



## Semi-Empirical Modelling of Surface Roughness in CNC End Milling

N K Mandal<sup>1</sup>, N K Singh<sup>2</sup>, UC Kumar<sup>3</sup> and V Kumar<sup>4</sup>

<sup>1, 3, 4</sup> National Institute of Technical Teachers' Training and Research

Block- FC, Sector-III, Salt Lake City, Kolkata-700106, India

<sup>2</sup> Indian School of Mines, Dhanbad-826004, Jharkhand, India

Phone Number: +91-033-66251973

\*Corresponding Author's E-mail: [nkmandal@nittrkol.ac.in](mailto:nkmandal@nittrkol.ac.in)

### Abstract

Surface roughness is an important parameter of product quality because it has direct effect on the fitting characteristics of the mating components, resistance to corrosion, frictional losses of fluid flow along pipelines, and above all the aesthetics of the product. There are basically two types of modelling: analytical modelling and empirical modelling. Scientists are believing on analytical modelling where mostly controllable factors of a process are taken into account and does not tell anything about uncontrollable factors like vibration, tool wear etc. whereas Engineers believe on Empirical modelling where both controllable and uncontrollable factors of a process are considered. In this research work, we have developed a semi-empirical model of surface roughness of a surface produced by CNC end milling process using Buckingham pi Theorem.

**Keywords:** Surface roughness, Semi Empirical model, CNC end milling, Buckingham pi theorem

### 1. Introduction

Among different types of milling processes, end milling is one of the most important and common metal cutting operations used for machining parts because of its capability to remove materials at faster rate with a reasonably good surface quality. Also, it is capable of producing a variety of configurations using milling cutter. Surface roughness is a key factor in the machining process while considering machining performance and that is why in many cases, industries are looking for maintaining the good surface quality of the machined parts. It also influences several functional attributes of a part, such as light reflection, heat transmission, coating characteristics, surface friction, fatigue resistance etc. However, the mechanism behind the formation of surface roughness is very dynamic, complicated and process dependent; therefore, it is very difficult to calculate its value through analytical formulae. Various theoretical models that have been proposed are not accurate enough and apply only to a limited range of processes and cutting conditions. Therefore, semi-empirical model for surface roughness for end milling machine for various cutting conditions and work material is being developed.

### 2. Fundamental Concepts of Modelling

Modelling is mathematical representation of physical behavior of components or collections of components. This representation must include descriptions of the individual components as well as descriptions of how the components interact. A good model will provide an accurate description of behavior, while at the same time remaining mathematically simple enough to permit easy calculation [2]. Modelling of a machining process are basically two types:

*Analytical model:* A mathematical, logical or mechanical representation of relationship based on theory, process, and system. Although the analytical models help to provide better insight into the underlying physical nature of the cutting process in machining but it is usually less satisfactory in modelling due to simplifications and assumptions. In case of any dynamic process like machining, there are two types of input parameters: controllable parameters and uncontrollable parameters. Controllable parameters include speed, feed, and depth of cut etc., the parameters which can be easily controlled. Uncontrollable parameters include machine vibration, tool wear which is transient in nature. Therefore, it can be opined that the difference between the value obtained from the analytical model and the value obtained from the experiments are due to these uncontrollable factors. Analytical model does not tell anything about these uncontrollable factors.

*Empirical model:* A mathematical, logical or mechanical representation of a relationship based on, observation or experience rather than theory or pure logic. As this type of modelling is based on experimental observation, it takes care both the controllable and uncontrollable factors. In that sense, empirical modelling is better than mathematical model. These are sometimes called statistical model. But other than these two, there is another types of model called hybrid modelling.

*Hybrid modelling:* This combines some of the analytical, numerical, empirical or artificial intelligence based methods; hybrid models can be created to predict industry-relevant process performance measures.

