



Fuzzy Nonlinear Controller for Stable Walking of Biped Robots

Omid Heydarnia, Behnam Dadashzadeh, Akbar Allahverdizadeh and M.R. Sayyed Noorani

School of Engineering-Emerging Technologies, University of Tabriz, Tabriz, Iran

Phone Number: +98-41-33393874

**Corresponding Author's E-mail: b.dadashzadeh@tabrizu.ac.ir*

Abstract

Control of underactuated biped robots is one of the challenging problems in the robotics field. The main difficulties of this problem that make it hard to control are its highly nonlinear dynamics, open-loop instability and impact event at the end of each step. Partial feedback linearization is a method to control underactuated systems but it is not robust against uncertainties and disturbances. This fact restricts the performance of the method to control walking or running of biped robots. In this paper we propose a fuzzy partial feedback linearization controller to improve this drawback mainly. Numerical simulations verify the effectiveness of the proposed method to generate stable bipedal walking gaits robustly against disturbances.

Keywords: *Fuzzy Control; Partial Feedback Linearization; Biped Robot; Underactuated system.*

1. Introduction

Researchers are still interested in biped robots because of their friendly and human-like appearance and their capability to move on uneven environments. Through three decades of research on biped robots, effective control methods were sought to improve stability and robustness of a biped robot walking like human or animals. Underactuated biped robots walking and running have been noticed by many researchers [18, 3]. Systems that have fewer numbers of actuators than the degrees of freedom (DOFs) are defined as underactuated systems. Control of these systems is more challenging than fully actuated systems and it is an open research problem. Second order sliding mode control [21], sliding mode tracking control [18] and optimal sliding mode [11] were proposed for underactuated systems. The hierarchical sliding mode is designed to overcome uncertainty and disturbances of a class of underactuated systems [23]. Partial feedback linearization was also proposed by Spong [14-15] as a method to control underactuated systems. A collocated form of the partial feedback linearization was applied in a flexible link manipulator and asymptotic stability of its zero dynamics was proved [25]. Biped robots can be divided into three classes: 1- passive bipeds, 2- underactuated bipeds, and 3- fully actuated bipeds. There are no actuators in passive robots and they use the gravity force to continue walking. Engineers usually use springs to improve the performance of these robots [17, 24]. A robot with point feet provides an example of a common underactuated biped, since there is no actuator on its ankle joint. While flat feet biped robots are often controlled by zero moment point (ZMP) stability criterion – showing unnatural and slow walking and running gaits [6] – However, underactuated robots demonstrate more natural motion dynamics and faster gaits. Tzafestas [22] showed that for a 5-link biped robot a sliding mode controller is more robust than a feedback linearization method. A robust tracking control algorithm was presented for underactuated biped robots by [16], making them self-balance in presence of disturbances. Then, stability criteria were derived base on the linearization of one dynamics of nonlinear motion equations. In [13], an

adaptive control scheme was developed for an underactuated bipedal locomotion system in which the recursive least square error method was used for parameter estimation. The stability analysis of a compass gait biped robot has been investigated based on the partial feedback linearization by Kochuvila et al. [19, 20]. They took into consideration the convergence of the biped's states to a reference limit cycle for both feedback linearization and partial feedback linearization. Poincare map is one of the best tools for analysing stability of periodic orbits of dynamic systems. It converts the hybrid dynamic model of a bipedal walking or running to a discrete map. The fixed point of the map is corresponding to a periodic walking or running gait of the robot. The method of Poincare map has been used to study the stability of underactuated systems in several studies [1, 4]. The Poincare map has indeed been used to study the stability of both passive and underactuated biped robots [1, 7].

Initiating by Zadeh's pioneering work [12], fuzzy logic has been utilized in control engineering for four decades. As a result, an Adaptive Network Based Fuzzy Interface System (ANFIS) control strategy has been proposed based on a hierarchy of walking gait planning and joint control level which does not require detailed kinematics and dynamics of the robot model [8]. Also, Fuzzy logic has been used to eliminate chattering phenomena in the classic sliding mode and was applied to a biped robot [2].

In this paper, a fuzzy partial feedback linearization controller is proposed in order to enhance the performance of controllers in stabilizing a desired walk gait. This method is applied in a compass gait as well as a five-link biped robot. Besides, two different methods for gait generating are examined to investigate the effect of gait generation methods on the controller performance. The remainder of this paper is organized as follows. Section II describes dynamics model of the biped robots, including dynamics models of stance phase and touch-down. Section III describes methods of generation of reference trajectories for walking. Section IV discusses the controller design using the fuzzy logic and its implementation in biped models. Section V provides simulation results of our control strategy for walking on the flat ground, for both compass gait and five-link biped robots. Finally, Section VI is conclusion and discussion about the effect of the fuzzy logic on the enhancement of the performance of the partial feedback linearization for stabilizing the biped robots in walking.

