

## Sparse Spectrum Sensing using Improved Sparsity aware Diffusion Adaptive Algorithms over Small Cell Networks

Amin Aliabadi, Mahdi Chehel Amirani and Changiz Ghobadi

*Dept. electrical engineering Urmia University, Urmia, Iran*

*Phone Number: +98-9126655962*

*\*Corresponding Author's E-mail: [a.aliabadi@urmia.ac.ir](mailto:a.aliabadi@urmia.ac.ir)*

### Abstract

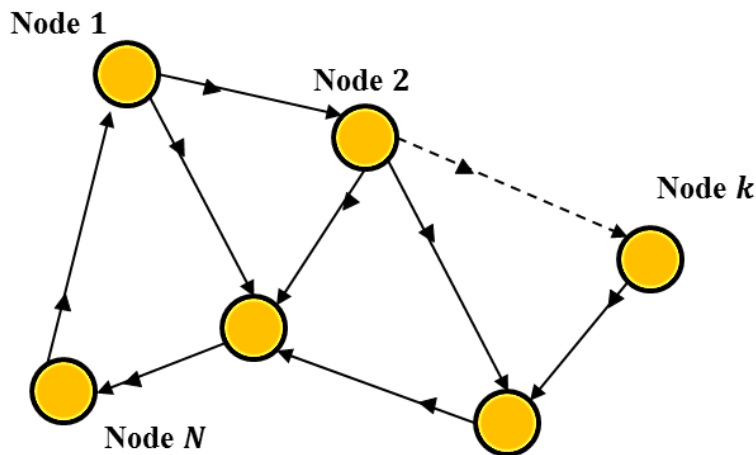
In this paper, the usage of diffusion adaptive networks is investigated in the spectrum sensing task. For this reason we consider the power spectral density of a small cell network which is modeled as a linear function of some basis functions. Our intention is to estimate the coefficients of these basis functions as accurately as possible. The tests in previous studies has shown that the vector of these coefficients is sparse, which means it contains a few non-zero elements and the rest are zero. Therefore, sparsity aware algorithms must be used in order to properly estimate this sparse vector. For this task, we proposed the cooperative version of the soft parameter function penalized normalized maximum correntropy criterion (SPF-NMCC) algorithm with diffusion strategy. Our simulation results showed better performances for this algorithm than all the previously proposed diffusion adaptation schemes in the sparse spectrum sensing task including diffusion reweighted zero attracting (DRZA-LMS) and diffusion  $l_0$ -norm algorithms.

**Keywords:** Adaptive networks, diffusion adaptation, spectrum sensing, correntropy-induced metric, small cell networks, least mean mixture norm. soft parameter function penalized normalized maximum correntropy criterion (SPF-NMCC).

### 1. Introduction

Adaptive sparse signal processing has gathered a lot of attention in the past few years due to the vast applications of this topic [1-5]. Sparse signals and systems can be modeled with long impulse responses with only a few valuable coefficients and the rest to be zero or close to zero. So far, numerous algorithms have been developed to estimate Sparse systems. For example in [1] the zero attracting least mean square (ZA-LMS) and reweighted zero attracting LMS (RZA-LMS) algorithms have been presented. In [2] the  $l_0$ -norm LMS algorithm has been proposed for sparse system identification. The authors in [3], [21] and [23] introduced the family of least mean fourth (LMF) algorithms to the sparse systems. Finally and much recently, an algorithm based on corentropy induced metric with least-mean mixture-norm (CIM-LMMN) in [4] and the soft parameter function penalized normalized maximum correntropy criterion (SPF-NMCC) algorithm was presented in [5], that had a better performance in identifying the sparse systems in comparison to the family of LMF algorithms. All of these algorithms (specially the SPF-NMCC algorithm) have excellent performances for estimating sparse variables, but none of these algorithms are presented in the cooperative and network form. One of the main and important usages of adaptive sparse signal processing is spectrum sensing for cognitive radio systems [9-10]. In cognitive radio systems, the primary users only use a part of

frequency spectrum and we need to analyze it for finding the gaps and unused parts in order to allocate them to the secondary or cognitive users. As we mentioned, at each time instant only certain parts of the spectrum are occupied and this forces the nature of the spectrum to be sparse. In other words the spectrum of primary users are sparse and by estimating them at each time instant we can find suitable places in the spectrum for our secondary users. It is important to mention that sometimes the spectrum of primary users shift through the frequency domain and also a primary user might be off at a time and turn on seconds later. This means that the spectrum sensing task must be done continuously. In newly emerging wireless communication systems, the spectrum estimation task is handled by small cell eNode B (SC-eNB) that is a low power base station capable of performing spectrum estimation using adaptive algorithms. These base stations work cooperatively and form an adaptive network to sense the spectrum. An example of such a network with  $N$  adaptive nodes, is depicted in Fig. 1.