*Semi empirical model:* It is part of hybrid modelling. It is defined by the underlying theoretical approach and the integral approximations that determine the types of interactions included. The implementation of a semi empirical model specifies the evaluation of all non-vanishing integrals and introduces the associated parameters. The integrals are either determined directly from experimental data or calculated exactly from the corresponding analytical formulas or represented by suitable parametric expressions.

End milling is an essential operation in several manufacturing processes in some industries, because of its capability to remove materials at faster rate with a reasonably good surface quality. Several researchers have attempted to improve the performance characteristics namely the surface roughness, cutting speed, and material removal rate. Surface roughness is a key factor in the machining process. Some researchers and along with their research work have been mentioned below.

Hua et al. [6] have applied Analysis of Variance (ANOVA) method to determine the most influential parameters concerning surface quality. It has been shown the effect of tool nose radius and feed on surface roughness. Tsai et al. [14] have established semi-empirical model of surface finish on work for various materials employing dimensional analysis based upon pertinent process parameters (such as peak current, pulse duration, electric polarity, and properties of materials) in the electrical discharge machining process. Ghani et al. [5] have applied Taguchi optimization methodology to optimize cutting parameters in end milling Doniavi et al. [4] have used response surface methodology (RSM) in order to develop empirical model for the prediction of surface roughness by deciding the optimum cutting condition in turning.

The authors have showed that the feed rate influences surface roughness remarkably. Thamizhmanii et al. [13] have studied the effect of cutting parameters on surface roughness (SCM440 alloy steel) by Taguchi method and found that Cutting speed has lesser role on surface roughness. Mata et al. [7] have analyzed the derivation of statistical models to predict roughness parameters during machining process of PEEK composites using PCD and K10 tools. Ozcakar et al. [8] have found that controllable surface roughness is the key factor in capacity & cost strategies. In order to maintain these goals, surface roughness is modelled by design of experiment techniques and the cutting conditions are optimized by the heuristic algorithms. Ramesh et al. [12] have analysed the influence of cutting conditions in turning of Duplex Stainless Steel 2205 and optimizes the turning conditions based on surface roughness. Arokiadass et al. [2] have investigated the effect of spindle speed, feed rate, and depth of cut and different % wt. of SiCp on surface roughness in end milling of LM25Al/SiCp. Patwari

et al. [11] have described mathematically the effect of cutting parameters on surface roughness in end milling of medium carbon steel. Arokiadass et al. [1] have focused on study and analyses of surface quality improvement in end milling operation of Al/SiCp metal matrix composite. Parmar et al. [10] have found the Surface finish is an important indicator of the milling operation in manufacturing process. The aim was to predict the surface roughness by using artificial neural networks. Cheng [3] have developed empirical models for estimating the surface roughness of machined components under various cutting speed by using regression analysis software. Phillips. et. al. [9] have adopted a Regression Analysis to construct a prediction model for surface roughness of EN19. Warfield et al. [15] have investigated the effect of process parameters on metal removal rate and surface roughness in milling of SAE52100 tool steel.

### 3. Mathematical Models of Surface Roughness

Recently a branch of science called surface science has been developed to study and analyse the surface contact area and their influence on the performance of various components, units and machines. A Surface can be described in simple turn to the outermost layer of an entity. An interface can be defined to be the transition layer between two or more entities that differ either chemically or physically or in both aspects. *Ideal Roughness*: Ideal surface roughness is a function of feed and geometry of the tool. It represents the best possible finish which can be obtained for a given tool shape and feed. It can be achieved only if the built-up-edge, chatter and inaccuracies in the machine tool movements are eliminated completely.

*Natural Roughness*: In practice, it is not usually possible to achieve conditions such as those described above, and normally the natural surface roughness forms a large proportion of the actual roughness. One of the main factors contributing to natural roughness is the occurrence of a built-up edge and vibration of the machine tool. Thus, larger the built up edge, the rougher would be the surface produced, and factors tending to reduce chip-tool friction and to eliminate or reduce the built-up edge would give improved surface finish.