2. Dynamics modelling

In this paper, we examine dynamics models of two planar biped robots for walking. The walk really includes two continuous phases, single support phase (SSP) and double support phase (DSP). However, it is assumed here that transition from SSP to DSP occurs instantaneously. So, the models describing our walker biped robots, consist of only one continuous phase called stance phase, and one discrete impact event at the touch down – i.e. collision of the landing leg with the ground.

The walking dynamics model of an n-DOF biped robot with point feet can be expressed as follows. Let q be a vector including generalized coordinates spanning an n-dimensional configuration space Q , and u be a vector including actuator forces, which has a dimension of $(n - 1)$. Then, we have:

$$\left. \begin{aligned} M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) &= B(q)u \quad \text{for} \quad q \notin S \\ q^+ &= \Delta_q q^- \\ \dot{q}^+ &= \Delta_{\dot{q}}(q^-)\dot{q}^- \end{aligned} \right\} \quad \text{whenever} \quad q^- \in S \tag{1}$$

where $M(q)$ is inertia matrix, $C(q, \dot{q})$ is matrix of Coriolis and centrifugal terms, $G(q)$ is related to gravity field, and $B(q)$ describes the effects of actuators on the generalized coordinates. The set of S represents switching surface which is chosen to be:

$$S = \{q \in Q, \dot{q} \in TQ \mid P_e^V = 0, P_e^H > 0\} \tag{2}$$

where P_e^V and P_e^H denote the vertical and horizontal position of the end of the swing leg, respectively.

Several assumptions are considered for the touch-down event [9]. It occurs instantaneously, without slip and rebound, and robot's actuators don't impose impulsive torques. Accordingly, to find the angular velocities just after the impact this method is followed. First, the equation of motion given by (1) is integrated over an infinitesimal time interval around the impact event. This establishes n equation with $(n + 2)$ unknowns. Those are n unknown velocities corresponding to the moment just after the impact, denoted by q^+ , in addition to 2 components of the ground reaction force. Hence, the Lagrange multiplier method is exploited to serve the assumption of being the impact without slip and rebound. In this way, the position coordinates of the end of the stand leg, (x_o, y_o) , are added to former generalized coordinates to form an augmented one denoted by $q_e = (q, x_o, y_o)$. Thus, by integrating the equation of motion described by q_e , and setting the velocity of the end of the landing leg just after the impact equal to zero, and some mathematical manipulating we find:

$$\dot{q}_e^+ = \begin{bmatrix} I_{1 \times N} & -M_e^{-1} J_p^T (J_p M_e^{-1} J_p^T)^{-1} J_p \end{bmatrix} \dot{q}_e^- \equiv \Delta_{\dot{q}_e}(q_e^-) \dot{q}_e^- \quad (3)$$

where M_e is inertia matrix corresponding to q_e , $J_p = \partial(x_p, y_p)/\partial q_e$ is Jacobian matrix of the end of the landing leg. The unknown velocities just after the impact q^+ are the first n elements of q_e^+ in (3). Details of the derivation can be seen in [9].

2.1. Compass gait biped robot

This robot has two degrees of freedom in which there is only one actuator on the hip joint. Then, this robot is an underactuated system. Dynamics model of a compass gait can be derived using Lagrange method and is available in literature [5, 10]. According to Fig. 1(a), we denote $q = (q_1, q_2)$ as a configuration variables vector of the robot. So, they are: $M(q) \in R^{2 \times 2}$, $C(q, \dot{q}) \in R^{2 \times 2}$, $G(q) \in R^{2 \times 1}$, $B(q) \in (1 \ 0)^t$.

2.2. Five-link Biped robot

One of the famous anthropomorphic models for biped robots is five-link biped robot model. Various prototypes have been developed by this model, such as RABBIT [5] and MABEL [10]. According to Fig. 2(b), $q = (q_1, q_2, q_3, q_4, q_5)$ is the generalized coordinates vector, as well as $M(q) \in R^{5 \times 5}$, $C(q, \dot{q}) \in R^{5 \times 5}$, $G(q) \in R^{5 \times 1}$, $B(q) \in (0_{1 \times 4} \ I_{4 \times 4})^t$.

