

## Suggested Algorithm for Estimation of Best Sectional Offer Price to Reach Maximum Revenue Considering Contingency on Transmission Lines

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### Abstract

In this paper, the best estimation way for 2-degree curve of offered price of power plant in the concentrated competitive market is proposed considering the effects of transmission lines contingency on this optimal method, electricity sales and the corresponding plant revenues. The lagrangian iterative method has been used to solve the optimization problem. Finally, the proposed algorithm has been implemented on a 30-bus IEEE system. The results show that, even the type of offered curve is dependent on the number of elements and breakdown points considering the constraint on adjacency of points and recognizing the best point, specifically in nuclear and renewable power plants where the piecewise curves have large degree of freedom and the variety of the domical curves, independent from the number of steam valves of power plant. So this methodology can have a significant effect on the revenues of these power plants.

**Keywords:** Optimization, Local marginal price, 30-bus IEEE system, Transmission lines.

### 1. Introduction

The input-output curves are obtained by some measurements on the various levels of generating unit's output power. Even if much precise measurements will be done, the consequent estimated points usually don't form a smooth line. So, a linear interpolation on this data as a 2-degree function will be adequately acceptable. The Fig.1 depicts a piecewise cost curve and corresponding marginal cost curve. Because any part of the corresponding incremental piecewise curve is linear, any part of the marginal cost curve will be constant. Due to this fact, the unit loading based on energy prices will be simpler. If the price will be equal to the value of one part of the marginal cost; at corresponding interval, this unit can select any value for the level of generation. Marginal cost in any breakdown point is equal to the slope of the next piece; because, it is common that limit-cost is defined as the cost of next 1-MW generation, not the cost of previous 1-MW generation [2]. Optimization in the market can be done by two methods; the first one is the customer's charge optimization and the second one is the cost optimization [5]. For example, in [4] simultaneous optimization of the energy and reserve have been done by heuristic differential algorithm for both previous methods [4]. In optimal dispatching of energy and reserve, considering piecewise prices of power plants, the load flow in the full electricity market has been presented [5]. The manner of piece wiseing the cost curve of units and its effect on

the optimization and increasing the outcome of the generating unit, besides; the public welfare level will not decrease and it can has a significant role in the outcome's share of the unit, that isn't considered as important as here. In this paper, because of the importance of this subject, it has been focused on finding the best breakdown points in order to maximize the revenue of corresponding generating unit, while minimizing the charges paid to power plants, by a new iterative algorithm. This iterative algorithm uses lagrangian method in internal loops. Dc load flow has been used in order to recognize that transmitted power either violates the standard allowed range or not. The simulation studies have been implemented on the 30-Bus IEEE test system. Different piecewise two-part and three-part curves and also the curve that has a constant marginal cost have been analyzed. In various load conditions and the variation in different busses, corresponding curves have been evaluated considering either the existence of constraints or not. Finally, the best suggested case has been chosen.

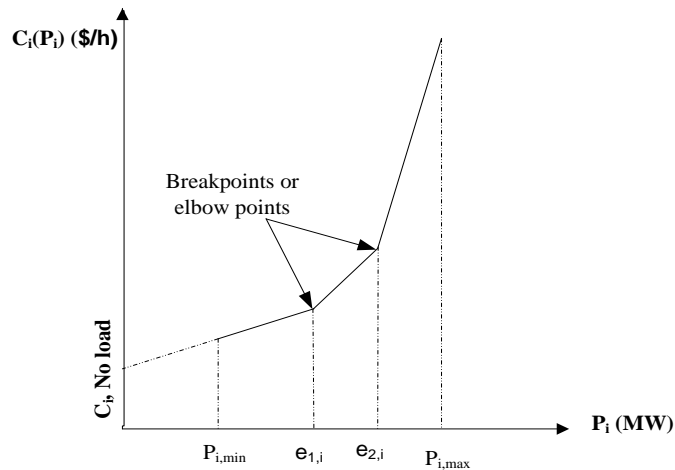


Figure 1: Linear piecewise curve of cost.