**Figure 1:** An example of the diffusion adaptive network

The network in Fig.1, follows a diffusion topology which means each node in this network can share information with  $\mathcal{N}_k$  neighboring nodes. There are other topologies suggested for adaptive networks like the incremental topology [6] and [24].

Recently the use of diffusion adaptive algorithms has been suggested for sparse system identification in [6-8]. Also, the sparse spectrum sensing using diffusion networks has been suggested in [9] and [10]. But, in these references only a few sparsity aware algorithms namely  $l_0$ -norm LMS and RZA-LMS algorithms have been extended to distributed mode. Also in [8] normalized least mean fourth algorithm was tested with diffusion strategy. However, as we mentioned in recent years so many other promising sparsity aware algorithms have been introduced. By extending these algorithms to the distributed mode we can acquire more suitable algorithms for sparse spectrum sensing. We could extend tens of sparsity algorithms with diffusion strategy and test them in the spectrum sensing task but for the sake of brevity and efficiency, we picked the most powerful and recently proposed sparsity aware algorithms and implemented them in the diffusion network for spectrum sensing.

We believe that the usage of adaptive networks for cognitive radio systems is a promising research area, due to the following reason: Adaptive networks involve simple agents that cooperate with one another to achieve an emergent, unified estimation for the system as a whole, producing a robust system capable of finding high-quality solutions for problems with a large search space. Spectrum [11] and channel estimation [24] is two of those tasks that need very precise results in order to improve the performance of wireless mobile systems.

The aim of this paper is to find a suitable algorithm for sparse spectrum sensing which performs better than ATC  $l_0$ -norm LMS and ATC RZA-LMS algorithms. The proposed algorithm is the ATC SPF-NMCC algorithm and we will show in the simulation results that this algorithm outperforms both of the previously presented algorithms.

The rest of this paper is organized as follows:

In part II. the spectrum sensing formulation with the sparsity aware diffusion algorithms is reviewed. In part III. The application of the proposed SPF-NMCC algorithm in the diffusion spectrum sensing task is performed. In part IV the simulation results for estimating a typical PSD are presented. And finally in part in V. the concluding remarks and future scope of the paper are given.en.

*Notation:* We used boldface letters for vector variables. Also we used the notation  $\mathbb{E}[\cdot]$  to denote expectation operation and notation  $(\cdot)^T$  to denote Transposition for vectors. Furthermore, the operator  $\|A\|$  denotes the norm of vector  $A$ .

## 2. Problem Formulation

Consider a diffusion adaptive network with  $N$  SC-eNB's as in Fig. 2 that is deployed to estimate an unknown weight vector  $\mathbf{z}^o$  with  $M$  entries [9].

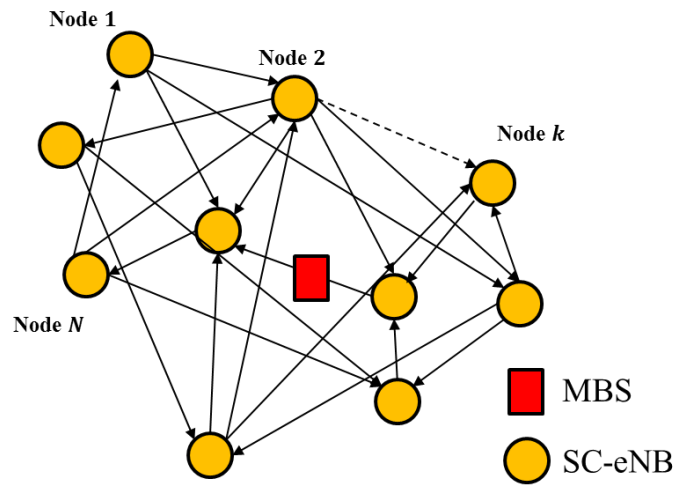


Figure 2: Diffusion network for the spectrum sensing application

We define  $\Phi_s(e^{j\omega})$  as the PSD of the signal transmitted by MBS which is shown with  $s(m)$ . This PSD can be modeled as a linear combination of several basis functions as:

