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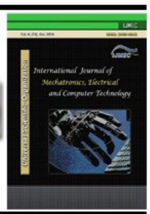

# Adaptive Fuzzy-Neural Control for a class of Nonlinear Time-Delay Systems using a State Observer

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## Abstract

In this paper, an observer-based adaptive fuzzy-neural control is presented for a class of nonlinear systems with unknown time delays. The state observer is first designed, and then the controller is designed via the adaptive fuzzy-neural control method based on observer states. Both the designed observer and controller are independent of time delays. Using an appropriate Lyapunov-Krasovskii functional, the uncertainty of the unknown time delay is compensated, and then the fuzzy-neural system is utilized to approximate the unknown nonlinear functions. Based on the Lyapunov stability theory, the constructed observer-based controller and the closed-loop system are proved to be asymptotically stable. The designed control law is independent of the time delays and has a simple form with only one adaptive parameter vector, which is to be updated on-line. Simulation results are presented to demonstrate the effectiveness of the proposed approach.

***Keywords*:** Adaptive fuzzy-neural control, state observer, nonlinear systems, time delay, stability.

## 1. Introduction

In most of the recent works in adaptive fuzzy controllers the parameters of the consequent part of the fuzzy controller were assumed free and were tuned by adaptive laws derived using the Lyapunov method, which also guaranteed stability of the system [1; 2]. It is well known that time delays are frequently encountered in real engineering systems. Time delay usually leads to unsatisfactory performance. Therefore, the problem of stabilization of time delay systems has received considerable attention over the past years [3; 4; 5]. To overcome this difficulty, the Lyapunov-Krasovskii functional is used for stability analysis and synthesis [6]. A typical approach for the analysis and synthesis of nonlinear system with time delay is the local linearization approach. First, a linearization model at the nominal operating point is obtained, and then a linear feedback control is designed for this linear model. In particular, some delay-independent stability conditions and stabilization approaches have been proposed for these linear delay differential equations [7; 8]. It is known that each local model is valid only for a certain range of operating conditions and these results can only guarantee the local stability of nonlinear systems with time delays. The adaptive neural controller was designed for a class of nonlinear time-delay systems [11; 12]. However, these adaptive neural control methods require a large number of neural weights to be adapted online simultaneously. This makes the learning time unacceptably lengthy. Fuzzy logic system is employed to approximate the unknown nonlinear functions, and then the adaptive law of adjustable parameters is obtained. In fact, most of the works in the fuzzy-neural control are dedicated to the control problem for the affine nonlinear systems, that is, systems characterized by inputs appearing linearly in the system input-output equation. In this paper, a novel dynamic model approximation method in first proposed to approximate the nonlinear dynamics. Then we combine the FLSs and NNs, and adaptive techniques proposed an adaptive fuzzy-neural observer for nonlinear systems.



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Because in many control problems, state variables may be partly unavailable. On the other hand, the problem of observer design for reconstructing state variables is a more involved issue in system with any kind of delay. In general, some sufficient conditions for the existence of an observer have been established, and computational algorithms for construction of the observers have been presented in [13; 14].

The main features of this paper are: 1) State observer is first designed, and the controller is designed via adaptive fuzzy-neural control method based on the observed states. 2) Both the designed observer and controller are independent of the time delay.

The paper is organized as follows. The problem under investigation and the fuzzy-neural system are introduced in Section 2. The observer-based adaptive fuzzy-neural control design is introduced in Section 3. The main result is presented in Section 4. Simulation results are provided in Section 5. Conclusions are given in Section 6.

## 2. Problem Statement and Preliminaries

### 2.1. Control problem statement

Consider the nonlinear time-delay dynamic system in the following form [13]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(x) + g(x)u(t) + h(x(t-\tau)) \\ y = x_1 \end{cases} \quad (1)$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \Re^n$ is the system state vector, $u \in \Re$ and $y \in \Re$ denote system control input and output, respectively.

Functions $f(x)$, $g(x)$ and $h(x(t-\tau))$ are unknown smooth functions, $\tau_i$ is an unknown time delay of the state variables, $\bar{y}_d$. It is assumed that the desired output trajectory and its derivatives $Y_d = [y_d, \dot{y}_d, \ldots, y_d^{(n-1)}]^T$ are measurable and bounded and $y_d^{(n-1)}$ denotes the $(n-1)-th$ derivative of $y_d$ with respect to time.