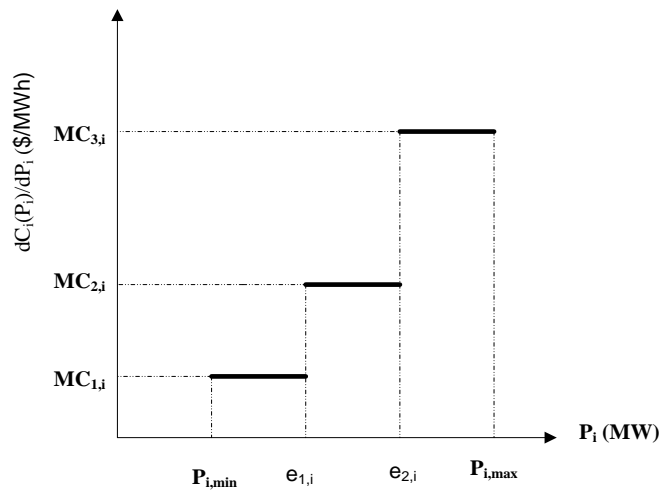


Figure 2: Incremental piecewise constant cost corresponding to the Figure 1.

## 2. Assumptions

### 2.1. Energy and reserve optimization

Since the energy and reserve optimization can be done either simultaneously or separately [3] and the major revenue of units are achieved through energy sales and the revenue yielded by reserves is low. So, because of the slight effect of simultaneous reserve and energy optimization on the type of piecewise curve, it is assumed that both optimization and piece wising the reserve curve have been

done separately and during piece wise the cost curve, the energy market has been considered, exclusively.

## 2.2. Computing the power transmitted through lines

In this study, in order to simplify the computations and reaching accurate convergence to optimal point, the optimization problem can be done based on a liberalized model (namely, dc load flow) instead of using a full model and an ac load flow.

Dc load flow comprises these assumptions:

- Resistance of any branch is negligible in comparison to its reactance.
- The voltage magnitude in any bus is equal to its nominal value.
- Difference between voltages angles in two ends of the line is negligible.

Since these assumptions are well-established in normal conditions of power system will not cause significant errors [2].

- The bus where the generator under study is located is selected as the slack bus.

Since selecting the slack bus is arbitrary, the previous assumption causes: first; the bus angle will be definite and the best optimal case (with regard to line constraints, specifically the lines connected to the bus under study) is fined simply, second; selecting this one as a reference may cause more freedom in generating power due to having swing bus.

- Since the prominent effect on the energy sale is correspond to the lines connected to the unit bus (between all lines), it is assumed that most critical constraint is considered, namely, the constraint on the one of the lines connected to the bus is considered.

## 3. Problem formulation

### 3.1. The combination of lagrangian and iterative algorithm

Usually, the cost function of any power plant is defined by a 2-degree curve using a few points and by applying the least squares method, as described below:

$$C_i = a_i \times P_i^2 + b_i \times P_i + c_i \quad (1)$$

Which any of coefficients  $(a_i, b_i, c_i)$  values are defined in terms of MWh/\$.

The objective function is defined as:

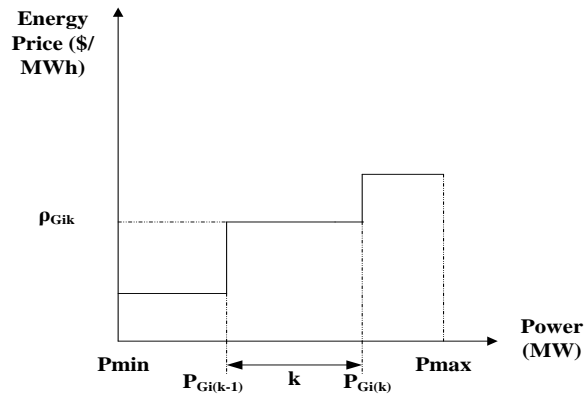
$$\begin{aligned} \text{Minimize } & \sum_{i=1}^N C_i(P_i) \\ \text{Subject to } & \sum_{i=1}^N P_i = L \end{aligned} \quad (2)$$