$$\Phi_s(e^{j\omega}) = \sum_{v=1}^{\mathcal{B}} b_v(e^{j\omega})z_v^o = \mathbf{b}_0^T(e^{j\omega})\mathbf{z}^o \tag{1}$$

where  $\mathcal{B}$  is the number of basis functions and  $\mathbf{z}^o = [z_1^o, \dots, z_{\mathcal{B}}^o]^T \in \mathbb{R}^{\mathcal{B}}$  is a weighting vector. The basis functions vector is given as  $\mathbf{b}_0^T(e^{j\omega}) = [b_1(e^{j\omega}), \dots, b_{\mathcal{B}}(e^{j\omega})]^T$ .

By defining  $H_k(e^{j\omega}, i)$  as the channel transfer function between the  $k$ th SC-eNB and the MBS, the PSD received in this SC-eNB is:

$$\mathbf{I}_k(e^{j\omega}, i) = |H_k(e^{j\omega}, i)|^2 \Phi_s(e^{j\omega}) + \sigma_{n,k}^2 = \sum_{v=1}^{\mathcal{B}} |H_k(e^{j\omega}, i)|^2 b_v(e^{j\omega})z_v^o + \sigma_{n,k}^2 = \mathbf{b}_{k,i}^T(e^{j\omega})\mathbf{z}^o + \sigma_{n,k}^2 \tag{2}$$

where  $\sigma_{n,k}^2$  is the channel noise power and we have  $\mathbf{b}_{k,i}^T(e^{j\omega}) = [ |H_k(e^{j\omega}, i)|^2 b_v(e^{j\omega}) ]_{v=1}^{\mathcal{B}}$ .

We assume that each node has the ability to perform periodogram spectrum estimation process and therefore at each time instant  $i$  has access to noisy measurements of the true spectrum  $\mathbf{I}_k(e^{j\omega}, i)$ . Our goal is to find the  $\mathbf{z}^o$  weighting vector using these noisy measurements. By considering channel

noise and measurement noise ( $v_k^q(i)$ ) we have the following linear relation for measurements in each SC-eNB at each time instant and frequency  $w_q$ :

$$d_k^q(i) = \mathbf{b}_{k,i}^T(e^{jw_q})\mathbf{z}^o + \sigma_{n,k}^2 + v_k^q(i) \tag{3}$$

where  $d_k^q(i)$  is a realization of measurements  $\mathbf{d}_k(i)$ . By neglecting the channel noise, we can write this equation more compactly as:

$$d_{k,i} = \mathbf{B}_{k,i}\mathbf{z}^o + v_{k,i} \tag{4}$$

where  $\mathbf{B}_{k,i} = [\mathbf{b}_{k,i}^T(e^{jw_q})]_{q=1}^{N_c} \in \mathbb{R}^{N_c \times \mathcal{B}}$  with  $N_c > \mathcal{B}$ . The aim of the network is to minimize the following cost function cooperatively by the measurements of all the nodes:

$$J(\mathbf{z}) = \sum_{k=1}^N \mathbb{E} \|\mathbf{d}_{k,i} - \mathbf{B}_{k,i}\mathbf{z}\|^2 + \gamma f(\mathbf{z}) \tag{5}$$

where  $f(\mathbf{z})$  is a regularization function for considering sparsity and  $\gamma$  is a weighting parameter. By applying sparsity aware diffusion adaptation algorithms to this problem we can have a cooperative solution. Each node can collect and combine the estimations from other nodes and after that perform the adaptation process, the name of this strategy is adapt then combine (ATC) cooperation.

In [9] two different sparsity aware algorithms have been applied to the PSD estimation problem with ATC strategy. The overall adaptation sequence is given as:

At each iteration  $i$  and each node  $k$  repeat:

$$\begin{cases} \psi_{k,i} = z_{k,i-1} + \mu_k \mathbf{B}_{k,i}^T [d_{k,i} - \mathbf{B}_{k,i}\mathbf{z}_{k,i-1}] - \mu_k \gamma \partial f(\mathbf{z}_{l,i-1}) \\ \mathbf{z}_{k,i} = \sum_{l \in \mathcal{N}_k} a_{l,k} \psi_{k,i} \end{cases} \tag{6}$$