Let $\hat{x} = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_n]^T \in \Re^n$ be the estimation of the system state vector. Define error vector $e$, estimation error vector $\hat{e}$, and observation error $\tilde{e}$, respectively as

$$\begin{aligned} e &= x - Y_d = [e_1, e_2, \cdots, e_n]^T \\ \hat{e} &= \hat{x} - Y_d = [\hat{e}_1, \hat{e}_2, \cdots, \hat{e}_n]^T \\ \tilde{e} &= e - \hat{e} = [\tilde{e}_1, \tilde{e}_2, \cdots, \tilde{e}_n]^T \end{aligned} \quad (2)$$

The filtered tracking error $e_s$, estimation error $\hat{e}_s$, and observation error $\tilde{e}_s$ are defined respectively as

$$e_s = \left(\frac{d}{dt} + \lambda\right)^{n-1} e_1 = [\Lambda_1^T \quad 1]e \quad (3)$$

$$\hat{e}_s = \left(\frac{d}{dt} + \lambda\right)^{n-1} \hat{e}_1 = [\Lambda_1^T \quad 1]\hat{e} \quad (4)$$

$$e_s = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{e}_1 = [\Lambda_1^T \quad 1]\tilde{e} \quad (5)$$

where $\Lambda_i = [\lambda_i^{n-1}, \lambda_i^{n-2}, \ldots, \lambda_i]^T$, $i = 1, 2, \ldots, n$ and $\lambda_i > 0$ are positive contants which can be specified by the designer.

The control objective is to design an observer based adaptive fuzzy-neural tracking controller for system (1) such that the system output $y$ tracks the desired reference signal $y_d$ while all the signals in the closed-loop system remain bounded.

**Remark 1:** The equality (4) has the ollowing properties:

1) When $\hat{e}_s = 0$, it defines a time-varying hyperplane in $\Re^n$ on which the estimated tracking error $\hat{e}_1$ converges to zero eventually.

2) When $\hat{e}_s$ is bounded, the estimated tracking error vector $\hat{e}_s$ is also bounded. These properties are helpful for stability analysis.

We have the following assumptions for the system's signals, unknown functions and reference signals.

**Assumption 1.** The desired trajectory vector $\bar{y}_d$ given by $\bar{y}_d = \left[Y_d^T \quad y_d^{(n)}\right]^T \in \Omega_d \subset \Re^{n+1}$ with $\Omega_d$ being a known compact set is continuous and available.

**Assumption 2.** The unknown time delay is bounded by a known constant, *i.e.*, $\tau_i \le \tau_{\max}$, $i = 1, 2, \ldots, n$.

### 2.2. Fuzzy-neural modelling

In this subsection, in order to develop an adaptive control law to adjust the parameters of fuzzy-neural networks so that estimation $e(t)$ converge to compact set, we will introduce a modelling method that can be used to approximate the nonlinear function $f(x)$.

The basic configuration of fuzzy logic systems consists of fuzzy logic and neural network. The fuzzy logic can be devided into some fuzzy IF-THEN rules and a fuzzy inference engine. From an input linguitic vector $x = [x_1, x_2, \ldots, x_n]^T \in \Re^n$ to an output linguistic variable, the fuzzy inference engine can perform a mapping $y \in \Re$ by using the fuzzy IF-THEN rules. The $i$ th fuzzy IF-THEN rule can be written as

$R^l: If\ x_1\ is\ F_1^l\ and\ x_2\ is\ F_2^l\ and \ldots and\ x_n\ is\ F_n^l,\ Then\ q\ is\ G^l$ where $F_1^l, F_2^l, \ldots, F_n^l$ and $G^l$ are fuzzy sets with membership variable which can be considered as output the fuzzy-neural system. By using product inference, center-average and singleton fuzzifier, the output of the fuzzy logic system can expressed as

$$q(x) = \frac{\sum_{l=1}^{M} \bar{q}^l \prod_{i=1}^{n} \mu_{F_i^l} x_i}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{F_i^l} x_i} \tag{6}$$

where $M$ the number of IF-THEN rules in the fuzzy rule base. The IF-THEN rules take the following form for $l = 1, 2, \ldots, M$.