*Roughness Average (Ra)*: This parameter is also known as the arithmetic mean roughness value, AA (arithmetic average) or CLA (center line average). Ra is universally recognized and the most used international parameter of roughness.

$$R_a = \frac{1}{L} \int_0^L |Y(x)| dx \quad (1)$$

where,  $R_a$ = arithmetic average deviation from the mean line.

$L$  = sampling length.

$Y$  = ordinate of the profile curve.

It is the arithmetic mean of the departure of the roughness profile from the mean line.

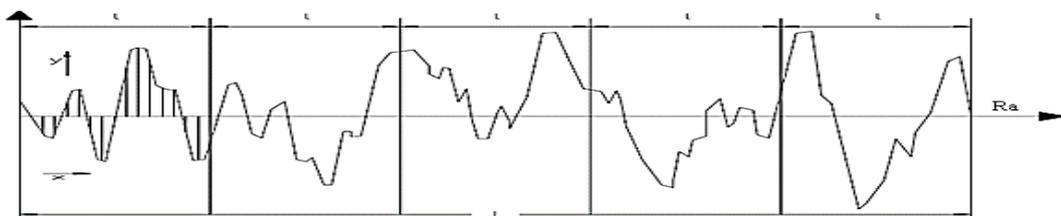


Fig.1 Surface profile of component

There are various factors which have direct effect on surface roughness which includes:

**Depth of Cut:** Increasing the depth of cut increases the cutting resistance and the amplitude of vibrations. As a result, cutting temperature also rises. Therefore, it is expected that surface quality will deteriorate. **Feed:** Experiments show that as feed rate increases surface roughness also increases due

to the increase in cutting force and vibration. Cutting Speed: It is found that an increase of cutting speed generally improves surface quality. Engagement of the Cutting Tool: This factor acts in the same way as the depth of cut. Cutting Tool Wears: The irregularities of the cutting edge due to wear are reproduced on the machined surface. Apart from that, as tool wear increases, other dynamic phenomena such as excessive vibrations will occur, thus further deteriorating surface quality.

Surface roughness is a result of tool-work piece replication based on the movement provided by the machine tool. Theoretically surface roughness can be predicted if tool geometry and cutting parameters are given. If depth of cut is sufficiently large then theoretical surface roughness is a function of tool feed and tool geometry. It represents the best possible finish which can be obtained for a given tool shape and feed. When tool shape straight- nose tool is considered then Surface roughness is the maximum height (peak to valley,  $R_{max}$ ) of the scallops, given by [20]

$$R_{max} = \frac{F}{\cot \alpha + \cot \beta} \tag{2}$$

where  $D$  is the depth of cut,  $F$  is the tool feed,  $\alpha$  is the major cutting edge angle, and  $\beta$  is the minor cutting edge angle.

When tool shape round-nosed tool with a nose radius  $R$  is considered then the maximum height of the scallops.

$$R_{max} = R - \left( R^2 - \frac{F^2}{4} \right)^{\frac{1}{2}} \approx \frac{F^2}{8R} \tag{3}$$

#### 4. Buckingham’s Pi-Theorem

This theorem states that if there are  $n$  variables in a dimensionally homogeneous equation and if these variables contain  $m$  primary dimensions, then the variables can be grouped into  $(n - m)$  non-dimensional parameters.” The non-dimensional groups are called pi-terms. Mathematically this can be represented by

$$f(x_1, x_2, x_3, \dots, x_n) = 0 \tag{4}$$

Where  $x$ 's are dimensional physical quantities (Such as velocity, density, viscosity, pressure and area etc.) pertinent to a physical phenomenon, then the same phenomenon can be described by  $(n - m)$  dimensional pi-terms.

$$\phi = [\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}] = 0 \tag{5}$$

Where  $m$  represents the fundamental dimensions (such as mass, length and time or force, length and time). Out of the given physical variables, one has to select any three variables which amongst them contain all the fundamental units of  $M$ ,  $L$  and  $T$ . These variables are not to form non-dimensional parameters amongst themselves. We have assumed the following parameters given in Table 1 for dimensional analysis of a CNC end milling process.