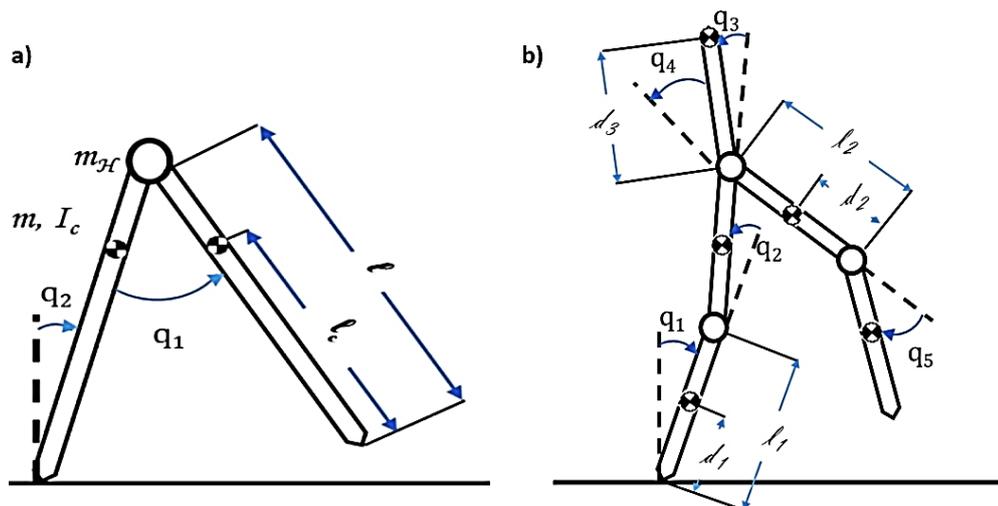


Figure 1: Models for biped robots: a) compass gait biped robot, b) five-link biped robot.

3. Reference trajectory of walking

In order to generate reference trajectories of walking for a biped robot, we consider here two convenient methods: 1- Poincare Map [21], 2- intellectual trajectory [22]. We examine both methods to understand the effects of reference trajectories on controller abilities. We use the Poincare map to find reference trajectory of the compass gait, and utilize the intellectual method to search for reference trajectories of the five-link biped robot.

3.1. Active Poincare map

Poincare map is a powerful mathematical tool to transform the problem of finding periodic orbits of an oscillating system into finding fixed points of a discrete map. A fixed point can be considered as an equilibrium point of a specific discrete nonlinear system. Active Poincare map can be used to find reference trajectories of a walking. This consists of finding an initial condition and control commands which enables the robot to walk periodically. Therefore, it is mathematically formulated as follows:

$$x(1) = x_0; \quad x(k + 1) = P(x(k), u(k)), \quad k = 1, 2, \dots \quad (4)$$

We use the active Poincare map to find reference trajectories of the compass gait biped robot. The robot's configuration just at the start of the stance phase, while both feet are on the ground, describes the Poincare section. We search for the fixed point of the walking Poincare map by using a nonlinear optimization method such that minimizes $er = x(k + 1) - x(k)$, where $x(k)$ is the state of the robot on the Poincare section. In this optimization we search for optimal values of initial states of the robot as well as 5 adjusting parameters utilized to form the motor commands required during the stance phase. The obtained reference trajectories resulted by this method are shown in Fig. 2.

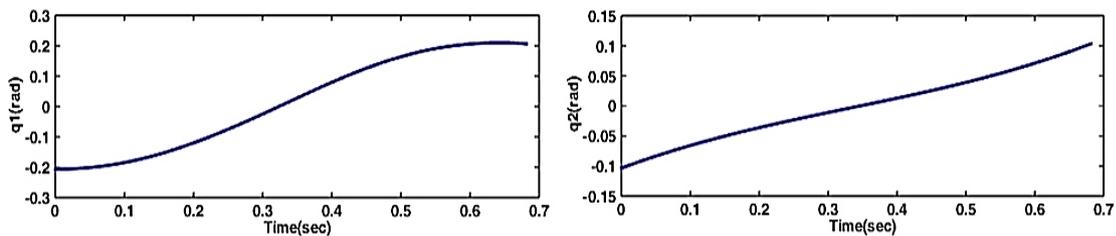


Figure 2: Joint reference trajectories for compass gait biped robot.

3.2. Intellectual method

We named the method “intellectual” to describe the reference trajectory proposed by an expert, without requiring an analytical method such as inverse kinematics, splines, and etc. We follow Tzafestas [22] in using this method to plan joint reference trajectories for the five-link biped robot. Those are planned so that torso always remains vertically, and the angular momentum of the robot about the support point increases by gravitational forces. Fig. 3 indicates the reference trajectories which are planned by this way.

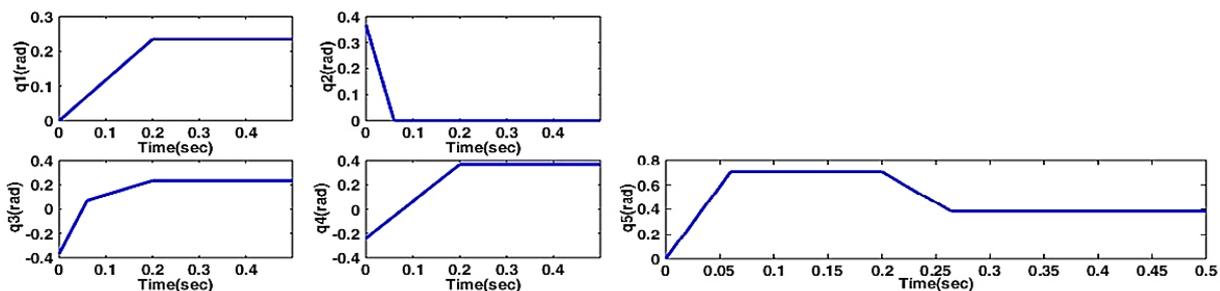


Figure 3: Joint reference trajectories for five-link biped robot.