$\mu_{F_i^l}$ is the membership function value of the fuzzy variable. Parameter $\bar{q}^l$ is the point at which $\mu_{G^l}(\bar{q}^l)$ achieves its maximum value and we assume that

$$\mu_{G^l}(\bar{q}^l) = 1 \tag{7}$$

equality (6) can be rewritten as

$$q(x) = \psi^T \xi(x) \tag{8}$$

where $\psi = [\psi_1, \psi_2, \ldots, \psi_M]^T$ is parameter vector, and $\xi(x) = \left[\xi^1(x), \xi^2(x), \ldots, \xi^M(x)\right]^T$ is a regressive vector with regressor $\xi^l(x)$, which is defined as a fuzzy basis function (FBF) of the form

$$\xi^l(x) = \frac{\prod_{i=1}^{n} \mu_{F_i^l\, x_i}}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{F_i^l\, x_i}} \tag{9}$$



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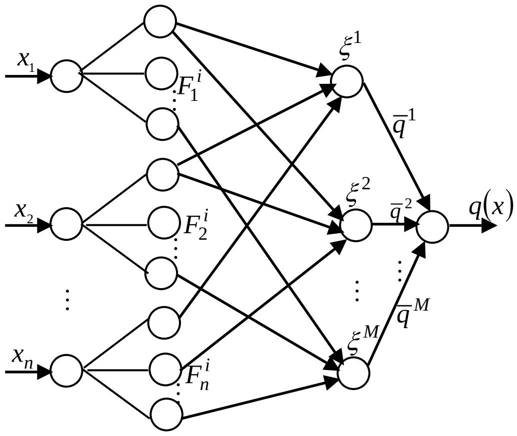


**Fig. 1.** Configuration of fuzzy neural networks

When the inputs are given into the fuzzy-neural network shown in Fig. 1, the truth value $\xi_l$ (layer III) of the antecedent part of the $i$th implication is calculated by (9). Among the commonly used defuzzification strategies, the outputs (layer IV) of the fuzzy-neural system are expressed as (8). The fuzzy logic approximator based on neural networks can be established. The approximator has four layers. At layer I, nodes which are input ones stand for input linguistic vector $x=[x_1, x_2, \ldots, x_n]^T$. At layer II, nodes represent the values of the membership function of total linguistic variables. Each node of layer II performs a membership function value. At layer III, nodes are the values of the fuzzy basis vector $\xi$. Each node of layer III performs a fuzzy rule. The links between layer III and layer IV are full connected by the weighting vector, $\psi=\left[\bar{q}^1, \bar{q}^2, \ldots, \bar{q}^M\ \right]^T$, i.e., the adjusted parameters. At layer IV, the outputs represent the values of the output $q(x)$ [23, 24].

## 3. Observer based Adaptive Neuro-Fuzzy Controller

As the state vector $x$ is unavailable, an observer needs to be introduced in the control structure. $\hat{x}=[\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]^T=\left[\hat{x}_1, \dot{\hat{x}}_1, \ldots, \hat{x}_1^{(n-1)}\right]^T$ will be used to estimate $x$. Our objective is to develop a state observer so that the estimated state vector $\hat{x}$ and the state vector $x$ can be approximate as possible. The observer proposed in this paper takes the following form:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + k_1(x_1-\hat{x}_1) \\ \dot{\hat{x}}_2 = \hat{x}_3 + k_2(x_2-\hat{x}_2) \\ \vdots \\ \dot{\hat{x}}_n = f(\hat{x})+g(\hat{x})u(t)+k_n(x_1-\hat{x}_1) \end{cases} \qquad (10)$$

where parameters $k_i$ $(i\in[1,\ldots,n])$ are the observer gains, which are selected to make sure that the characteristic polynomial $s^n+k_n s^{n-1}+k_{n-1}s^{n-2}+\ldots+k_1=0$ is Hurwitz.

**Assumption 3.** For $1\le i\le n$, the signs of $g(\hat{x})$ are known, and there exist unknown positive constants $b$ and $c$ such that $0<b\le|g(\hat{x})|\le c<\infty$, $\forall\ x\in\Re^i$. Without loss of generality, it is assumed that $g(\hat{x})\ge b>0$.