**Table 1.** Dimension of important parameters

Factor	Symbol	Unit	Dimension
Surface roughness	$R_a$	$\mu\text{m}$	$L$
Speed	$N$	rad/s	$T^{-1}$
Feed	$F$	mm/s	$LT^{-1}$
Depth of cut	$D$	mm	$L$

According to Buckingham  $\pi$  theorem, the relationship of surface roughness of work can be expressed as follows

$$R_a = f(N, F, D) \quad (6)$$

Since the dimensionless homogeneous equation has 4 variables and only two fundamental dimensionless coefficients, the solution can be expressed as

Let  $n$  = Total number of variable, and  $m$  = number of fundamental quantities  $n = 4$ , and  $m = 2$ ,  
Number of  $\pi$  terms =  $(n-m) = 4-2 = 2$

So,

$$\pi_1 = R_a(N^{a_1}, D^{b_1}) \quad (7)$$

$$M^0 L^0 T^0 = L(T^{-a_1}, L^{b_1}) \quad (8)$$

By equating the powers of fundamental unit on both side of equation (5.3)

$$a_1 = 0 \text{ and } b_1 = -1$$

By putting value  $a_1$  and  $b_1$ ,  $\pi_1$  can be expressed as,

$$\pi_1 = R_a(N^0, D^{-1}) \quad (9)$$

$$\pi_1 = \frac{R_a}{D} \quad (10)$$

Other  $\pi$  term is,

$$\pi_2 = F(N^{a_2}, D^{b_2}) \quad (11)$$

$$M^0 L^0 T^0 = LT^{-1}(T^{-a_2}, L^{b_2}) \quad (12)$$

By equating the powers of fundamental unit on both side of equation (6.3)

$$a_2 = -1 \text{ and } b_2 = -1$$

By putting value  $a_2$  and  $b_2$ ,  $\pi_2$  can be expressed as,

$$\pi_2 = F(N^{-1}, D^{-1}) \quad (13)$$

$$\pi_2 = \frac{F}{ND} \quad (14)$$

Equation 5.1 can be expressed as both in term  $\pi_1$  and  $\pi_2$

$$\pi_1 = f(\pi_2) \quad (15)$$

$$\frac{R_a}{D} = f\left(\frac{F}{ND}\right) \quad (16)$$

It can finally be expressed as

$$R_a = KD \left(\frac{F}{ND}\right)^T \quad (17)$$

where, K is constant

T is the power index of the corresponding dimensionless bracket

Based on experimental results, the coefficients and the power indexes in equation 17 have been calculated for each work with the help of MATLAB shown in Table 6.7.

### 5. Experimentation

27 no. of experiments were conducted on a CNC milling machine (Model: BMV35 TC20) made by Bharat Fritz Werner Ltd. installed at CAD/CAM Laboratory of Mechanical Engineering Department, NITTR Kolkata, West Bengal, India. The Stainless steel plate of 250 mm × 75mm × 25mm size, copper plate of size 200mm × 100mm × 20mm and aluminium 200mm × 100mm × 20mm have been used as work piece (Fig. 2.-a and Fig. 2-b) material for the present experiments.



Fig. 2-a. Cutting tool



Fig. 2-b. Work pieces

The cutting tool used in experiments is PVD (physical vapour deposition) coated cemented carbide tool. Properties of this cutting tool are given in Table 2.

Table 2. Characteristics of typical tool coating

Coating	Colour	Hardness[Hv]	Friction coefficient	Temperature oxidation[°C]
TiAlN	Black	2800	0.3	850

In the present investigation four input factors considered (speed, feed, depth of cut and work material) and each factors has three levels shown in Table 3.

