1 &= \frac{dm}{dt} \\ \dot{x}_2 &= \frac{di_{Lo}}{dt} \\ \dot{x}_3 &= \frac{dv_l}{dt} \end{aligned}\right\} \quad (36)$$

Therefore, (32) through to (34) become

$$\dot{x}_1 = \frac{1}{T_i} v_{o,ref} - K_f \frac{v_{dc}}{K} \quad (37)$$

$$\dot{x}_2 = \frac{v_a}{L_o K} - \frac{1}{L_o} x_3 \quad (38)$$

$$\dot{x}_3 = \frac{1}{C_o} x_2 - \frac{1}{r_l C_o} x_3 \quad (39)$$

Eqn. (37) through to (39) are the state variable eqns. In matrix form, state equations are often written as

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad \text{(State equation)} \quad (40)$$

$$y(t) = Cx(t) + Du(t) \qquad \text{(Output equation)} \quad (41)$$

Therefore, (37) through to (39) can be written in the form of (40) as

$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \\ \dot{X}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{L_o} \\ 0 & \frac{1}{C_o} & -\frac{1}{R_l C_o} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{K_f}{K} \\ 0 & 0 \\ \frac{i_{Lo}}{R_l C_o} & 0 \end{bmatrix} \begin{bmatrix} U_1(t) \\ U_2(t) \end{bmatrix} \quad (42)$$

Eqn. (42) can be rewritten as

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -{}^{1}/_{L_o} \\ 0 & {}^{1}/_{C_o} & -{}^{1}/_{R_l C_o} \end{bmatrix} x(t) + \begin{bmatrix} 0 & -{}^{K_f}/_{K} \\ 0 & 0 \\ {}^{i_{Lo}}/_{R_l C_o} & 0 \end{bmatrix} u(t) \quad (43)$$

From (43)

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -{}^{1}/_{L_o} \\ 0 & {}^{1}/_{C_o} & -{}^{1}/_{R_l C_o} \end{bmatrix} \tag{44}$$

$$B = \begin{bmatrix} 0 & -{}^{K_f}/_{K} \\ 0 & 0 \\ {}^{i_{Lo}}/_{R_l C_o} & 0 \end{bmatrix} \tag{45}$$

$$y = v_l = x_3 = C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \tag{46}$$

A is the coefficient matrix of TUPS, B is the driving matrix, C is the output matrix, D is the transmission matrix and equals zero.

5. Ricatt Equations of TUPS

It is necessary to seek a control u which makes the satellite system, $v(x(t), u(t), t)$ expressed by (40) to optimally trace a path $x(t)$ which minimizes a performance index expressed as

$$J = \int_{t_i}^{t_f} s(x(t), u(t), t)dt \tag{47}$$

Assuming the performance index has a minimum value $g(x, t)$ defined over the time interval t_i(initial time) to t_f(final time) such that

$$g(x, t) = \min_u \int_{t_i}^{t_f} s(x, t)dt \tag{48}$$

Within defined time interval the following is true

$g(x, t_i) = g\big(x(0)\big)$ and $g\big(x, t_f\big) = 0$. If eqn. (48) is considered to be a Lyapunov function, its derivative can be equated to the differentials obtained through the application of chain rule to the same eqn. Therefore, we get

$$\frac{\partial g(x,t)}{\partial t} + \left(\frac{\partial g(x,t)}{\partial t}\right)^T v(x, u) = -s(x(t), u(t), t) \tag{49}$$

$$\frac{\partial g(x,t)}{\partial t} = -\min_u \left(\left(\frac{\partial g(x,t)}{\partial t}\right)^T v(x, u) + s(x(t), u(t), t)\right) \tag{50}$$

Eqn. (50) is referred to as Hamilton-Jacobi equation, where the superscript T means transpose of the matrix in question. Let the performance index of eqn. (47) be the quadratic type such that

$$J = \int_{t_i}^{t_f} (x^T N x + u^T R u)dt \tag{51}$$

Substituting (40) and (51) in (50) we have

$$\frac{\partial g(x,t)}{\partial t} = -\min_u \left(\left(\frac{\partial g(x,t)}{\partial t}\right)^T (Ax + Bu) + x^T N x + u^T R u\right) \tag{52}$$