**Remark 2.** It should be emphasized that bounds $b$ and $c$ are only required for analytical purposes, their true values are not necessarily known since they are not used for controller design.

**Assumption 4.** The unknown functions $f(.)$, $i=1,2,\ldots,n$ can be expressed as a fuzzy-neural system of the form (8), i.e.,

$$f(\hat{x})=f(\hat{x}/\theta_f)=\theta_f^T\,\xi(\hat{x}) \qquad (11)$$

where $\theta_f^T$ is the estimation of the unknown parameter vector and $\xi(\hat{x})$ is the associated fuzzy basis function.

From (1) and (3), we obtain

$$\dot{e}_s = f(x) + g(x)u(t) + h(x(t-\tau)) + v_1 \qquad (12)$$

where $v_1 = [0 \quad \Lambda_1^T]\ e - y_d^{(n)}$

From (4) and (10), we also obtain

$$\dot{\hat{e}}_s = \theta_f^T\ \xi(\hat{x}) + g(\hat{x})u + v_2 \qquad (13)$$

where $v_2 = [0 \quad \Lambda_2^T]\ \hat{e} - y_d^{(n)} + k_n e_1.$

Subtracting (13) from (12) yields

$$\dot{\tilde{e}} = f(x) - f(\hat{x}) + (g(x) - g(\hat{x}))u + (v_1 - v_2) + h(x(t-\tau)) \qquad (14)$$

Define the minimun approximation error as

$$\omega = (f(x) - f(\hat{x})) + (g(x) - g(\hat{x}))u. \qquad (15)$$

Equality (14) can be rewritten as

$$\dot{\tilde{e}} = \omega + v_3 + h(x(t-\tau)) \qquad (16)$$

where

$$v_3 = v_1 - v_2.$$

If the smooth function is chosen as

$$V_{es} = \frac{1}{2}\left(\hat{e}_s^2 + \tilde{e}_s^2\right) \qquad (17)$$

then its time derivative along (13) and (14) is given by

$$\dot{V}_{es} = \hat{e}_s\left(\theta_f^T\ \xi(\hat{x}) + g(\hat{x})u + v_2\right) + \tilde{e}_s(\omega + h(x(t-\tau)) + v_3) \qquad (18)$$

By the triangular inequality, we have

$$\tilde{e}_s\ h(x(t-\tau)) \le \frac{\tilde{e}_s^2}{2} + \frac{h^2(x(t-\tau))}{2} \qquad (19)$$

subtituting (19) into (18) yields

$$\dot{V}_{es} \le \hat{e}_s\left(\theta_f^T \xi(\hat{x}) + g(\hat{x}) + v_2\right) + \tilde{e}_s\left(v_3 + \omega + \frac{\tilde{e}_s}{2}\right) + \frac{h^2(x(t-\tau))}{2} \qquad (20)$$

In order facilitate the procedure in the presence of the unknown time-delay, the following Lyapunov-Krasovskii functional is considered:

$$V_U = \frac{1}{2}\int_{t-\tau}^{t} h^2(x(\tau))\,d\tau \qquad (21)$$

Then it follows from (20) and (21) that

$$\dot{V}_{es} + \dot{V}_U \le \hat{e}_s\left(\theta_f^T \xi(\hat{x}) + g(\hat{x})u + L(Z)\right) \qquad (22)$$

where

$$L(Z) = v_2 + \frac{h^2(x)}{2\,\hat{e}_s} + \frac{\tilde{e}_s}{\hat{e}_s}\left(v_3 + \omega + \frac{\tilde{e}_s}{2}\right) \qquad (23)$$

with $Z = [\hat{x}^T \quad y_d^T]^T \in \Omega_Z \subset \Re^{2n+1}$ and $\Omega_Z$ being a compact set. Because of containing unknown function $h(x(t))$, the last two terms in (23) cannot be used directly to construct the control law $u$. In addition, the last two terms in (23) cannot be approximated by the fuzzy-neural system because it is not well defined when $\hat{e}_s = 0$. To make the fuzzy approximation efficient, we define compact sets $\Omega_Z^0$ and $\Omega_{C_s} \subset \Omega_Z$ as