In this relation we define the error value as

$$e_{k,i} = [d_{k,i} - \mathbf{B}_{k,i}\mathbf{z}_{k,i-1}] \tag{7}$$

This vector can be used for explaining the application of various sparsity aware adaptive algorithms in this task. Here we describe the two algorithms that previously presented in [9]:

A. ATC reweighted zero attracting algorithm

If we substitute the last part of equation (6) with the following relation we get reweighted zero attracting (RZA) diffusion algorithm (or ATC RZA-LMS):

$$\partial f(\mathbf{z}) = \text{diag} \left\{ \frac{1}{\varepsilon + |\mathbf{z}_1|}, \dots, \frac{1}{\varepsilon + |\mathbf{z}_B|} \right\} \text{sign}(\mathbf{z}) \tag{8}$$

The non-cooperative version of the RZA-LMS algorithm has been originally proposed in [1] with the following equation:

$$\partial f(\mathbf{z}) = \varepsilon \frac{\text{sign}(\mathbf{z})}{1 + \varepsilon |\mathbf{z}|} \tag{9}$$

It is important to mention that the main parameters of this algorithm are  $\mu_{RZA}$ ,  $\gamma_{RZA}$  and  $\varepsilon$ .

B. ATC  $l_0$ -norm LMS attracting algorithm

Likewise, if we substitute the last part of (6) with the following relation, we have  $l_0$ -norm diffusion algorithm (or  $l_0$ -norm ATC):

$$\partial f(\mathbf{z}) = \beta \cdot \text{diag}\{e^{-\beta|z_1|}, \dots, e^{-\beta|z_B|}\} \text{sign}(\mathbf{z}) \quad (10)$$

The non-cooperative version of this algorithm has been given in [2]. The main parameters of this algorithm are  $\gamma_{l_0\text{-norm}}$  and  $\beta$ .

These were the presented algorithms for spectrum sensing task till now, in the next part we apply a new algorithm to the diffusion spectrum sensing.

### 3. Applying the proposed algorithm

In the previous part, the  $l_0$ -norm LMS and RZA-LMS algorithms were extended to the distributed scheme using ATC strategy and applied to the spectrum sensing. Here we apply the ATC version of our proposed algorithm to this task.

The non-cooperative version of soft parameter function penalized normalized maximum correntropy criterion (SPF-NMCC) algorithm was proposed in [5].

$$\mathbf{w}(i) = \mathbf{w}(i-1) + \mu_{SPF} \frac{\exp\left(-\frac{e^2(i)}{2\sigma^2}\right) e(i)\mathbf{x}(i)}{\|\mathbf{x}(i)\|^2} - \gamma_{SPF} S'_\beta(\mathbf{w}(i-1)) \quad (11)$$

The normalization operation in this algorithm is performed by dividing the second part with  $\|\mathbf{x}(i)\|^2$ . Also in this relation we have:

$$S'_\beta(\mathbf{w}(i-1)) = (1 + \beta^{-1})(1 - e^{-\beta|\mathbf{w}(i-1)|}) \quad (12)$$

Also,  $\mu_{SPF}$ ,  $\beta$  and  $\gamma_{SPF}$  are the algorithm parameters. If we extend this algorithm using ATC strategy and apply it to the spectrum sensing, the weight update equation of this algorithm can be given as:

At each iteration  $i$  and each node  $k$  repeat:

$$\boldsymbol{\psi}_{k,i} = \mathbf{z}_{k,i-1} + \mu_{k\text{-SPF}} \frac{\exp\left(-\frac{e_{k,i}^2}{2\sigma^2}\right) e_{k,i} \mathbf{B}_{k,i}^T}{\|\mathbf{B}_{k,i}^T\|^2} [d_{k,i} - \mathbf{B}_{k,i} \mathbf{z}_{l,i-1}] - \gamma_{SPF} S'_\beta(\mathbf{z}_{k,i-1}) \quad (13)$$

This part is the adaptation process of the ATC strategy, for the combination procedure we have:

$$\mathbf{z}_{k,i} = \sum_{l \in N_k} a_{l,k} \boldsymbol{\psi}_{l,i} \quad (14)$$

Now we are ready to give our simulation results using these sparsity aware diffusion algorithms.

### 4. Simulation results

To run our simulations we consider a network with 10 connected SC-eNB's ( $N = 10$ ) as in Fig. 2. As we mentioned in part II, the channel noise is neglected and only the measurement noise is considered with the variance of 0.001. Overall, 4 algorithms are tested in the simulations, 1) the ATC diffusion algorithm which is not sparsity aware, 2) the ATC  $l_0$ -norm LMS algorithm, 3) the ATC RZA-LMS algorithm and finally, 4) the proposed ATC SPF-NMCC algorithm.