Where $A(t)$, $B(t)$, $N(t)$ *and* $R(t)$ are continuously differentiable functions. Also, $N(t)$ is a symmetric semi-definite matrix *and* $R(t)$ is a symmetric positive definite matrix. Let us consider a square symmetric matrix, (Ricatti matrix), M such that

$$g(x,t) = x^T M x \tag{53}$$

From (53) we can write

$$\frac{\partial g(x,t)}{\partial x} = 2Mx \tag{54}$$

$$\left[\frac{\partial g(x,t)}{\partial x}\right]^T = 2Mx^T \tag{55}$$

$$\frac{\partial g(x,t)}{\partial t} = x^T\left(\frac{\partial M}{\partial t}\right)x \tag{56}$$

Therefore, (52) becomes

$$\frac{\partial g(x,t)}{\partial t} = -\min_u(2Mx^T(Ax+Bu) + x^T Nx + u^T Ru) \tag{57a}$$

Differentiating (57a) with respect to u and equating to zero, we have

$$2MBx^T + 2Ru^T = 0 \tag{57b}$$

The optimal value of the control u^o is obtained as

$$u^o = -R^{-1}MB^T x = -kx \tag{58}$$

Where

$$k = R^{-1}MB^T \tag{59}$$

Substituting (56) and (58) in (57), we have

$$x^T\left(\frac{\partial M}{\partial t}\right)x = -x(N + 2MA - 2MBR^{-1}B^T M + MBR^{-1}B^T M)x^T \tag{60}$$

$$x^T\left(\frac{\partial M}{\partial t}\right)x = -x(N + 2MA - MBR^{-1}B^T M)x^T \tag{61}$$

Eqn. (61) simplifies to

$$\frac{\partial M}{\partial t} = \dot{M} = MBR^{-1}B^T M - A^T M - MA - N \tag{62}$$

Eqn. (62) is the matrix Ricatti equations for the satellite system. The solutions converge to constant values as integration is performed in the reverse direction. If t_f is further away from t_i, then matrix Ricatti equations reduce to

$$A^T M + MA + N - MBR^{-1}B^T M = 0 \tag{63}$$

Eqn. (63) represent a group of simultaneous equations. Let us consider a TUPS system which will minimize the performance index J such that

$$J = \int_0^\infty (x_1^2 + x_2^2 + u_1^2 + u_2^2)dt \tag{64}$$

$$MA = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1/L_o \\ 0 & 1/C_o & -1/R_l C_o \end{bmatrix} \quad (65)$$

$$MA = \begin{bmatrix} 0 & M_{13}/C_o & -\left(M_{12}/L_o + M_{13}/R_l C_o\right) \\ 0 & M_{23}/C_o & -\left(M_{22}/L_o + M_{23}/R_l C_o\right) \\ 0 & M_{33}/C_o & -\left(M_{32}/L_o + M_{33}/R_l C_o\right) \end{bmatrix} \quad (66)$$

$$A^T M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1/C_o \\ 0 & -1/L_o & -1/R_l C_o \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \quad (67)$$

$$A^T M = \begin{bmatrix} 0 & 0 & 0 \\ M_{31}/C_o & M_{32}/C_o & M_{33}/C_o \\ -\left(M_{21}/L_o + M_{31}/R_l C_o\right) & -\left(M_{22}/L_o + M_{32}/R_l C_o\right) & -\left(M_{23}/L_o + M_{33}/R_l C_o\right) \end{bmatrix} \quad (68)$$

$$N = \begin{bmatrix} N_{11} & 0 & 0 \\ 0 & N_{22} & 0 \\ 0 & 0 & N_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (69)$$

$$R = \begin{bmatrix} r_{11} & 0 \\ 0 & r_{22} \end{bmatrix} = 1$$

$$MBR^{-1}B^T M =$$

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} 0 & -K_f/K \\ 0 & 0 \\ i_{Lo}/R_l C_o & 0 \end{bmatrix} 1 \begin{bmatrix} 0 & 0 & i_{Lo}/R_l C_o \\ -K_f/K & 0 & 0 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \quad (70)$$

$$MBR^{-1}B^T M =$$

$$\begin{bmatrix} \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{13}^2 + \left(\frac{K_f}{K}\right)^2 M_{11}^2 & \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{13}M_{32} + \left(\frac{K_f}{K}\right)^2 M_{11}M_{12} & \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{13}M_{33} + \left(\frac{K_f}{K}\right)^2 M_{11}M_{12} \\ \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{23}M_{31} + \left(\frac{K_f}{K}\right)^2 M_{11}M_{21} & \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{23}^2 + \left(\frac{K_f}{K}\right)^2 M_{12}^2 & \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{23}M_{33} + \left(\frac{K_f}{K}\right)^2 M_{13}M_{21} \\ \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{31}M_{33} + \left(\frac{K_f}{K}\right)^2 M_{11}M_{31} & \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{33}M_{32} + \left(\frac{K_f}{K}\right)^2 M_{12}M_{31} & \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{33}^2 + \left(\frac{K_f}{K}\right)^2 M_{13}^2 \end{bmatrix} \quad (71)$$