$$\Omega_Z^0 = \Omega_Z - \Omega_{C_S} \quad (24)$$

$$\Omega_{C_s} = \left\{ \hat{e}_s : | \hat{e}_s | < C_s \right\} \quad (25)$$

where $C_s$ is a positive design constant that can be chosen arbitrarily small and the sign "-" in (24) denotes the complement of set $\Omega_{C_s}$ in set $\Omega_Z$ . Moreover, it has been proven that $\Omega_Z^0$ is a compact set on which the unknown function $L(Z)$ is continuous. Therefore, the fuzzy system (8) can be used to approximate $L(Z)$ over the compact set $\Omega_Z^0$ such that

$$L(Z) = \theta_L^T \xi(Z) + \delta(Z) \quad (26)$$

where $\delta(Z)$ is the approximation error and satisfies $|\delta(Z)| \le \varepsilon$ with $\varepsilon$ being an arbitrarily small constant.

## Main Results

In this section, the boundedness of the closed-loop signals is proved using the Lyapunov function approach [9].

**Theorem 1.** For the nonlinear system (1), if the adaptive fuzzy-neural control is chosen as

$$u = \begin{cases} -\alpha_1 \hat{e}_s - \hat{\theta}_f^T \xi(\hat{x}) - \hat{\theta}_L^T \xi(Z) - \dfrac{\hat{e}_s}{2}, & \hat{e}_s \in \Omega_Z^0 \\ 0, & \hat{e}_s \in \Omega_{C_s} \end{cases} \quad (27)$$

where $\alpha_1 > 0$ is any positive constant, and the parameters are updated by

$$\dot{\hat{\theta}}_f = \begin{cases} \gamma_1 \xi(\hat{x}) \hat{e}_s, & \hat{e}_s \in \Omega_Z^0 \\ 0, & \hat{e}_s \in \Omega_{C_s} \end{cases} \quad (28)$$

$$\dot{\hat{\theta}}_L = \begin{cases} \gamma_2 \xi(Z) \hat{e}_s, & \hat{e}_s \in \Omega_Z^0 \\ 0, & \hat{e}_s \in \Omega_{C_s} \end{cases} \quad (29)$$

where $\gamma_1$ and $\gamma_2$ are positive design parameters, then for any initial conditions $Z(0), \hat{\theta}_f(0)$ and $\hat{\theta}_L(0)$, all the signals in the closed-loop system are bounded, and the estimated tracking error $\hat{e}_s$ will stay in the compact set $\Omega_{C_s}$ finally.

**Proof.** To show that $\Omega_Z^0$ is a domain of attraction, we first find a Lyapunov function candidate $V(t) > 0$ such that $\dot{V}(t) \le 0$, $\forall\ \hat{e}_s \notin \Omega_Z^0$. For $\hat{e}_s \notin \Omega_Z^0$, let us consider the following Lyapunov candidate:

$$V = V_{es} + V_U + \frac{r_1}{2\gamma_1} \tilde{\theta}_f^T \tilde{\theta}_f + \frac{r_2}{2\gamma_2} \tilde{\theta}_L^T \tilde{\theta}_L^T. \quad (30)$$

where $r_1$ and $r_2$ are positive constants, $\tilde{\theta}_f = \hat{\theta}_f - \theta_f^*$, and $\tilde{\theta}_L = \hat{\theta}_L - \theta_L^*$, where the "*" denotes the optimal estimations.

**Assumption 5.** The optimal estimations $\theta_f^*$ and $\theta_L^*$ of the parameters of $\theta_f$ and $\theta_L$ are assumed to have the forms $\| \theta_f^* \| = \alpha \| \theta_f \|$ and $\| \theta_L^* \| = \beta \| \theta_L \|$, where $\alpha$ and $\beta$ are arbitrary constants.