Which  $C_i$  is the generation cost of any unit dependent on the generation value corresponding to  $P_i$ , it is clear that; by neglecting the losses, summation of these generations should be equal to the network load. In order to do the most suitable piecewise estimation, it is assumed that in the worst case, the ability of the other plants to have piecewise pricing is much flexible; so that, the marginal price in any point is dependent to the power at that point. As a result; even in the worst case, the best operation point can be found; despite the variation of the other power plants, the operation point is recognized as the most effective point. So, depending on the type of corresponding power plant that is power plant No #1 here, incremental cost of any unit in three, two or one power intervals have certain values that will be equal to Lagrange factor, just after the first derivation based on the following relations. The another unit's power can be evaluated, simply; by a linear relation consequent of derivation of the expression with respect to its unit power

$$l = \sum_{i=1}^N C_i(P_i) + meu - \sum_{i=1}^N P_i \tag{3}$$

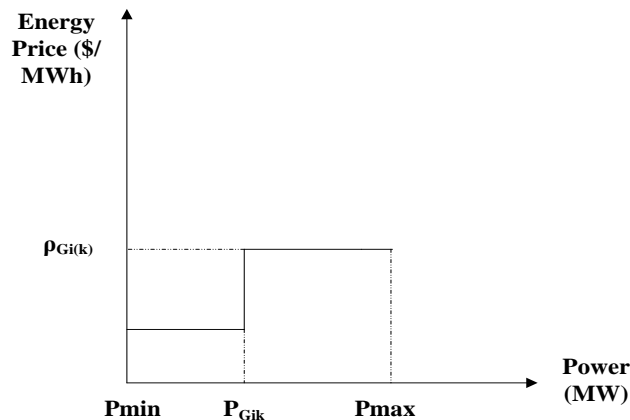
Where  $meu = IC_1$

$IC_1$  is dependent on the slope of piecewise curves and the output power of unit No#1, which is addressed regarding to the Figure 2 –Figure 4 in different intervals.

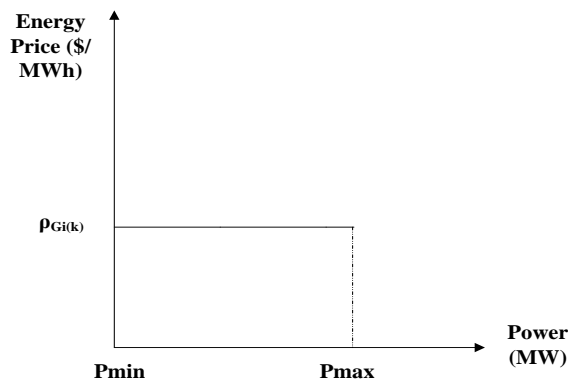


**Figure 3:** Three-piece estimation corresponding to three different values of marginal cost;  $IC_1(1)$ ,  $IC_2(1)$ ,  $IC_3(1)$ , respectively, ( $IC_1(2) = \rho_{Gi(k)}$ ).

In these figures,  $k=md$  shows the minimum distance between breakpoints, because adjacency of units in some power plans (specially steam power plants) has no applicable justification and it is opposed by market operator (MO). In the other words, near adjacency of points cause error in the estimation and lead to unreasonable interpolation.



**Figure 4:** Three-piece estimation by two different cost values  $IC_1(1)$ ,  $IC_1(2)$ , where ( $IC_1(2) = \rho_{Gi(k)}$ ).



**Figure 5:** Uniform estimation considering a marginal cost  $IC_1(1)$  which  $IC_1(1) = \rho_{Gi(k)}$  values of marginal cost;  $IC_1(1)$ ,  $IC_2(1)$ ,  $IC_3(1)$ , respectively, ( $IC_1(2) = \rho_{Gi(k)}$ ).