4. Fuzzy partial feedback linearization control

Fully feedback linearization in underactuated systems is often impossible. So, an alternative way called “partial feedback linearization” has been proposed recently, to linearize the controllable part of an underactuated system through a feedback loop, so that the remaining zero dynamics is stable. The partial feedback linearization method is classified into three categories: 1- Collocated form, 2- Non-collocated form, and 3- Task space form. In a collocated form, the control law linearizes the dynamics of actuated degrees of freedom. The non-collocated form refers to linearize the dynamics of passive degrees of freedom, and in a task space form a combination of some active and passive degrees of freedom are linearized and controlled.

Here, we use a collocated partial feedback linearization and exploit the fuzzy logic to enhance capabilities of the control method. General formulization of underactuated systems is given by:

$$\begin{cases} M_{aa}\ddot{q}_a + M_{ao}\ddot{q}_o + C_a(q, \dot{q}) + G_a(q) = \tau \\ M_{ao}^T \ddot{q}_a + M_{oo}\ddot{q}_o + C_o(q, \dot{q}) + G_o(q) = 0 \end{cases} \quad (5)$$

where q_a and q_o represent actuated and unactuated coordinates, respectively. Note that because the total inertial matrix of M is uniformly positive definite, then both parts of M_{aa} and M_{oo} are also positive definite, and hence invertible. Thus, it can be obtained as follows:

$$\ddot{q}_o = -M_{oo}^{-1} (M_{ao}^T \ddot{q}_a + C_o + G_o) \quad (6)$$

Substituting (6) into first equation of (5), the actuated dynamics of the system can be expressed as:

$$D(q)\ddot{q}_a = \tau - H(q, \dot{q}) \quad (7)$$

where,

$$\begin{aligned} D(q) &= M_{aa} - M_{ao}M_{oo}^{-1}M_{ao}^T \\ H(q, \dot{q}) &= C_a + G_a - M_{ao}M_{oo}^{-1}(C_o + G_o) \end{aligned} \quad (8)$$

A feedback linearization control law can linearize the actuated subsystem. Hence, it can be defined for (7) according to:

$$\tau = Dv + H \quad (9)$$

where, v is an additional outer loop control input. Similar to the method applied by [14], v can be defined as the below, with $K_p, K_d > 0$:

$$v = \ddot{q}_a^d - K_d(\dot{q}_a - \dot{q}_a^d) - K_p(q_a - q_a^d) \quad (10)$$

The complete dynamics model of the system can be rewritten as:

$$\begin{aligned} \ddot{q}_a &= v \\ M_{oo}\ddot{q}_o + C_o + G_o &= -M_{ao}^T v \end{aligned} \quad (11)$$

Let's follow the method of [14] to define new variables as the below:

$$\begin{aligned} z_1 &= q_a - q_a^d, & z_2 &= \dot{z}_1 = \dot{q}_a - \dot{q}_a^d \\ \eta_1 &= q_o, & \eta_2 &= \dot{\eta}_1 = \dot{q}_o \end{aligned} \quad (12)$$

Let be $z = (z_1 \ z_2)$ and $\eta = (\eta_1 \ \eta_2)$. Then, by substituting these new variables into (11) we have:

$$\dot{z} = Az \equiv \begin{bmatrix} O & I \\ -K_p & -K_d \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}; \quad \dot{\eta} = \Omega(z, \eta, t) \quad (13)$$

The matrix A is Hurwitz because we supposed $K_p, K_d > 0$. Also, $\eta = \Omega(z = 0, \eta, t)$ is a zero dynamics of the system. The stability of this zero dynamics has been discussed in [14].

Because of high nonlinearity of a biped robot and presence of an impact at the end of each step, design a controller for a point feet biped robot is more difficult than other underactuated systems. Then, properly adjusting the parameters of a controller considered for such robots is very important. Hence, we exploit a supervisory fuzzy controller to adjust the parameters of the main controller, i.e. the partial feedback linearization, as shown in Fig. 4.