The time derivative of (30) is given by

$$\dot V = \dot V_{es} + \dot V_U + \frac{r_1}{\gamma_1}\tilde\theta_f^T\,\dot{\hat\theta}_f + \frac{r_2}{\gamma_2}\tilde\theta_L^T\,\dot{\hat\theta}_L^T. \qquad (31)$$

Now, using (22), (28) and (29), we get

$$\dot V \le \hat e_s\, g(\hat x)u(t) + \hat e_s\hat\theta_f^T\,\xi(\hat x) + \hat e_s\,\hat\theta_L^T\xi(Z) + \hat e_s\,\delta(Z) + r_1\,\tilde\theta_f^T\hat e_s\,\xi(\hat x) + r_2\,\tilde\theta_1^T\,\hat e\,\xi(Z). \qquad (32)$$

using Assumption 2 and the following triangular inequality

$$\hat e_s\,\delta(Z) \le \frac{b\,\hat e_s^2}{2} + \frac{\varepsilon^2}{2\,b} \qquad (33)$$

it can be easily verified that

$$\dot V \le -\alpha_1 b\,\hat e_s^2 - b\,\hat e_s\,\hat\theta_f^T\,\xi(\hat x) - b\,\hat e_s\,\hat\theta_L^T\xi(Z) - \frac{b\,\hat e_s^2}{2} + \frac{\varepsilon^2}{2b} + \hat e_s\hat\theta_f^T\,\xi(\hat x) + \hat e_s\hat\theta_L^T\,\xi(Z) + r_1\left(\hat\theta_f^T - \theta^{*T}_f\right)\hat e_s\,\xi(\hat x) + r_2\left(\hat\theta_L^T - \theta^{*T}_L\right)\hat e_s\,\xi(Z) \qquad (34)$$

where the control law (27) has been substituted in (34)

Equality (34) can be reduced to

$$\dot V \le -\alpha_1 b\,\hat e_s^2 + \frac{\varepsilon^2}{2b} + \left(\frac{r_1 - b + 1}{r_1}\hat\theta_f^T - \theta^{*T}_f\right) r_1\,\hat e_s\,\xi(\hat x) + \left(\frac{r_2 - b + 1}{r_2}\hat\theta_L^T - \theta^{*T}_L\right) r_2\;\hat e_s\,\xi(Z). \qquad (35)$$

By using assumption (5), (35) becomes

$$\dot V \le -\alpha_1 b\,\hat e_s^2 + \frac{\varepsilon^2}{2b}. \qquad (36)$$

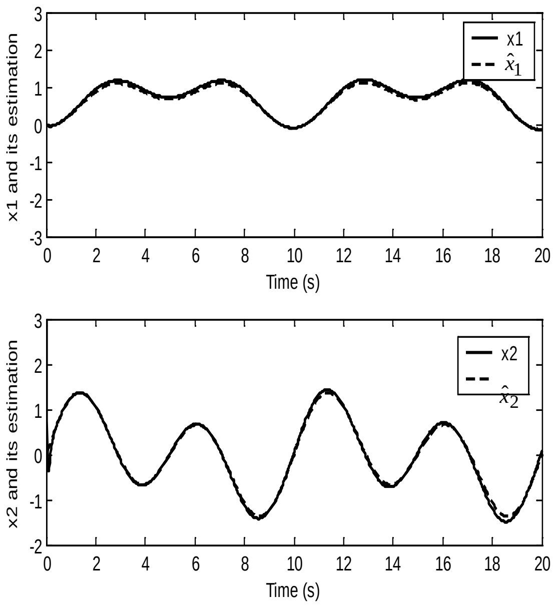


**Fig. 2.** System states and their estimations

As $\varepsilon$ is a small positive constant representing the approximation error in $L(Z)$, we conclude that $V(t)$ is a Lyapunov function. Therefore $\hat{e}_s(t)$, $\hat{x}_1(t)$, $\hat{\theta}_f^T$ and $\hat{\theta}_L^T$ are bounded. In addition, the domain $\Omega_Z^0$ is attractive in the sense that $\hat{e}_s$ will be driven to $\Omega_Z^0$ in a finite time, and then afterwards stays within it. For $\hat{e}_s \in \Omega_{C_s}$ since $\hat{e}_1 = \hat{x}_1 - y_d$, $\hat{\theta}_f = 0$, and $\hat{\theta}_L = 0$, $\hat{x}_1$ is bounded, $\hat{\theta}_f^T$ and $\hat{\theta}_L^T$ are not unchanged in bounded values. We can readily conclude that the tracking error $\hat{e}_s \in \Omega_{C_s}$ while all the other closed-loop signals are bounded.