It is obvious that, in any interval, the values of marginal cost are equal to the lagrange factor or  $meu$ . So, the power of any unit except the unit No#1 is dependent to the marginal cost of unit No#1. In the other words, by assuming  $P_1$  for unit No#1, we have:

$$IC_1 = meu \quad (4)$$

For  $P_2, P_3, \dots, P_N$ :

$$P_i = \frac{meu - b_i}{2 \times a_i} \quad (5)$$

It is obvious that, all ' $P_i$ ' values should be in permitted range which will be introduced during algorithm description [1]. In the figure 2, the numbers of algorithm's unknown variables are two variables,  $e_1$  and  $e_2$ , respectively. In the figure 3 the number of unknown variables is one and the figure 4 hasn't any unknown variable and it is depicted in order to have a comparison. It can be written that:

$$IC = \frac{C(P_{\max}) - C(P_{\min})}{P_{\max} - P_{\min}} \quad (6)$$

So, finding these optimal points is the final aim of this study.

### 3.2. The algorithm discription

It is clear that, considering a lower price for unit No#1 another unit's prices decrease and  $P_1$  increases. If price of this unit increases, another units prices increases and the power of unit No#1 decreases, desperately in order to keep the balance between generation and demand. Here, the goal is to finding the best break-down point of curve that minimizes total cost to increase welfare level, besides; maximum revenue reach to unit No#1. So following steps should be passed in the suggested algorithm:

Step 1: the variable "answer" is 1 or 0, when the line constraints are considered the value is 1 and when the constraints are ignored 0 is replaced. Besides, when "answer=1" allowed bound of corresponding variable should be edited in the program.

Step 2: for different values of  $e_i$  (corresponding break-down point) with the condition of satisfying the minimum distance between points. By a stochastic step for all of the proposed states starting from  $P_{\min}$  upward or from  $P_{\max}$  downward some guesses about these power values is considered. It is obvious the power of unit No#1 between the intervals dependent to these values have different marginal prices.

Step 3: by this assumption the main price curve  $IC_1$  which is equal to lagrange factor is found.

Step 4: by relation (5) the power of another units is found and feasibility of lagrange relation; considering the constraints and bounds that limit the power to be below maximum generation and up the minimum generation. If the constraints have been violated the values are fixed on the boundary values.

Step 5: the value of  $P_1$  is found by condition (2) with regard to values of  $P_i$  ( $i=2,3,\dots,N$ ) which  $N$  is the number of buses. If this value was negative, the power of the another units are high and the lower marginal cost should be considered in the previous operating points. If this value has been violated the allowed interval, as we have started from higher marginal cost; another suggested marginal cost was not been available, so we fix it on the maximum value and the value of exceed power lead to difference between lagrange factors, so:

$$meu_i = 2 \times a_i \times P_i + b_i \quad (7)$$

The exceed power is divided between power plants which have lower values of  $meu_i$ .

Step 6: any revenue value of power plant corresponding to any cost is recorde

Step 7: if the variable of “answer” is 0 go to the last step, else; go to the step 8.

Step 8: the admittance matrix ( $Y_{bus}$ ) of the network is computed using the received data and dc load flow is done using the resultant generated powers and the load of all buses. The active powers passed from the lines are computed using dc load flow.

Step 9: the passing powers of lines is compared to the allowed values. If the power passing through none of lines isn't exceeded the allowed value, go to the end step, else, the power of the line which is located on the bound is fixed on its bound. Considering the assumptions of this study, using this value of power, the angle in second side of bus j can be computed as:

$$\Delta j = 0 - f_{\max}(i, j) \times X_{i,j} = -f_{\max}(1, j) \times X_{1,j} \quad (8)$$

Step 10: the generated power of bus j is equal to the value which fixes the angle at its previous value and because these angles value have the optimal values, so the optimal state of  $P_j$  is computed by multiply the load flow coefficients matrix by this situation's angles, the condition of (7) is obtained.

Step 11: fix the pi in its value in the step 11, for another buses repeat the steps 1 to 9 in order to get the power of another buses. In step 8, it is regarded the number of unknown bus angles is reduced by one, so the coefficients matrix has (n-1)-1 rows and columns. It is clear that the slack bus isn't considered in this matrix. Also, in the step 9 the bound is satisfied and we go the final step.