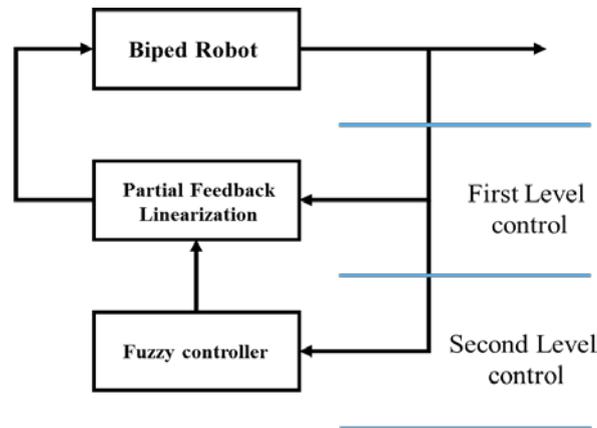


Figure 4: General scheme of the fuzzy partial feedback linearization control.

Let parameterize the gain diagonal matrixes of K_p and K_d so that the tracking error equation established based on the control law of (10) is being critically damped. Then, we set:

$$k_p = \lambda^2, \quad k_d = 2\lambda \tag{14}$$

Moreover, if we consider λ_{min} and λ_{max} as the lower and upper bounds of the λ parameter, respectively, then to facilitate computations, it is normalized as $\lambda' = (\lambda - \lambda_{min}) / (\lambda_{max} - \lambda_{min})$.

To design the fuzzy supervisory controller, we should consider a fuzzy set for each input variable. For each fuzzy set a membership function is also defined. Then an output fuzzy set is considered, too. The center of the membership functions of the output fuzzy set would optionally be located at $2(j + k + \dots + l) / n(N - 1)$, where $s = j + k + \dots + l$ is the index of the output fuzzy set, denoted by Y^s , and j, k, \dots, l are the linguistic-numeric indices of the input fuzzy sets, and N is the number of membership functions of each input variable, and n is the number of input variables.

Here, the tracking error e and its derivation \dot{e} corresponding to the actuated degrees of freedom of the robot are considered as input variables of the fuzzy controller, and the λ' parameter adjusting the gains of the main controller is its output. The membership functions of input variables (e, \dot{e}), and the membership functions of the output linguistic variable λ' are shown in Fig. 5(a) and Fig. 5(b), respectively. The fuzzy rule base is designed according to Table 1. Thus, a general form to describe the fuzzy rules is as follows:

$$\text{If } e(t) \text{ is } E^j \text{ and } \dot{e}(t) \text{ is } D^k \text{ then } \lambda' \text{ is } Y^s$$

where, E^j, D^k and Y^s are all triangular membership functions that has been depicted in Fig. 5.

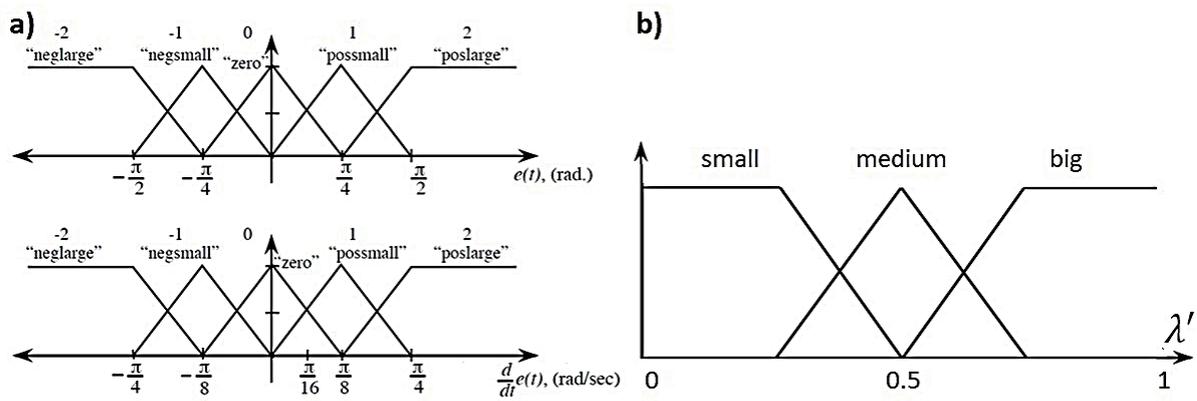


Figure 5: a) Membership functions of input variables, b) Membership functions of output variable.

Table 1: Fuzzy rule base table.

λ'		$e(t)$ (j index)				
		E^{-2}	E^{-1}	E^0	E^1	E^2
$\frac{d}{dt} e(t)$ (k index)	D^{-2}	B	B	B	M	S
	D^{-1}	B	B	M	M	S
	D^0	B	M	S	M	B
	D^1	S	M	M	B	B
	D^2	S	M	B	B	B

5. Simulation results

Validity of our proposed method to control an underactuated biped robot is examined via numerical simulations. We apply the two-level controller on models of the compass gait biped robot and the five-link biped robot. As mentioned before, two different methods to generate the walking reference trajectories are utilized, each one for one of the models. By this way, the performance of the proposed controller can be examined by different models of underactuated biped robots and different methods of reference trajectory generation.