For the ATC  $l_0$ -norm LMS algorithm consider the  $\gamma = 2.2 \times 10^{-3}$  and  $\beta = 50$ . For the ATC RZA-LMS algorithm we adjusted the values as  $\mu_{ATC-RZA} = 0.013 = \mu_{ATC\ diffusion} = \mu_{ATC\ l_0-LMS}$ ,  $\gamma = 0.48$  and  $\varepsilon = 5$ . Finally, for the ATC version of SPF-NMCC algorithm the parameters are  $\mu_{SPF} = 0.3$ ,  $\gamma_{SPF} = 2.5 \times 10^{-5}$ ,  $\sigma = 1000$  and  $\beta = 6$ .

For presenting our simulation results first we consider estimating a typical spectrum which is shown in Fig. 3:

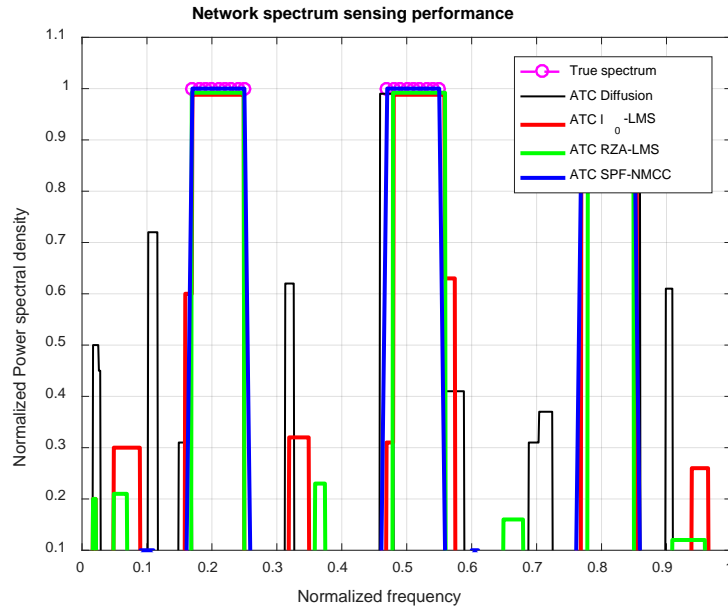


Figure 3: Diffusion network for the spectrum sensing application

The power of the true spectrum which is sparse is normalized to 1. Also only some parts of the spectrum are considered to be occupied following the scenario in cognitive radio systems.

As it can be seen in Fig. 3, the wrongly estimated spectrum levels are very low for the ATC RZA-LMS and ATC SPF-NMCC algorithms, but the second algorithm estimates the true spectrum more accurately.

Next we compare the performance results with the steady state mean square deviation (MSD) criterion which shows the distance between true basis function coefficients and the estimated coefficients after convergence. The average MSD for this paper is defined as follows:

$$MSD = \lim_{i \rightarrow \infty} \mathbb{E} \sum_{k=1}^N \|z_{k,i} - z^o\|^2 / N \tag{15}$$

This criteria helps us to expect the level of error in our estimation tasks. In order to obtain the MSD levels of the proposed algorithms we run another simulation using the same parameters in the previous simulation and presented the results for all the mentioned algorithms in Fig. 4.

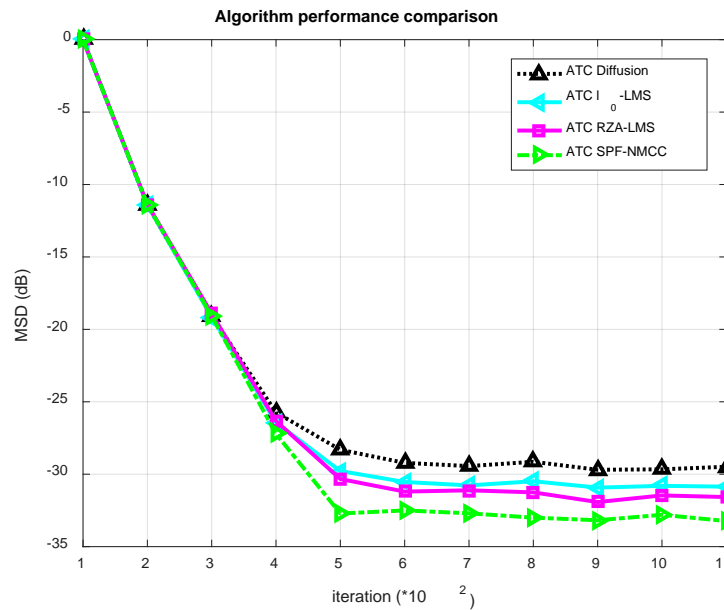


Figure 4: Performance of ATC diffusion algorithms in sparse spectrum sensing.