Final step: in any iteration, the new revenue is computed and it is compared with the previous one. If this revenue is more, it should be fixed on its maximum value and all of the parameters that lead to this consequent should be recorded, also, this process is continued for all of estimations. Once the iterations are ended (nor=number of repeat=100), the mean value of all best states are computed or the state with maximum revenue (Max total) is selected as the best piece-wise offer to interpolate in this system.

#### 4. Simulation

The proposed algorithm is implemented to find the best point of unit No#1 in the IEEE 30-Bus system which is introduced in [6]. Total available load in this system is 283.4 MW. The power and 2-degree curves of all units are considered in “g” matrix. The minimum permitted distance between two points is considered to be 30 MW and when considering the constraints, it is assumed that the line between Bus #1 and Bus #2 is capable to carry the power of 1.2 p.u. (namely 120-MW in the bases of 100 kVA) [4]. The optimization goal is to find the piece-wise estimation of the unit No#1 in the system for all conditions, and to compare the consequent revenue of units.

By starting from Pmin, incrementally and stochastically and by starting from the Pmax descending and stochastically is implemented for the curves depicted in the figure 3-5, and the optimal state in any curve have been selected. The dc load flow procedure is based on the study in [1].

In a system without transmission constraints, if all of the generators are assumed to have fixed marginal costs, all of the generators, except one generator are located on the zero power or maximum power point. The exception is corresponding to the marginal generator which equalize between generation and demand. It is said that this generator is carrying part-load because it is responsible for supplying exceed estimated power, which is determinative of the marginal cost [2].

Based on the previous assumptions, the exceed 1-MW power in the bus is supplied by its generator and the local marginal price (LMP) is equal to the incremental cost of unit.

## 5. Results

Since incremental cost of unit is considered just for the unit No#1 (that is considered piecewise); and the incremental cost of another units is dependent on the generation point; the condition which forces all units except the slack unit not to locate on the point of zero generation or maximum generation is not established and it is perhaps that in addition to the slack unit, some of the another ones carry negligible power. By starting from the Pmin or Pmax and elapsing 100 stochastic incremental or decreasing iterations between this two values for any curve, the results have been presented in table 1, by neglecting any constraints on the lines. The results for such problem considering the line constraints have been presented in table 2.

**Table 1:** The interpolated values for the curve and consequent revenue by neglecting the constraints.

Estimation type	The optimal interpolation values for linearization	The output power of 1-MW unit	Revenue (\$/h)
Three-piece	$e_1 = 80, e_2 = 170$	185.5143	628.4296
Two-piece	$e = 170$	185.5570	628.4282
Constant	-----	122.1429	525.2143

**Table 2:** The interpolated values for the curve and consequent revenue considering the constraints.

Estimation type	The optimal interpolation values for linearization	The output power of 1-MW unit	Revenue (\$/h)
Three-piece	$e_1 = 80, e_2 = 170$	172.0836	582.9333
Two-piece	$e = 170$	172.0836	582.9333
Constant	-----	122.1429	525.2143