Table 3. Variable parameter and level in experiment

Symbol	Parameters	Level 1	Level 2	Level 3
A	Speed (rpm)	2500	3000	3500
B	Feed (mm/min)	500	600	700
C	Depth of cut (mm)	0.10	0.15	0.20
D	Work materials	SS	Cu	Al

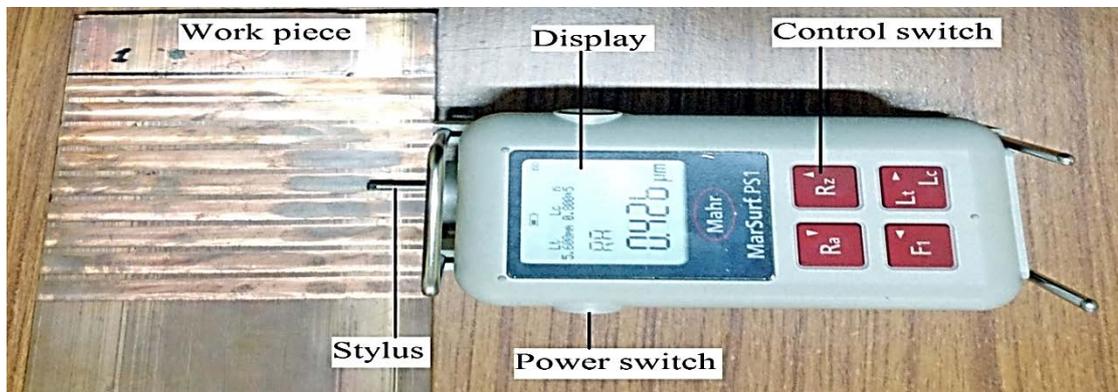
Blocks of copper, aluminium, and stainless steel have been cleaned with sand paper and then a moist cloth to remove any dust or rust particles sticking to it. The top surface of the block has been then milled using old tool on CNC milling machine. Then the sides of the block have been machined in

the same way. The next step facing has been done from one end to another end of blocks along with width as shown in Fig. 3.



**Fig. 3.** End milling of work piece

In this work, surface roughness has been measured by “MarSurf PS1” Surface profilometer manufactured by Mahr GmbH Gottingen (Fig. 4). The MarSurf is a shop floor type surface-roughness measuring instrument, which traces the surface of various machine parts and calculates the surface roughness based on roughness standards, and displays the results in  $\mu\text{m}$ . The work piece is attached to the detector unit of the PS1 which traces the minute irregularities of the work piece surface. The vertical stylus displacement during the trace is processed and digitally displayed on the display of the PS1.



**Fig. 4.** Set up for surface roughness measurement

A scientific approach to plan the experiments is a necessity for efficient conduct of experiments. By the statistical design of experiments, the process of planning the experiment is carried out, so that appropriate data will be collected and analysed by statistical methods resulting in valid and objective conclusions. There are two aspects of an experimental problem: design of the experiments and the statistical analysis of the data.

## 6. Results and Discussion

The experiments have been conducted to study the effect of process parameters over the output response such as surface roughness. The experimental results for surface roughness are given in Table

4. Total 27 experiments have been conducted using Taguchi experimental design methodology and each experiment is simply repeated three times for obtaining S/N values.

**Table 4.** Condition for model

S.N.	Model	Speed (rpm)	Feed (mm/min)	DOC (mm)	Work Material
1	P	2000	400	0.05	SS
2	P	2500	500	0.10	SS
3	P	3000	600	0.20	SS
4	P	3500	700	0.15	SS
5	P	4000	700	0.25	SS
6	Q	2000	700	0.05	Cu
7	Q	2500	600	0.15	Cu
8	Q	3000	700	0.10	Cu
9	Q	3500	500	0.20	Cu
10	Q	4000	400	0.25	Cu
11	R	2000	500	0.05	Al
12	R	2500	700	0.20	Al
13	R	3000	500	0.15	Al
14	R	3500	600	0.10	Al
15	R	4000	500	0.20	Al