## Simulation Results

The following example is given to verify the effectiveness of the proposed approach. We consider the following nonlinear time-delay system:

$$\begin{cases} \dot{x} = x_2 \\ \dot{x}_2 = \dfrac{1-e^{-x_1}}{1+e^{-x_1}} + \left(x_2^2 + 2\,x_1\right)\sin x_2 - 0.5\,x_1 u(t) + h(x(t-\tau)) \\ y = x_1 \end{cases} \qquad (37)$$

where $x_1$ and $x_2$ denote the state variables, $u$ is the system control input, $y$ is the system output, the time-delay term is $h(x(t-\tau)) = 2\,x_1(t-\tau)\,x_2(t-\tau)$. In this example, we choose $\tau = 2$ with the bound $\tau_{\max} = 2$ and the desired reference signal $y_d = \sin(t) + \sin(0.5\,t)$. The desired observer takes the following form:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + k_1\,(x_1 - \hat{x}_1) \\ \dot{\hat{x}} = \hat{x}_1 + \dfrac{1-e^{-\hat{x}_1}}{1+e^{-\hat{x}_1}} + \left(\hat{x}_2^2 + 2\hat{x}_1\right) - 0.5\hat{x}_1 u(t) + k_2\,(x_1 - \hat{x}_1) \end{cases} \qquad (38)$$

The control objective is to design an adaptive fuzzy-neural tracking controller for system (37) such that the system output $y$ tracks the desired reference signal $y_d$ while all the signals in the closed loop system remain bounded. Vector Z is defined as $Z = [x_1,\ \hat{x}_2,\ y_d,\ \dot{y}_d,\ \ddot{y}_d]^T$.

Seven Gaussian membership functions with centers evenly spaced between $[-1.5,\ 1.5]$ for variables $x_1$, $\hat{x}_2$, $y_d$, $\dot{y}_d$ and $\ddot{y}_d$ are chosen as

$$\begin{cases} \mu_{F_i^1} = \exp\left(-\dfrac{0.5\,(Z_i - 1.5)^2}{4}\right) \\ \mu_{F_i^2} = \exp\left(-\dfrac{0.5\,(Z_i - 1)^2}{4}\right) \\ \mu_{F_i^3} = \exp\left(-\dfrac{0.5\,(Z_i - 0.5)^2}{4}\right) \\ \mu_{F_i^4} = \exp\left(-\dfrac{0.5\,Z_i^{\,2}}{4}\right) \\ \mu_{F_i^5} = \exp\left(-\dfrac{0.5\,(Z_i + 0.5)^2}{4}\right) \\ \mu_{F_i^6} = \exp\left(-\dfrac{0.5\,(Z_i + 1)^2}{4}\right) \\ \mu_{F_i^7} = \exp\left(-\dfrac{0.5\,(Z_i + 1.5)^2}{4}\right) \end{cases} \quad \text{for } i = 1, 2, \ldots, 5 \qquad (39)$$

For the nonlinear system, seven fuzzy rules in the following format are employed

$R^l$ : *If* $x_1$ *is* $F_1^l$ *and* $\hat{x}_2$ *is* $F_2^l$ *and* $y_d$ *is* $F_3^l$ *and* $\dot{y}_d$ *is* $F_4^l$ *and* $\ddot{y}_d$ *is* $F_5^l$ , *then* $y$ *is* $G^l$ , $l = 1, 2, \ldots, 7.$ (40)

Denoting $D_1 = \sum_{l=1}^{7} \prod_{i=1}^{2} \mu_{F_i^l} \hat{x}_i$ and $D_2 = \sum_{l=1}^{7} \prod_{i=1}^{5} \mu_{F_i^l} Z_i$ , we can write the FBFs which are used to generate approximations for both $f_i(x_i)$ and $L(Z)$ as

$$\xi(x_i) = \left[ \frac{\prod_{i=1}^{2} \mu_{F_i^1}(\hat{x}_i)}{D_1}, \ldots, \frac{\prod_{i=1}^{5} \mu_{F_i^7}(\hat{x}_i)}{D_1} \right]^T \quad (41)$$

$$\xi(Z_i) = \left[ \frac{\prod_{i=1}^{5} \mu_{F_i^1}(Z_i)}{D_2}, \ldots, \frac{\prod_{i=1}^{5} \mu_{F_i^7}(Z_i)}{D_2} \right]^T \quad (42)$$

Let $k_1 = 10$, $k_2 = 5$, $\alpha_1 = 25$, $\gamma_1 = \gamma_2 = 0.1$, $C_S = 10^{-4}$, the initial conditions be $[x_1, \hat{x}_2]^T = [0, -0.5]^T$ and $\lambda_i = 5$ .