5.1. Compass Gait biped robot

A dynamics model for the compass gait biped robot was derived by [10]. Here, we use this model with the same physical characteristics considered by [10]. A non-constrained optimization approach (i.e. OPTIMSEARCH) is utilized to search for a fixed point of the Poincare map of the compass gait biped robot. Then, by finding a stable periodic solution corresponding to the fixed point obtained by the optimization method, the time histories of the solution is served as the reference trajectories should be tracked via of the fuzzy partial feedback linearization control method.

In order to examine the performance of the controller, we make about 30% deviation in the robot's initial states with respect to the reference trajectory at the beginning of the stance phase. Results of the numerical simulation obtained by MATLAB are shown in Fig. 6. These results validate the ability of the fuzzy partial feedback linearization controller in stabilizing the walking of the robot against the deviations from the reference trajectory. Similar numerical simulations of the walking of the compass gait biped robot are done in which a conventional partial feedback linearization control is applied. This controller is able to stabilize the walking against the deviations in the initial states when imposed up to 20%.

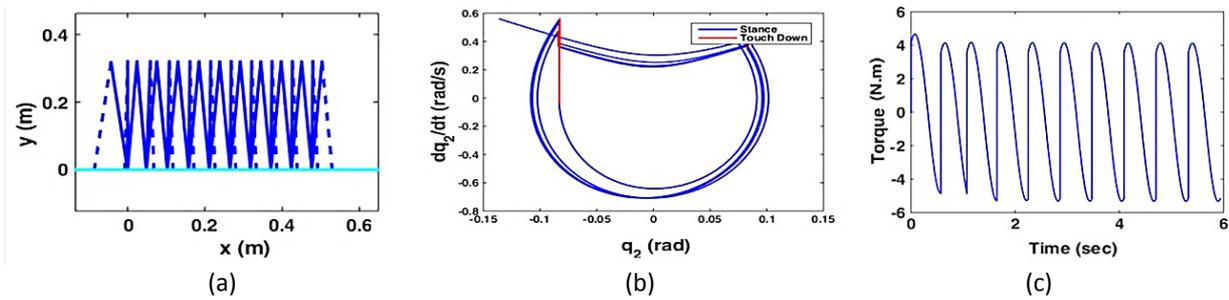


Figure 6: The simulation results obtained for the compass gait biped robot. a) Stick diagram of 10 steps walking, b) The evolution of state variables of one of legs in the phase plane towards forming a stable limit cycle, c) Profile of torque required on the hip joint.

5.2. Five-link Biped robot

For the dynamics model and characteristics of the five-link biped robot which simulated here, it is referred to [22]. Similarly, we apply both convenient as well as fuzzy partial feedback linearization controllers in this model of biped robot and we obtain similar results to compass gait biped robot. However, in both cases the time histories shown in Fig. 3 are served as joint reference trajectories. The stick diagram of a stable walking resulted by our controller in simulation is shown in Fig. 7.

In the phase portraits shown by Fig. 8(a), absolute angles of the links belonging to the stand leg with respect to the vertical axis has been considered to investigate the stability of the limit cycles well. Clearly, they are taken as $\theta_1 = q_1$ and $\theta_2 = q_1 - q_2$.

Fig. 8(b) shows required actuator torques at the hip and the knee joints. These torque profiles are comparable to the results obtained by a non-fuzzy controller reported by [22], and their magnitudes are reasonable for a biped robot with the mass of 50 kilograms and the height of 1.3 meters. Fig. 9 shows updates of the control parameter λ in each step that converges to a value after 10 steps of walking.

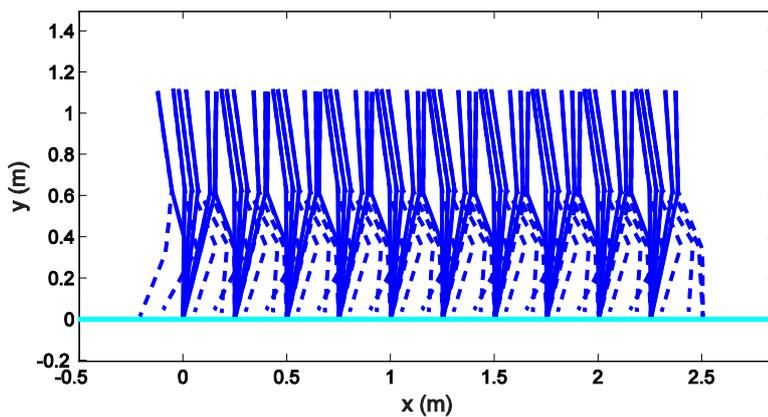


Figure 7: Stick diagram of 10 steps walking of the five-link biped robot.