**Table 5.** Experimental results of surface roughness

Exp. No.	R1	R2	R3	S/N Ratio	Mean
1	0.868	0.869	0.870	1.2196	0.869
2	0.869	0.870	0.868	*	*
3	0.870	0.869	0.868	*	*
4	0.425	0.426	0.427	7.4118	0.426
5	0.426	0.425	0.427	*	*
6	0.427	0.426	0.425	*	*
7	0.173	0.172	0.174	15.2390	0.173
8	0.172	0.173	0.174	*	*
9	0.174	0.172	0.173	*	*
10	0.649	0.648	0.647	3.7685	0.648
11	0.648	0.649	0.647	*	*
12	0.647	0.648	0.649	*	*
13	0.816	0.815	0.817	1.7662	0.816
14	0.815	0.816	0.817	*	*
15	0.817	0.815	0.816	*	*

16	0.438	0.439	0.437	7.1705	0.438
17	0.439	0.437	0.438	*	*
18	0.437	0.438	0.439	*	*
19	0.504	0.505	0.503	5.9514	0.504
20	0.503	0.505	0.504	*	*
21	0.505	0.504	0.503	*	*
22	0.679	0.680	0.678	3.3626	0.679
23	0.680	0.678	0.679	*	*
24	0.678	0.680	0.679	*	*
25	0.797	0.798	0.796	1.9708	0.797
26	0.798	0.797	0.796	*	*
27	0.796	0.798	0.797	*	*

In Table 6, the formula which has been developed for surface roughness implemented on three different work materials SS, Cu and Al. Work materials have been represented by P, Q and R respectively. For each samples five different value of speed, feed and DOC has been taken. In Table 6 comparisons between experimental results and model predictions for surface roughness done. In Table 6, error is defined as

$$Error = \left| \frac{\text{experimental results} - \text{predictions}}{\text{experimental results}} \right| \times 100(\%) \tag{6.13}$$

**Table 6.** Computation between experimental data and model prediction of surface roughness

S.N.	Experimental value $R_a$ ( $\mu m$ )	Predicted value $R_a$ ( $\mu m$ )	Differences	Error (%)
1	0.930	0.925	0.005	0.53
2	0.869	0.868	0.001	0.11
3	0.816	0.815	0.001	0.12
4	0.797	0.837	0.040	5.00
5	0.710	0.709	0.001	0.14
6	0.775	0.773	0.002	0.25
7	0.426	0.426	0.000	0.00
8	0.438	0.438	0.000	0.00
9	0.504	0.225	0.279	55.0
10	0.156	0.144	0.012	7.69
11	1.049	1.036	0.013	1.23
12	0.873	1.099	0.226	25.0

13	0.648	0.647	0.001	0.15
14	0.679	0.678	0.001	0.14
15	0.479	0.474	0.005	1.04

**Table 7.** Coefficient and index of model of surface roughness of various work materials

Coefficient/index	Stainless steel	Copper	Aluminum
K	30.2743	13.6634	13.6624
T	1.0903	1.1482	1.0426

A Semi-empirical model of surface roughness of work has been established with model parameters consisting of speed, feed and depth of cut. Based on experimental results, the coefficients and the power indexes of equation 12 have been tabulated for each work shown in Table 7. For different work material coefficients and the power indexes are different. It cannot be represented by a single set of coefficients and power indexes for various works in CNC end milling.

### Conclusions

The effects of the process parameters viz. speed, feed, depth of cut and work material, on response characteristic (surface roughness), have been studied. The optimal set of process parameter has been obtained for surface roughness. Buckingham’s Pi-Theorem has been applied for dimensional analysis developing the mathematical models in the form of equations correlating the dependent parameters with the independent parameters (speed, feed, depth of cut) in CNC end milling machine. Based on experimental results, the coefficients and the power index have been calculated. A semi-empirical model of surface roughness of work in CNC end milling machine has established. The final results have shown that the model is dependent on work materials, therefore for different work material coefficients and the power indexes are different. It cannot be represented by a single set of coefficients and power indexes for various works in CNC end milling. Comparison has been prepared for study of experimental result and prediction based on model for surface roughness of work. According to the error analysis, the developed model is reasonably accurate.

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