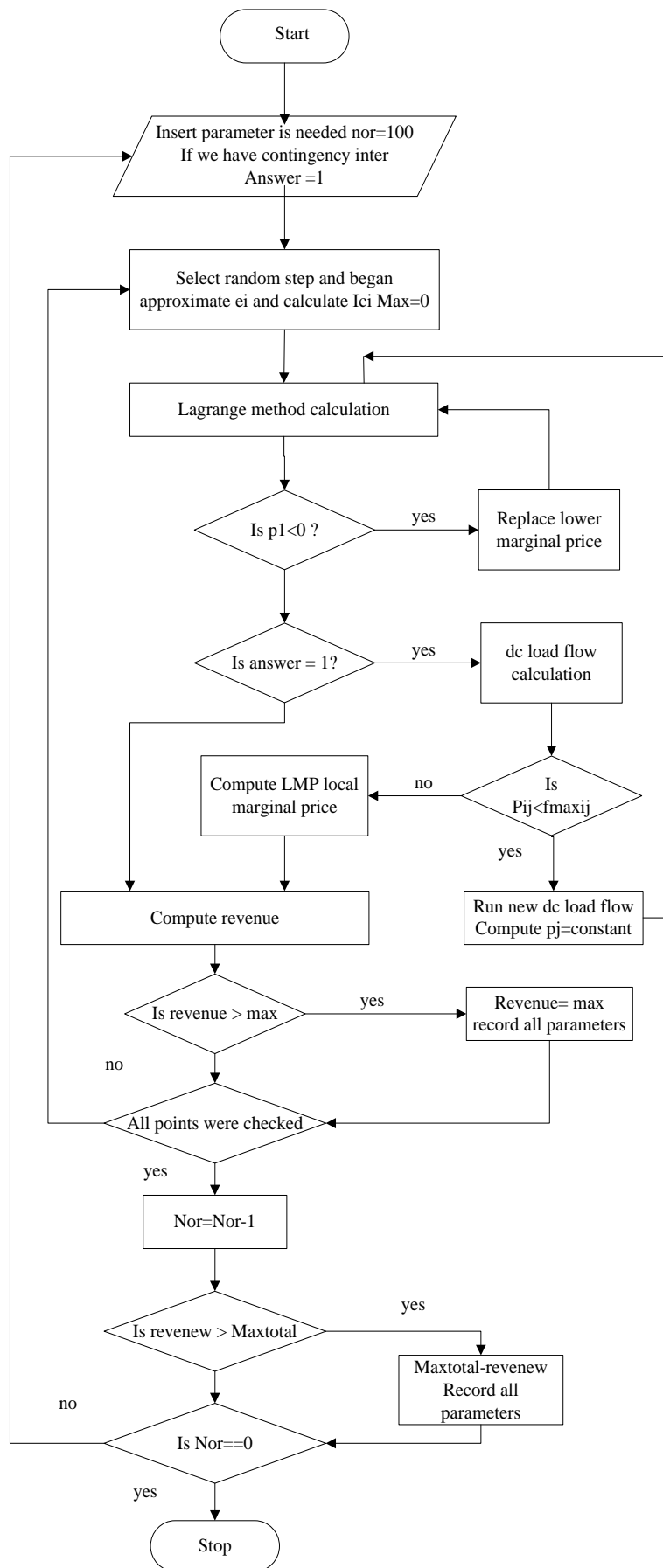
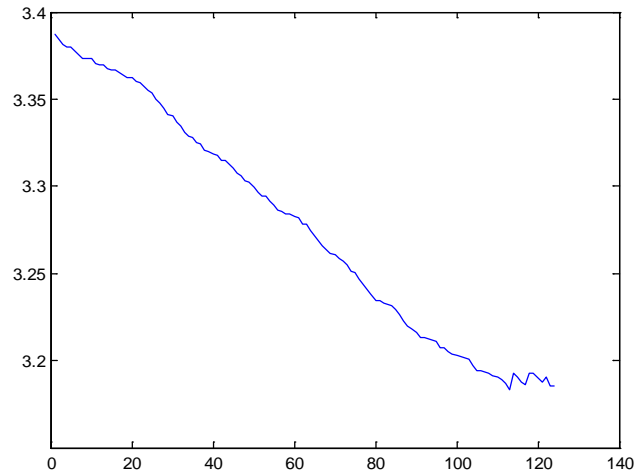
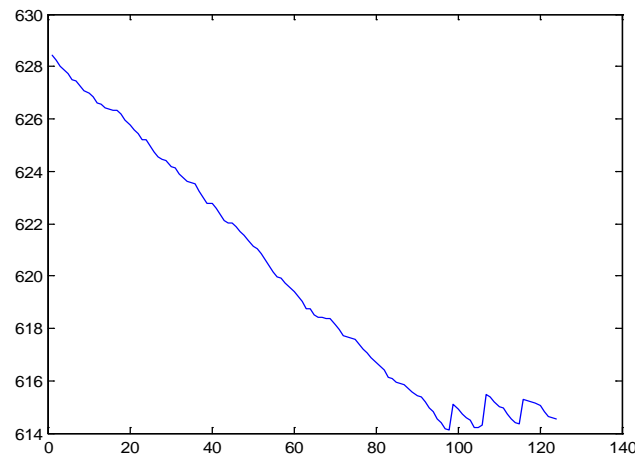


Figure 6: Flowchart of resolving the Lagrange and iteration problem.