As we can see in Fig. 4, the proposed ATC SPF-NMCC algorithm outperforms all other tested sparsity aware algorithms both in convergence speed and the estimation error.

## Conclusion and future scope

In this paper, the performance of some sparsity-aware diffusion algorithms were tested on the spectrum sensing of small cell SC-eNB networks. As we showed the coefficient vector of this power spectral density (PSD) presents the sparsity feature and needs to be tracked in real time, because of the variations in its sparsity. The tested algorithms in this paper for spectrum sensing are fully distributed and can handle the sparsity cooperatively. Our simulation results showed that the ATC RZA-LMS algorithm is better than ATC  $l_0$ -norm LMS algorithm for spectrum sensing and the proposed ATC SPF-NMCC algorithm works better than both of these algorithms. In future works we will test other sparsity aware algorithms in the distributed PSD estimation task. Also, the proposed ATC SPF-NMCC algorithm can be used in other sparse system identification tasks like channel estimation

## References

- [1] Y. Chen, Y. Gu, and A. O. Hero, "Sparse LMS for system identification," IEEE Int. Conf. on Acoust., Speech and Signal Process. (ICASSP), pp. 3125-3128, April 2009.
- [2] Y. Gu, J. Jin, and S. Mei, " $l_0$ -norm constraint LMS algorithm for sparse system identification," IEEE Signal Process. Lett., vol. 16, no. 9, pp. 774-777, Sep. 2009.
- [3] G. Gui, W. Peng, and F. Adachi, "Adaptive system identification using robust LMS/F algorithm," Int. J. Commun. Sys., first published online, DOI: 10.1002/dac.2517, pp. 1-8, Feb. 2013.
- [4] Y. Li, Z. Jin, Y. Wang, R. Yang, "A robust sparse adaptive filtering algorithm with a correntropy induced metric constraint for broadband multi-path channel estimation," Entropy 2016, vol. 18, 380.
- [5] Y. Li, Y. Wang, R. Yang, F. Albu, "A Soft Parameter Function Penalized Normalized Maximum Correntropy Criterion Algorithm for Sparse System Identification," Entropy 2017, 1, 45.
- [6] M. O. Bin Saeed, and A. U.H.Sheikh, "Sparse system identification over adaptive networks," 1st international conference on signal processing and their applications, pp. 1-5, Feb. 2013.
- [7] P. Lorenzo, A. H. Sayed "Sparse distributed learning based on diffusion adaptation", IEEE Transactions on Signal Processing, vol. 61. pp. 1419-1433, 2013