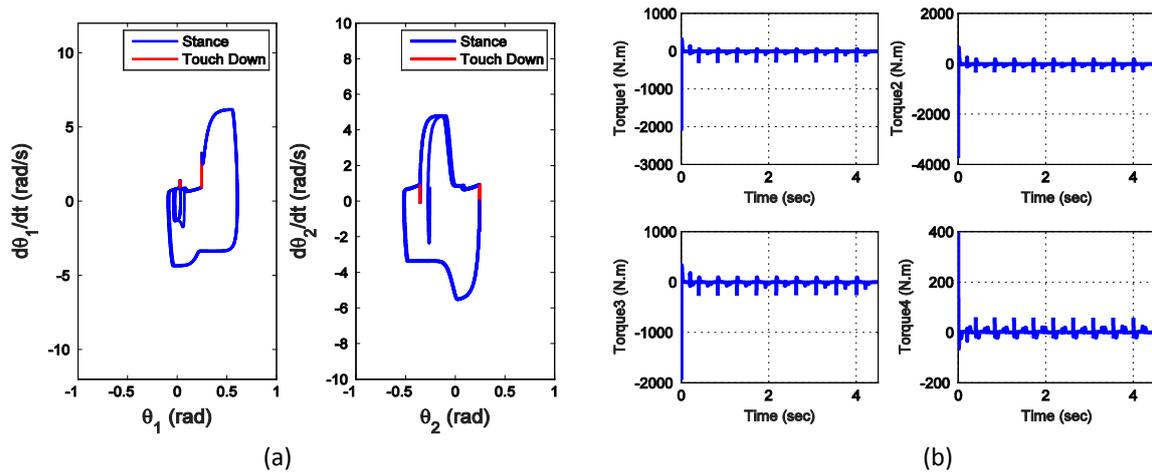


Figure 8: a) The evolution of link variables belonging to the shin and thigh of the stand leg in phase plane towards forming a stable limit cycle, b) Profiles of the torques required on the joints of five-link biped robot.

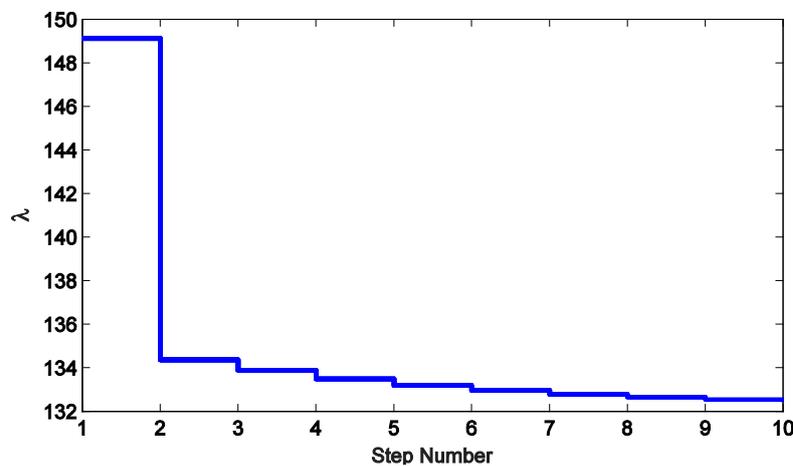


Figure 9: a) Variation on the control parameter λ during walking steps

CONCLUSION

A fuzzy partial feedback linearization controller were proposed and applied in two models of underactuated walking biped robots, i.e. the compass gait biped robot and the five-link biped robot. Specially, we exploit the fuzzy logic as a supervisory in second level of our control plan to enhance the performance of a partial feedback linearization controller. According to numerical simulations done by MATLAB, the fuzzy partial feedback linearization controller showed a better performance in comparison with a convenient partial feedback linearization one. Two methods for generating reference trajectories of a walking gait was particularly examined; the active Poincare map method was used for the compass gait biped robot and an intellectual method was utilized in the five-link. This control scheme was able to stabilize the desired walking gait in both models of a biped robot. This was investigated by the limit cycle of some selected joints or links having oscillatory movements. The basin of attraction was larger when we used the fuzzy partial feedback linearization controller.