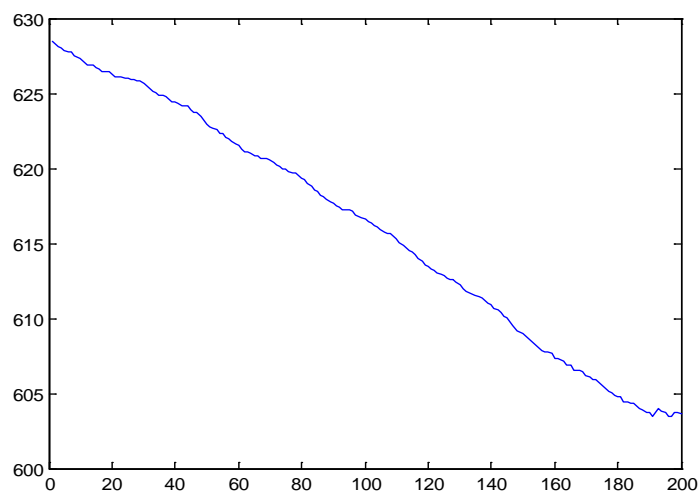




**Figure 7:** Marginal cost of unit No#1, as the interpolation started from  $P_{max}$  toward  $P_{min}$ , neglecting the transmission constraints.



**Figure 8:** The revenue variation in 30-Bus system, using three piece estimation. Vertical axis is revenue and horizontal axis is the iteration, the estimation started from  $P_{max}$  toward  $P_{min}$ , considering the transmission constraints.



**Figure 9:** The revenue variation of unit No#1, two-piece estimation, vertical axis is revenue and horizontal axis is started from  $P_{max}$ , descending.

## Discussion

It can be seen that, the resultant optimal point lead to the best state for the system. As we are looking forward maximizing the revenue, in companion with not increasing the welfare level, we are searching to maximize the ratio of unit No#1 revenue to the difference of this revenue from the total payment. With regard to figure 7, the reason why  $e_2$  is located in its maximum point is that by starting from  $P_{max}$  and reducing the approximation toward the lower points lead to decrease in the unit's cost, besides; because of the equality between demand and generation, the load should be supplied from the units which have upper costs. So the dominator is reduced in the (6), so the answer is getting away from the optimal answer. When the constraints are neglected, because of the limit on the line 1-2 the power of unit No#1 is decreased. In this situation, the generators carry a negligible load. In the other hand, without limit on the LMP price, the cost of any unit for sell in all busses is identical. It is seen that, piece wising the prices in the market may lead to higher benefits and further revenues for the unit in any working hour.

The figure 8 shows variation of the unit No#1 revenue, by three-piece estimation. The unit No#1 has the upper possible point while doing the estimation, because of the lower marginal cost. But in lower powers, because of the ineffectiveness of  $a_i$  coefficient, revenue isn't much and selects a lower proposed price in order to reduce the total paid cost. Fluctuations of the curve are related to the violating from the rated power of power plant or generation under permitted value of power plan power that are fixed on the bounds and lead to instantaneous decrease or increase of the price. In the figure8 the fluctuations of price are more than the fluctuations in the figure 7 because of three-piece wising and instantaneous jump of price in the revenue curve. This estimation isn't dependent on the demand variations, significantly. If the system's load increases to 4 p.u. the capacity is fixed on the upper bound (200-MW) and the best estimation considering allowed ranges in MW appears in the previous point. The reason of invariability can be found in the cost curve and optimality of the total payment.

## Conclusion

The proposed method prepares a suitable estimation of the best offer that a unit can attain for energy selling. In order to have a good effectiveness for the methodology, the reserve and transmission cost and minimization of the pay to customers can be analyzed considering several constraints in the main lines and considering active losses in different situations of load variation. The proposed price in most situations can lead to most probable profit in comparison to all another prices.

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