- [8] M. Hajiabadi and H. Z. Jafarian, "Distributed adaptive LMF algorithm for sparse estimation in gaussian mixture noise", 7th international symposium on telecommunications (IST), pp. 1046-1049, Sept. 2014.
- [9] P. Di Lorenzo, Sergio Barbarossa and A. H. Sayed, "Distributed Spectrum Estimation For Small Cell Networks based on Sparse Diffusion Adaptation," IEEE Signal Processing letters, vol. 61, no. 6, pp. 1–11, 2013.
- [10] R. A. Valles, Sina Maleki and G. Leus, "Distributed wideband spectrum sensing for cognitive radio networks," IEEE International Conference on Acoustic, Speech and Signal Processing, pp. 7263-7267, 2014.
- [11] E. Axell, G. Leus, E. G. Larsson, and H. V. Poor, "Spectrum sensing for cognitive radio: State-of-the-art and recent advances," IEEE Signal Processing Magazine, vol. 29, no. 3, pp. 101–116, 2012.
- [12] Z. Li, F.R. Yu, and M. Huang, "A distributed consensus-based cooperative spectrum-sensing scheme in cognitive radios," IEEE Trans. on Vehicular Technology, vol. 59, no. 1, pp. 383– 393, 2010.
- [13] K. Seung-Jun, E. Dall'Anese, and G.B. Giannakis, "Cooperative spectrum sensing for cognitive radios using kriged kalman filtering," IEEE Journal of Selected Topics in Signal Processing, vol. 5, no. 1, pp. 24–36, 2011.
- [14] P. Di Lorenzo, S. Barbarossa, and A.H. Sayed, "Bio-inspired decentralized radio access based on swarming mechanisms over adaptive networks," IEEE Trans. on Signal Processing, vol. 61, no. 12, pp. 3183–3197, 2013.
- [15] J.A. Bazerque, and G.B. Giannakis, "Distributed spectrum sensing for cognitive radio networks by exploiting sparsity," IEEE Trans. on Signal Processing, vol. 58, no. 3, pp. 1847–1862, 2010.
- [16] D.D. Ariananda, and G. Leus, "Cooperative compressive wideband power spectrum sensing," in Proc. of the Forty Sixth Asilomar Conference on Signals, Systems and Computers (ASILOMAR'12), 2012, pp. 303–307.
- [17] P. Paysarvi-Hoseini, and N.C. Beaulieu, "Optimal wideband spectrum sensing framework for cognitive radio systems," IEEE Trans. on Signal Processing, vol. 59, no. 3, pp. 1170– 1182, 2011.
- [18] O. Mehanna, and N.D. Sidiropoulos, "Frugal sensing: Wideband power spectrum sensing from few bits," IEEE Transactions on Signal Processing, vol. 61, no. 10, pp. 2693–2703, 2013.
- [19] M. O. Bin Saeed, and A. Zerguine, "A Variable Step Size Strategy for Sparse System Identification," 10th international multi-conference on systems, signals and devices, pp. 1-4, March 2013.
- [20] G. Gui, A. Mehdniya and F. Adachi, "Adaptive Sparse Channel Estimation Using Reweighted Zero-Attracting Normalized Least Mean Forth", IEEE international conference on communications, pp. 368-373, Aug. 2013.
- [21] Y. Li, Y. Wang, A. Jiang, "Sparse Channel Estimation Based on a p-Norm-Like Constrained Least Mean Fourth Algorithm" International conference on wireless communications and signal processing, pp. 1-4, Oct. 2015.
- [22] G. Gui, "Fast NLMF-Type Algorithms for Adaptive Sparse System Identifications", Asia-Pacific signal and information processing association annual summit and conference, pp. 958-962, Dec. 2015.
- [23] A. Bazdar, A. Aliabadi, E. Mostafapour, Ch. Ghobadi, "Non-stationary Sparse System Identification over Adaptive Sensor Networks with Diffusion and Incremental Strategies," International journal of information & communication tech. research, vol. 8, No. 4, pp. 11-17, Autumn 2016.
- [24] S. Haykin, K. J. Ray Liu, Handbook on Array processing and sensor networks. Canada: Wiley, 2009.



## Autor(s)



**Amin Ali Abadi** was born in 1984, Tehran, Iran. He received his B.S. degree from Hamedan University and the M.S. degree from Boushehr University, in 2010 and 2012, respectively, both in electrical engineering. He is currently pursuing the Ph.D. degree with the Department of Electrical Engineering, Urmia University. His research interests include stochastic and adaptive signal processing, wireless cellular and adaptive networks.

**email:** [a.aliabadi@urmia.ac.ir](mailto:a.aliabadi@urmia.ac.ir)

**Phone Number:** (+98-9126655962)



**Mehdi Chehel Amirani** received the B.Sc. degree in Electronic Engineering from Urmia University, Iran, in 1993 and the M.Sc. and Ph.D. degrees in Communications engineering from Iran University of Science and Technology (IUST) in 1998 and 2009, respectively. In 2009, he joined the department of electrical engineering at Urmia University, where he is currently an associate professor. His research interests include pattern recognition, digital signal processing, and wireless communication.



**Changiz Ghobadi** was born on 1 June, 1960 in Iran. He received the B.Sc. degree in Electrical and Electronic Engineering and M.Sc. degree in Electrical and Telecommunication Engineering from Isfahan University of Technology, Isfahan, Iran and Ph.D. degree in Electrical-Telecommunication from University of Bath, Bath, UK in 1998. From 1998 he was an Assistant Professor and now is full Professor in the Department of Electrical Engineering, of Urmia University, Urmia, Iran. His current research interests are in antenna design, Propagation and adaptive filters.