Simulation results are shown in figs. 2-4, respectively. Fig. 2 shows the system states $x_1$ and $x_2$ and their estimations $\hat{x}_1$ and $\hat{x}_2$ . Fig. 3 shows the system output $y$ and the reference signal $y_d$ .

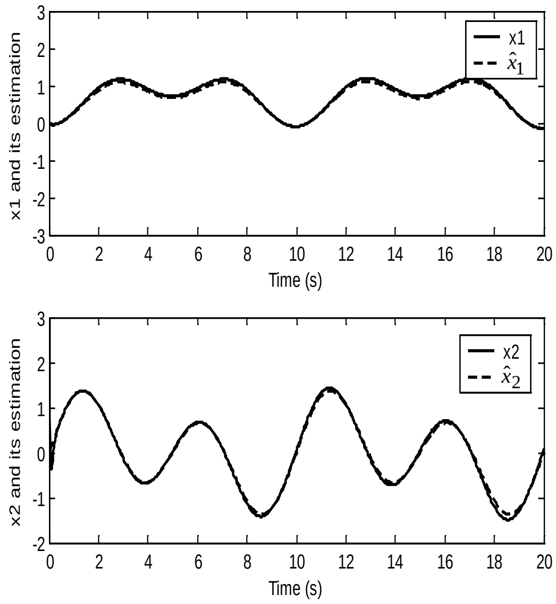


**Fig. 2.** System states and their estimations

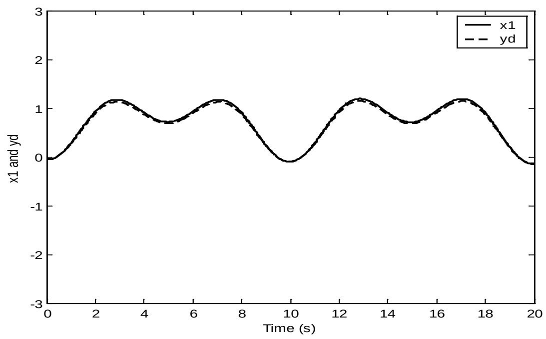


**Fig. 3.** System output $y$ and its trajectory $y_d$

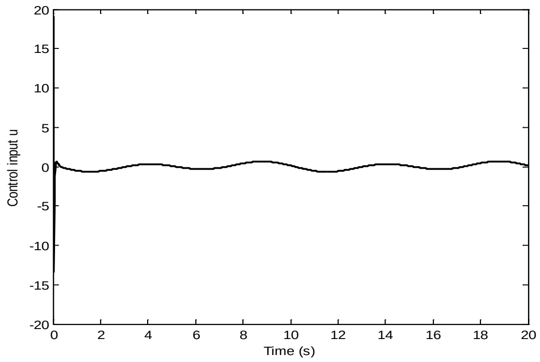


**Fig. 4.** Control input $u$

From Fig. 3, we can see that the good performance has been achieved. Fig. 4 shows the control input signal $u$. From the simulation results, it can be seen that the proposed controller not only guarantees the boundedness of all the signals in the closed loop system, but also achieves the good tracking performance.

## Conclusion

An adaptive fuzzy-neural control algorithm using a state observer for the nonlinear systems with unknown time delay has been proposed in this paper. Since the state variables of nonlinear systems are assumed to be unknown, the state observer is first designed to estimate state variables, via which fuzzy-neural control schemes and the Lyapunov-Krasovskii functional are formulated. Based on the Lyapunov stability theorem, it is rigorously proved that the stability of the closed-loop system is assured and the tracking performance is achieved. The proposed control scheme guarantees the semi-global boundedness of all the signals in the closed-loop system and the good tracking performance. Moreover, the suggested adaptive fuzzy-neural controller contains only one adaptive parameter. This makes our design scheme easier to be implemented in practical applications. Simulation results show that the overall control system guarantees that all involved signals are uniformly ultimately bounded, and the tracking performance index is achieved.

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