References

- [1] A. D. Kuo, "Stabilization of lateral motion in passive dynamic walking," *The International journal of robotics research*, vol. 18, pp. 917–930, 1999.
- [2] A. Takhmar, M. Alghooneh, and S. A. A. Moosavian, "Chattering Eliminated and Stable Motion of Biped Robots Using a Fuzzy Sliding Mode Controller," *Majlesi Journal of Electrical Engineering*, vol. 7, pp. 31–42, 2012.
- [3] B. Dadashzadeh, M. Mahjoob, M. Nikkhah Bahrami, and C. Macnab, "Stable active running of a planar biped robot using Poincare map control," *Advanced Robotics*, vol. 28, pp. 231–244, 2014.
- [4] B.-K. Cho and J.-H. Oh, "Running pattern generation with a fixed point in a 2D planar biped," *Int. J. Humanoid Robotics*, vol. 6, pp. 241–264, 2009.
- [5] Ch. Chevallereau, G. Abba, Y. Aoustin, F. P., E. R. Westervelt, C. Canudas-de-Wit, and J. W. Grizzle, "RABBIT: A Testbed for Advanced Control Theory," *Control Systems, IEEE*, vol. 23(5), pp. 57–79, 2003.
- [6] Ch. Chevallereau, G. Bessonnet, G. Abba, and Y. Aoustin, "Bipedal robots: modeling, design and walking synthesis," John Wiley & Sons, 2013.
- [7] C. E. Bauby and A. D. Kuo, "Active control of lateral balance in human walking," *Journal of biomechanics*, vol. 33, pp. 1433–1440, 2000.
- [8] C. Zhou and K. Jagannathan, "Adaptive network based fuzzy control of a dynamic biped walking robot," In *Intelligence and Systems*, 1996., IEEE International Joint Symposia on, pp. 109–116. IEEE, 1996.
- [9] E. R. Westervelt, J. W. Grizzle, Ch. Chevallereau, J. H. Choi, B. Morris, "Feedback control of dynamic bipedal robot locomotion," 1st ed., CRC Press: Taylor & Francis Group, 2007.
- [10] J.W. Grizzle, J. Hurst, B. Morris, H.-W. Park, and K. Sreenath, "MABEL, A New Robotic Bipedal Walker and Runner," In *American Control Conference*, 2009., pp. 2030–2036, IEEE, 2009.
- [11] K. R. Muske, H. Ashrafiuon, S. Nersesov, and M. Nikkhah, "Optimal sliding mode cascade control for stabilization of underactuated nonlinear systems," *J. Dynamic Systems, Measurement, and Control*, vol. 134, pp. 021020, 2012.
- [12] L.A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8(3), Pages 338–353, 1965.
- [13] L. M. Liu and W. Liang, "Adaptive asymptotic stable biped locomotion," in *Control Conference (CCC)*, 2014 33rd Chinese, pp. 8375–8380, IEEE, 2014.
- [14] M. W. Spong, "Partial feedback linearization of underactuated mechanical systems," in *Intelligent Robots and Systems, IROS 94, Proceedings of the IEEE/RSJ/GI Int. Conf. on*, pp. 314–321, 1994.
- [15] M. W. Spong, "Underactuated mechanical systems," in *Control Problems in Robotics and Automation*, ed.: Springer, pp. 135–150, 1998.
- [16] M. Nikkhah, H. Ashrafiuon, and F. Fahimi, "Robust control of underactuated bipeds using sliding modes," *Robotica*, vol. 25, pp. 367–374, 2007.
- [17] R. M. Alexander, "Three uses for springs in legged locomotion," *The International Journal of Robotics Research*, vol. 9, pp. 53–61, 1990.
- [18] R. Yu, Q. Zhu, G. Xia, and Z. Liu, "Sliding mode tracking control of an underactuated surface vessel," *IET control theory & applications*, vol. 6, pp. 461–466, 2012.
- [19] S. Kochuvila, S. Tripathi, and T. Sudarshan, "Control of a Compass Gait Biped Robot Based on Partial Feedback Linearization," in *Advances in Autonomous Robotics*, ed.: Springer, pp. 117–127, 2012.
- [20] S. Kochuvila, S. Tripathi, and T. Sudarshan, "Stability of an Underactuated Passive Biped Robot Using Partial Feedback Linearization Technique," *Applied Mechanics and Materials*, vol. 394, pp. 456–462, 2013.
- [21] S. Kurode, P. Trivedi, B. Bandyopadhyay, and P. Gandhi, "Second order sliding mode control for a class of underactuated systems," in *Variable Structure Systems (VSS)*, 2012 12th International Workshop on, pp. 458–462, IEEE, 2012.
- [22] S. Tzafestas, M. Raibert, and C. Tzafestas, "Robust sliding-mode control applied to a 5-link biped robot," *Journal of Intelligent and Robotic Systems*, vol. 15, pp. 67–133, 1996.
- [23] W. Wang, J. Yi, D. Zhao, and D. Liu, "Design of a stable sliding-mode controller for a class of second-order underactuated systems," *IEE Proceedings-Control Theory and Applications*, vol. 151, pp. 683–690, 2004.
- [24] Y. Akutsu, F. Asano, and I. Tokuda, "Passive dynamic walking of compass-like biped robot with dynamic absorbers," in *Intelligent Robots and Systems, IROS 2014, Proceedings of IEEE/RSJ Int. Conf. on*, pp. 4855–4860, 2014.
- [25] Z. Zhang and S. S. Ge, "Partial feedback linearization control of space flexible-link manipulator with geometric nonlinearity," in *Information and Automation (ICIA)*, 2014 IEEE International Conference on, 2014, pp. 1102–1107.