Using (66), (68), (69) and (71), twelve simultaneous equations could be generated as follows

$$-\left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{13}^2 - \left(\frac{K_f}{K}\right)^2 M_{11}^2 = 0 \quad (72)$$

$$M_{13}/C_o - \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{13}M_{32} - \left(\frac{K_f}{K}\right)^2 M_{11}M_{12} = 0 \quad (73)$$

$$-\left(M_{12}/L_o + M_{13}/R_l C_o\right) - \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{13}M_{33} - \left(\frac{K_f}{K}\right)^2 M_{11}M_{12} = 0 \quad (74)$$

$$M_{31}/C_o - \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{23}M_{31} - \left(\frac{K_f}{K}\right)^2 M_{11}M_{21} = 0 \quad (75)$$

$$M_{23}/C_o + M_{32}/C_o + 1 - \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{23}^2 - \left(\frac{K_f}{K}\right)^2 M_{12}^2 = 0 \quad (76)$$

$$-\left(M_{22}/L_o + M_{23}/R_l C_o\right) + M_{33}/C_o - \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{23}M_{33} - \left(\frac{K_f}{K}\right)^2 M_{13}M_{21} = 0 \quad (77)$$

$$-\left(M_{21}/L_o + M_{31}/R_l C_o\right) - \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{31}M_{33} - \left(\frac{K_f}{K}\right)^2 M_{11}M_{31} = 0 \quad (78)$$

$$M_{33}/C_o - \left(M_{22}/L_o + M_{32}/R_l C_o\right) - \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{33}M_{32} - \left(\frac{K_f}{K}\right)^2 M_{12}M_{31} = 0 \quad (79)$$

$$-\left(M_{32}/L_o + M_{33}/R_l C_o\right) - \left(M_{23}/L_o + M_{33}/R_l C_o\right) - \left(\frac{i_{Lo}}{R_l C_o}\right)^2 M_{33}^2 - \left(\frac{K_f}{K}\right)^2 M_{13}^2 = 0 \quad (80)$$

From eqn. (72)

$$M_{13} = M_{31} = \mp \frac{R_l C_o}{i_{Lo}} \quad (81)$$

The positive value of eqn. (81) is considered. Also, from eqn. (73)

$$M_{32} = M_{23} = \frac{1}{C_o}\left(\frac{R_l C_o}{i_{Lo}}\right)^2 \quad (82)$$

6. Discussions

Eqn. (28) expresses the principal idea of fluctuating input methodology. The fundamental output voltage component and frequency spectrum of TUPS are not dependent on the dc voltage, v_{dc}, of the inverter section of TUPS. The control dynamics of TUPS system are sufficiently represented by first-order differential equations of (32) through (34). The state variable representation allows the system, driving, and output matrices to be extracted. The system matrix is a function of C_o, R_l, and L_o, while the driving matrix is dependent on feedback factor, carrier signal amplitude, and integrator time constant. Hamilton-Jacobi approach provides the solution for a whole family of initial conditions by state feedback instead of only one specific set of initial conditions. The Hamilton-Jacobi approach yields a reflection of closed-loop solution in the form of Ricatti equations. Since TUPS is a second-order system the scalar R, is made constant at unity and performance index is best minimized with matrix of eqn. (69). The system is less sensitive to N_{11}.

Conclusion

Hamilton-Jacobi Representations of Trapezoidal Uninterruptible Power Supply System has been considered. TUPS is considered to be a control process with inputs and outputs. Hamilton-Jacobi approach has been used to generate a set of simultaneous equations which reflect the characteristic of TUPS optimal control law based on a predicted performance index. In this analysis, modeling of the dynamical nature of the system provides a basis for completely dependent optimal control inputs ($r_l$and v_{dc}) and stability analysis. It is obvious that the scalar factor R has the potential to make available the states which must be estimated in presence of the considered disturbances and measurement noise. Consequently, this can improve the performance of TUPS through a robust optimal control law.

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